

$$A(x) = \sum_{k=0}^{\infty} C^k F(x^{2^k})$$

| $C, F(\xi)$ | a_{2n} | a_{2n+1} |
|---|-----------------------------|---------------------------------------|
| $1, \frac{\xi^2}{1-\xi^2}$ | $a_n + 1$ | 0 |
| $1, \frac{\xi}{1-\xi}$ | $a_n + 1$ | 1 |
| $1, \frac{\xi}{1+\xi}$ | $a_n - 1$ | 1 |
| $1, \frac{\xi^2}{1+\xi}$ | $a_n + 1$ | -1 |
| $-1, \frac{\xi}{1-\xi}$ | $-a_n + 1$ | 1 |
| $2, \frac{\xi}{1-\xi}$ | $2a_n + 1$ | 1 |
| $3, \frac{3}{1-\xi}$ | $3a_n + 3$ | 3 |
| $2, \frac{\xi}{1-\xi^2}$ | $2a_n$ | 1 |
| $3, \frac{\xi}{1-\xi^2}$ | $3a_n$ | 1 |
| $4,$ | $4a_n + 3$ | 7 |
| $1, \frac{\xi}{1-2\xi^2}$ | a_n | 2^n |
| $2, \frac{\xi}{(1-\xi)^2}$ | $2a_n + 2n$ | $2n + 1$ |
| $2, \frac{2\xi}{(1-\xi)^2}$ | $2a_n + 4n$ | $4n + 2$ |
| $1, \frac{\xi}{(1-\xi^2)^2}$ | a_n | n |
| $1, \frac{(1-\xi^2)^2}{2\xi}$ | a_n | $2n + 1$ |
| $2, \frac{\xi_3 - \xi_2 + \xi + 1}{\xi_3^2}$ | $2a_n + 1$ | n |
| $2, \frac{(1-\xi_3^2)(1-\xi_2^2)^2 + 3\xi}{2\xi}$ | $2a_n$ | switch trailing 0s, $n + 2^{v_2} - 1$ |
| $2, \frac{(1-\xi^4)^2}{(1-2\xi-2\xi^2)}$ | $a_n + 2^{2n}$ | $4n + 3 / 8n + 2$ |
| $1, \frac{\xi(1+2\xi-2\xi^2)}{(1-2\xi)(1-\xi^2)}$ | $a_n + 1 - (n + 1 \bmod 3)$ | $a(a(n)) = 2n$ |
| $1, \frac{\xi}{1+\xi+\xi^2}$ | $a_n + 1 - (n \bmod 3)$ | $2^{2n+1} - 1$ |
| | | $A045654 - 1$ |
| | | $A084091$ |

