Aug 4, 1970

Dear Neil:

Thanx for your two letters, the return of the Stein's' table, and the computer printouts for discordant permutations, and the enlarged table for the essent. series series-parallel nos. I hope you checked the latter by the congruences (at the top of the page I sent)

Here are some comments

1. The values of the rook polynomials for permutation discordant with 3 (cyclical) permutations are at disposal for $n = 0, 1, 2$, so $R_2 = 1 + 6x + 5x^2$ is as good as $1 + 6x + 3x^2$ and suited me better at one time. Note that in problem 25 of chapter 8 of Comb. Analysis other initial conditions are used.

Sugai

2. The only things I notice about Sugai's paper is that the coefficients as written sum to $m-1$, and that the table looks like it might be simpler if new rows were intercalated, say

1				
1	2			
1	4	1		
1	6	6	1	
1	8	17	8	1

But what the right new rows are escapes me.

3. I have just written to Comtet telling him of the error you found in his Exercise on Schröder's fourth problem and also the following. Exercise 20 of Chapter V (Stirling nos) is titled "Nombre de «formules de Fubini»" which is $a_n = \sum k!\, S(n,k) = \sum \Delta^k 0^n$, which are assigned to certain Cayley trees in your table of sequences, and also appear in Touchard No. 9 (of my memorial note), and O A Gross in the reference I sent you

I supplied all the latter references for him. I also mentioned with due credit your discovery that the Schröder numbers are also those of $Z_n(1,1,\dots 1)$ in Comb. Identities.

4. I found a reprint of Touchard's paper and to my surprise a connection with Bessel polynomials (C.I. p.77 problem 10 of chap. 2). The connection appears in the gen. function (not in C.I)

$$\exp[x^{-1}(1-\sqrt{1-2xt})] = \sum_0 y_{n-1}(x)\, t^n/n! \qquad \underline{y_{-1}(x)=1}$$

Touchard's result may be rewritten (replace c_n by C_n)

$$\exp[\sqrt{1-2t}-1] = -\sum C_n\, t^n/n!$$

Hence

$$+C_n = y_{n-1}(-1)(-1)^{n-1}$$

The recurrence for the Bessel's is

$$y_{n+1}(x) = (2n+1)\,x\,y_n(x) + y_{n-1}(x)$$

$y_0 = 1 \qquad y_2 = 1+3x+3x^2$

$y_1 = 1+x \qquad y_3 = 1+6x+15x^2+15x^3$

Hence

806, but change name!

$C_n = (2n-3)C_{n-1} + C_{n-2}$

n	0	1	2	3	4	5	6	7	8
C_n	-1	1	0	1	5	36	329	3655	47844

If your table includes C_n, shouldn't it also include $y_n(1)$

1515 ✓ rename?

n	0	1	2	3	4	5	6	7	8
$y_n(1)$	1	2	7	37	266	2431	27007	353522	5329837

Congruences (p a prime greater than 2)

$y_{p+k}(1) \equiv y_k \mod p$

$C_{p+k} \equiv -C_k \mod p \qquad C_{2p+k} \equiv C_k \mod p \quad$ period $2p$.

$C_2 \equiv 0\ (2) \quad C_3 \equiv 1\ (2) \quad C_4 \equiv 1\ (2)$ & $C_{2n} \equiv 0\ (2)$ $C_{2n+1} \equiv C_{2n+2} \equiv 1\ (2)$

n	0	1	2	3	4	5	6	7	
mod 2 C_n	-1	1	1 . 0	1	1 . 0	1	1		period 3
mod 2 $y_n(1)$	1	0	1 . 1	0	1 . 1	0	1		n 3

JR

5. I think I remember that Pauling's C-dosage was 3000 units a week. 200 units a day is common.

6. I enclose the memo of Discordant Perm. I did for Schroder

7. I can't believe your computer printout is right!

Yours
John