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3 pages

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Thurs. June 20, 1991

Neil,

Here is another sequence you might consider entering into your revision of "A Handbook of Integer Sequences":

~~Essentially~~ twice ~~[illegible]~~ 0670

1, 2, 26, 150, 1082, 9366, ...
n=0, n=1, n=2, ...

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These whole numbers equal $\sum_{k=1}^{\infty} \frac{k^n}{2^k}$ exactly!

n=0 is obvious:	$1 = \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots$	n=0
n>0 is <u>less</u> obvious:	$2 = \frac{1}{2}+\frac{2}{4}+\frac{3}{8}+\frac{4}{16}+\frac{5}{32}+\cdots$	n=1
	$6 = \frac{1}{2}+\frac{4}{4}+\frac{9}{8}+\frac{16}{16}+\frac{25}{32}+\frac{36}{64}+\frac{49}{128}+\cdots$	n=2

The sequence can be forced out of RIORDAN's Problem 2.2a in his original, famous, "An Intro. to Combinatorial Analysis" 1958, J Wiley, p38

Prob. 2.2a says to do:

define $a_n(x) = (1-x)^{n+1} \sum_{k=0}^{\infty} k^n x^k$

using $D = \frac{d}{dx}$, show $a_n(x) = n x a_{n-1}(x) + x(1-x) D a_{n-1}(x)$

and $a_0(x)=1$, $a_1(x)=x$, $a_2(x)=x^2+x$, $a_3(x)=x^3+4x^2+x$

... $a_n(x)$ has integral coefficients which RIORDAN calls Eulerian numbers ~~and~~ displays in a table on p. 215 of the same book.

To show $\sum_1^{\infty} \frac{k^n}{2^k}$ is integral,

calculate: $\sum_1^{\infty} \frac{k^n}{2^k} = \sum_0^{\infty} \frac{k^n}{2^k} = \frac{a_n(x)}{(1-x)^{n+1}} = \frac{a_n(\frac{1}{2})}{(\frac{1}{2})^{n+1}} = 2^{n+1} a_n(\tfrac{1}{2})$

which is guaranteed to be integral (and even for $n \geq 1$)

since: (a) $a_n(x)$ has integral coefficients

and (b) $a_n(x)$ has degree n.

Hope you can stuff the sequence into your revision
Steve Snover

ps. Should I publish this little note somewhere? What do you suggest?

Sev

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AT&T

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July 24, 1991

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Dear Steve:

Thanks for your letter of June 20 and the nice sequence

$$\sum_{1}^{\infty} \frac{k^n}{2^k}$$

(sequence A629 ~~A6790~~, by the way). Here is an explicit formula for it, which also shows it is an integer:

$$\sum_{k=1}^{n} \sum_{j=0}^{k} 2^{n+1-k} (-1)^j \binom{n+1}{j} (k-j)^n .$$

This follows from what you did, using a formula for the Eulerian numbers. (See Comtet, Analyse Combinatoire, Vol. 2, p. 84, for example. This will be in the English edition too but I don't have a copy of that at hand.)

Using this I computed a few terms—see enclosed. Note that it begins

1,2,26,150

(you had 1,2,6,150,..., no doubt just a slip of the pen).

I didn't try to simplify this expression, but there may be a nicer form.

Best regards,

N. J. A. Sloane

Encl.

Enclose "95 15"

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     1

     2

     6

     26

     150

     1082

     9366

     94586

     1091670

     14174522

     204495126

     3245265146

     56183135190

     1053716696762

     21282685940886

     460566381955706

     10631309363962710

     260741534058271802

     6771069326513690646

     185603174638656822266

     5355375592488768406230
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