| $A(x) = \frac{1}{1-x} \left(B(x) + \sum_{k=0}^{\infty} C^k F(x^{2^k}) \right)$ | | | |
|---|----------------------------------|-----------------------|--|
| $B(x)$ | $C, F(\xi)$ | a_{2n} | a_{2n+1} |
| 1+ | $1, \xi$ | $a_n + 1$ | $a_n + 1$ |
| | $[1] a_n + 1$ | $a_n + 1$ | bin. length of n , A000523 + 1 |
| 1+ | $1, \xi$ | $2a_n + 1$ | bin. length of $2n + 1$ |
| | $[1] 2a_n$ | $2a_n + 1$ | a_{n-1} OR n |
| 1+ | $2, \xi$ | $2a_n$ | $2 \cdot 2^{\lfloor \lg n \rfloor}$ |
| | $[1] 2a_n$ | | runs of length 2^k |
| | $-1, \xi$ | $-a_n + 1$ | msb, $2^{\lfloor \lg n \rfloor}$ |
| | $2, \xi(1 - \xi)$ | $[0, 1] 2a_n$ | $1 + 2^{\lfloor \lg n + 1 \rfloor}$ |
| 2, | $[2] 2a_n - 1$ | $2a_n - 1$ | A053644 |
| | $\frac{x-2x^2}{1-x} + 2, 3\xi^2$ | $[0, 1] 2a_n + 1$ | A076877 |
| | | $2a_n$ | A054429 |
| | | | A000027 |
| | $2, \frac{\xi}{1+\xi}$ | $2a_n$ | (A035327) |
| | $2, \frac{\xi^2}{1+\xi^2}$ | $2a_n + 1$ | |
| 1+ | $2, \frac{\xi+2\xi^2}{1+\xi}$ | $2a_n$ | $-(n+1) + 2 \cdot 2^{\lfloor \lg n \rfloor}$ |
| | $[1] 2a_n + 1$ | | $-(n+1) + 4 \cdot 2^{\lfloor \lg n \rfloor}$ |
| 1+ | $2, \frac{2\xi+\xi^2}{1+\xi}$ | $2a_n$ | $n + 2^{\lfloor \lg n \rfloor}, (A004761)$ |
| | $[1] 2a_n$ | | $n + 2 \cdot 2^{\lfloor \lg n \rfloor}, (A004760)$ |
| 1+ | $2, \frac{2\xi+\xi^2}{1+\xi}$ | $2a_n$ | A004755 |
| | $[1] 2a_n$ | | A004756 |
| 2+ | $2, \frac{3\xi+2\xi^2}{1+\xi}$ | $2a_n$ | A004757 |
| | $[1] 2a_n$ | | A004758 |
| 3+ | $2, \frac{4\xi+3\xi^2}{1+\xi}$ | $2a_n$ | A004759 |
| | $[1] 2a_n$ | | A0079946 |
| 2, | $2, \frac{2a_n}{1+\xi}$ | $[1] 2a_n - 1$ | A062050 |
| | $[1] 2a_n$ | | A006257 |
| | $2a_n - 1$ | $2a_n + 1$ | A004762 |
| | $(2a_n)$ | $(2a_n + 1)$ | A004763 |
| 2, | $2a_n + 1 + 3[n > 1]$ | $2a_n + 1 + 5[n > 0]$ | A079251($n + 1$) - 2 |
| | $(2a_n + +[n > 1])$ | $(2a_n + +[n > 0])$ | A034702 |

$$A(x) = \frac{1}{1-x} \left(B(x) + \sum_{k=0}^{\infty} C^k F(x^{2^k}) \right)$$

| $B(x)$ | $C, F(\xi)$ | a_{2n} | a_{2n+1} | | | |
|--|---------------------------------|-------------------------------|---|---------------|---------|--|
| $1, \frac{\xi}{1+\xi^2}$ | $a_n + [n \text{ odd}]$ | $a_{n+1} + [n \text{ even}]$ | $e_1(\text{Gray}(n)), A037834 + 1$ | 15,15* | A005811 | |
| $2, \frac{\xi^2}{1+\xi^2}$ | $2a_n + [n \text{ odd}]$ | $2a_{n+1} + [n \text{ even}]$ | $n \text{ XOR } \lfloor \frac{n}{2} \rfloor, Gray \text{ code}$ | 15 | A003188 | |
| $2, \frac{\xi^4 - \xi^3 + \xi^2}{1+\xi^2}$ | $[0, 0] 2a_n + [n \text{ odd}]$ | $2a_{n+1} + [n \text{ even}]$ | “derivative” of n | | A038554 | |
| $1, \frac{1}{1+\xi^2}$ | $a_n + 2n$ | $a_n - 2n - 1$ | | | A071413 | |
| | | | | | | |
| $1, \frac{\xi}{(1+\xi)(1+\xi^2)}$ | a_n | $a_n + [n \text{ even}]$ | Runs of 1s in binary | | A069010 | |
| $1, \frac{\xi^2}{(1+\xi)(1+\xi^2)}$ | $a_n + [n \text{ odd}]$ | a_n | counting 10 in binary | | A033264 | |
| $1, \frac{\xi^3}{(1+\xi)(1+\xi^2)}$ | a_n | $a_n + [n \text{ odd}]$ | counting 11 in binary | | A014081 | |
| $1, \frac{\xi^2(1+\xi+\xi^2)}{(1+\xi)(1+\xi^2)}$ | $a_n + 1$ | $a_n + [n \text{ odd}]$ | # incr. bin. repr. | | A033265 | |
| $1, \frac{\xi^4}{(1+\xi)(1+\xi^2)}$ | $a_n + [n \text{ even}]$ | a_n | counting 00 in binary | | A056973 | |
| $1, \frac{\xi(1+\xi^2+\xi^3)}{(1+\xi)(1+\xi^2)}$ | $a_n + [n \text{ even}]$ | $a_n + 1$ | # incr. bin. repr., A037809 + 1 | | | |
| $1, \frac{\xi^2}{(1+\xi)(1+\xi^2)}$ | $[0, 0] a_n$ | $a_n + [n \text{ even}]$ | counting 01 in binary | | | |
| | | | counting 111 in binary | | | |
| | | | counting 1111 in binary | | | |
| $2, \frac{3\xi - \xi^3}{(1+\xi)(1+\xi^2)}$ | $2a_n$ | $a_n + [n \equiv 7 \pmod{8}]$ | Reversing bin. rep. of $-n$ | | A048724 | |
| $2, \frac{\xi(\xi^2+4\xi+1)}{(1+\xi)(1+\xi^2)}$ | $2a_n$ | $2a_{n+1} - 2(-1)^n + 1$ | Reversing bin. rep. of $-n$ | | A065621 | |
| 1+ | | | | | | |

$$A(x) = \frac{1}{1-x} \left(B(x) + \sum_{k=0}^{\infty} C^k F(x^{2^k}) \right)$$

| $B(x)$ | $C, F(\xi)$ | a_{2n} | a_{2n+1} | |
|--|-------------|----------------|-------------------------|--|
| $1, \frac{\xi}{1-\xi}$ | | $a_n + 2n$ | $a_n + 2n + 1$ | 2^{a_n} divides $(2n)!$, $2n - e_1(2n)$ |
| $1,$ | | $a_n + 3n$ | $a_n + 3n + 2$ | den. in $(1-x)^{-1/4}$, $3n - e_1(n)$ |
| $1,$ | | $a_n + n + 1$ | $a_n + n + 1 + [n > 0]$ | cube subgraphs, $n + \lfloor \lg n \rfloor$ |
| $1,$ | | $a_n + n - 1$ | $a_n + n$ | eigenvalues, $n - 1 - \lfloor \lg n \rfloor$ |
| $1,$ | | $a_n + 2n - 1$ | $a_n + 2n + 1$ | Connell seq., $2n - 1 - \lfloor \lg n \rfloor$ |
| $1,$ | | $a_n + 3n - 2$ | $a_n + 3n + 1$ | Connell seq., $3n - 2 - 2\lfloor \lg n \rfloor$ |
| $2, \frac{\xi}{1-\xi}$ | | $2a_n + 2n$ | $2a_n + 2n + 1$ | A049039 |
| $-1, \frac{\xi}{1-\xi}$ | | $-a_n + 2n$ | $-a_n + 2n + 1$ | A050487 |
| $-2, \frac{\xi}{1-\xi}$ | | $-2a_n + 2n$ | $-2a_n + 2n + 1$ | A080804 |
| $1, \frac{\xi}{1-\xi^2}$ | | $a_n + n$ | $a_n + n + 1$ | A083058 |
| $2, \frac{\xi}{1-\xi^2}$ | | $2a_n + n$ | $2a_n + n + 1$ | A08277 |
| $-1, \frac{\xi}{1-\xi^2}$ | | $-a_n + 2n$ | $-a_n + 2n + 1$ | A050292 |
| $-2, \frac{\xi}{1-\xi^2}$ | | $-2a_n + 2n$ | $-2a_n + 2n + 1$ | A063694 |
| $1, \frac{\xi^2}{1-\xi^2}$ | | $a_n + n$ | $a_n + n + 1$ | N |
| $2, \frac{\xi^2}{1-\xi^2}$ | | $2a_n + n$ | $2a_n + n + 1$ | A004514/2 |
| $-1, \frac{\xi^2}{1-\xi^2}$ | | $-a_n + n$ | $-a_n + n + 1$ | A006520($n - 1$) |
| $1, \frac{\xi^2}{1-\xi^2}$ | | $a_n + n$ | $a_n + n$ | — |
| $-2, \frac{\xi^2}{1-\xi^2}$ | | $-2a_n + 2n$ | $-2a_n + 2n$ | A068639 |
| $-2, \frac{\xi^2}{1-\xi^2}$ | | $-2a_n + 5n$ | $-2a_n + 5n + 2$ | A011371 |
| $1, \frac{\xi^2(1+2\xi-\xi^2)}{(1-\xi^2)^2}$ | | $a_n + n^2$ | $a_n + n^2 + 2n$ | remove even-pos. bits binary counter minimum cost addition chain |
| | | | | 9 A063695 A057300 A005766 |

$$A(x) = \frac{1}{1-x} \left(B(x) + \sum_{k=0}^{\infty} C^k F(x^{2^k}) \right)$$

| $B(x)$ | $C, F(\xi)$ | a_{2n} | a_{2n+1} | |
|-----------------------------------|------------------------|---------------|---|-----------------------------------|
| $1, \frac{\xi}{1+\xi}$ | a_n | $a_n + 1$ | e_1 | A000120 |
| $1, \frac{\xi}{1+\xi}$ | $a_n + 1$ | a_n | e_0 | A023416 |
| $1, \frac{\xi + \xi^2}{1+\xi^2}$ | $a_n + 2$ | $a_n + 1$ | $A061313(n+1)$ | |
| $1, \frac{2\xi + \xi^2}{1+\xi^2}$ | $a_n + 1$ | $a_n + 2$ | $A056792 + 1, A014701 + 2$ | A056791 |
| $1, \frac{\xi^2 - \xi}{1+\xi}$ | $a_n + 1$ | $a_n - 1$ | $e_0 - e_1$ | A037861 |
| $1, \frac{\xi^4}{1+\xi}$ | $[0, 0, 0, 0] a_n + 1$ | a_n | $e_0(n) + A079944(n-2) + 1$ | A083661 |
| $-1, \frac{\xi}{1+\xi}$ | $-a_n$ | $-a_n + 1$ | alternating bit sum | A065359 |
| $-1, \frac{\xi^2}{1+\xi}$ | $-a_n + 1$ | $-a_n$ | | A083905 |
| $-2, \frac{\xi}{1+\xi}$ | $-2a_n$ | $-2a_n + 1$ | | A053985 |
| $3, \frac{\xi}{1+\xi}$ | $3a_n$ | $3a_n + 1$ | $A003278 - 1, A033159 - 2, A033162 - 3$ | A005836 |
| $3, \frac{2\xi}{1+\xi}$ | $3a_n$ | $3a_n + 2$ | | A005824 |
| $3, \frac{2\xi}{1+\xi}$ | $[3]3a_n$ | $3a_n + 6$ | $3 \not\equiv \sum_0^n \binom{2k}{k}$ | A081601 |
| $3, \frac{\xi}{1+\xi}$ | $3a_n$ | $3a_n + 6$ | $A055246 - 1$ | |
| $1+$ | $3, \frac{\xi}{1+\xi}$ | $[1]3a_n - 2$ | $3a_n - 1$ | $a_n - 1$ in ternary= n in bin. |
| $3, \frac{\xi}{1+\xi}$ | $[2, 3]3a_n - 4$ | $3a_n - 3$ | $A003278 + 1$ | (A033159) |
| $3, \frac{\xi^2}{1+\xi}$ | $[3]3a_n - 6$ | $3a_n - 5$ | $A003278 + 2$ | A033162 |
| $3, \frac{\xi^2}{1+\xi}$ | $3a_n + 1$ | $3a_n$ | | A083904 |
| $4, \frac{\xi}{1+\xi}$ | $4a_n$ | $4a_n + 1$ | Moser-de Bruijn | A000695 |
| $4, \frac{\xi}{1+\xi}$ | $4a_n$ | $4a_n + 3$ | double bitters | A001196 |

$$A(x) = \frac{1}{(1-x)^2} \left(B(x) + \sum_{k=0}^{\infty} C^k F(x^{2^k}) \right)$$

| $B(x)$ | $C, F(\xi)$ | a_{2n} | a_{2n+1} | |
|-----------------------|--------------------------|--------------------------------|------------------------|---|
| $1, \xi$ | | $a_n + a_{n-1} + 2n$ | $2a_n + 2n + 1$ | $n \lceil \lg n \rceil - 2^{\lceil \lg n \rceil} + 1$ |
| $2, \xi(1-\xi)$ | | $2a_n + n$ | $a_n + a_{n-1} + n$ | $n + \min a_k, a_{n-k}$ |
| $1, \xi^2(1-\xi)$ | | $2a_n + 2a_{n-1} + 1$ | $4a_n + 1$ | A003314 |
| $1, \xi^2(1-\xi)$ | | $a_n + a_{n-1} + 1$ | $[n > 0](2a_n + 1)$ | A063915 |
| $1, \xi^2(1-\xi)$ | | $a_n + a_{n-1} + 3 - 2[n < 2]$ | $[n > 0](2a_n + 3)$ | $A_{6165}(n) - 1, A_{66997}$ |
| $2-$ | $1, \xi^2(1-\xi)$ | $[1]a_n + a_{n-1} - 1$ | $2a_n - 1$ | $A079945(n-2)$ |
| $2-$ | $1, \xi^2(1-\xi)$ | $[2]a_n + a_{n-1} - 1$ | $2a_n - 1$ | $A060973(n+1) + 1$ |
| $-1+$ | $1, 2\xi^2(1-\xi)$ | $[-1]a_n + a_{n-1} + 2$ | $2a_n + 2$ | $A0080776 - 2$ |
| $2+$ | $1, 2\xi^2(1-\xi)$ | $[2]a_n + a_{n-1} + 2$ | $2a_n + 2$ | $A005942(n+2) - 2$ |
| $\frac{3}{2x} +$ | $2, 3/2\xi$ | $(4a_n)$ | $(2a_n + 2a_{n+1})$ | $A073121 - 2$ |
| | | $(2a_n + 2)$ | $(a_n + a_{n-1} + 2)$ | Aronson-like |
| $\frac{x^2+x}{1-x} -$ | $1, \frac{\xi}{1-\xi}$ | $a_n + a_{n-1} + 2n^2 + n$ | $2a_n + 2n^2 + 3n + 1$ | $A077071(n)/2$ |
| | $1, \frac{\xi}{1-\xi}$ | $a_n + a_{n-1} + n - 1$ | $2a_n + n$ | $A_{788} - n$ |
| | $-1, \frac{\xi}{1-\xi}$ | $-a_n - a_{n-1} + n^2 + n$ | $-2a_n + n^2 + 2n + 1$ | $\sum A068639$ |
| | $1, \frac{\xi}{1+\xi}$ | $a_n + a_{n-1} + n$ | $2a_n + n + 1$ | |
| | $2, \frac{\xi}{1+\xi}$ | $2a_n + 2a_{n-1} + 3n - 2$ | $4a_n + 3n$ | $n(n-1)/2$ |
| | $-1, \frac{\xi}{1+\xi}$ | $-(a_n + a_{n-1}) + n$ | $-2a_n + n + 1$ | A000788 |
| | $1, \frac{\xi_2}{1+\xi}$ | $a_n + a_{n-1} + n$ | $2a_n + n$ | A005536 |
| | $2, \frac{\xi}{1-\xi^2}$ | $2(a_n + a_{n-1}) + n^2 + n$ | $4a_n + n^2 + 2n + 1$ | $A059015 - 1$ |
| | $2, \frac{\xi}{1+\xi^2}$ | $2(a_n + a_{n-1} + [n/2])$ | $4a_n + n + 1$ | $A022560$ |
| | | | | A048641 |