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5-9-94

Dear Neil:

Thanks a lot for return of the letter stating that your "Encyclopedia" does not have the algorithm for 1,3,5,15,17,51,85,255,257,771,1285,3855,...

It does now, and YOU are the co-discoverer!

The enclosed article is one of the most important discoveries in Pascal Triangle number theory in mathematical history.

If you want to be part of the the discovery, you will have to write up the article and submit to a journal of your choice, your name first, mine second.

Or, if you positively insist on not being a part of this, then you can quote me if the formula (really a set of rules) is integrated into the enclopedia.

Worst case scenario: you completely ignore this whole project, which would not go well with the mathematics "gods" since this is definitely worth publishing. I especially will be very disappointed if you do not participate fully, with your name first. If not, then so be it. In any event, no need to get back to me on this. If you like the cooperative venture, simply write up the article as you see fit. If not, then I won't hear from you.

Sincerely,

Gary W. Adamson

P.S. some type of computer work would also be beneficial, such as another computer printout of Pascal's Triangle, MOD 2. It doesn't have to be the one on the cover.

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May 12, 1994

Mr. Gary W. Adamson
PO Box 124571
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Dear Gary:

Thank you for your letter of May 9, 1994. You really mustn't put my name on a document without asking me first, you know. As it happens I don't want to be involved with that project. Sorry!

Yours sincerely,

N. J. A. Sloane

ALGORITHM FOR GENERATING nth ROW OF PASCAL'S TRIANGLE, MOD 2, FROM n

by

N.J.A. Sloane, PhD, &
Gary W. Adamson

```
                     1
                    1 1
                   1 0 1
                  1 1 1 1
                 1 0 0 0 1
                1 1 0 0 1 1
               1 0 1 0 1 0 1
              1 1 1 1 1 1 1 1
             1 0 0 0 0 0 0 0 1
            1 1 0 0 0 0 0 0 1 1
           1 0 1 0 0 0 0 0 1 0 1
          1 1 1 1 0 0 0 0 1 1 1 1
         1 0 0 0 1 0 0 0 1 0 0 0 1
        1 1 0 0 1 1 0 0 1 1 0 0 1 1
       1 0 1 0 1 0 1 0 1 0 1 0 1 0 1
      1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
     1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
    1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1
   1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1
  1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1
 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 1
1 1 0 0 1 1 0 0 0 0 0 0 0 0 0 0 1 1 0 0 1 1
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nth row = $\ldots(a_3^{g_1})(a_2^{g_2})(a_1^{g_3})\cdots$ (Binary)

Computer generated Pascal's Triangle, MOD 2, by Malcolm Lichtenstein of San Diego, CA.

ALGORITHM FOR GENERATING THE nth ROW OF PASCAL'S TRIANGLE, MOD 2, FROM n.

Introduction: Since each row of PT, MOD 2 can be seen as a Binary number and converted to its Decimal counterpart, we can reverse the procedure and find the sequence of bits in the nth row from n.

1	Row 0	= 1 Decimal
1 1	1	= 3
1 0 1	2	= 5
1 1 1 1	3	=15
1 0 0 0 1	4	=17

The goal of finding the nth row from n depends upon finding the general rule for the series 1, 3, 5, 15, 17, 51, 85, 255, 257, 771, 1285, 3855, 4369, 13107, 21845, 65535, 65537, 196611, 327685...

Ratios of term n+1 to n reveal the series: 3, 5/3, 3, 17/15, 3, 5/13 3, 257/255, 3, 5/3, 3, 17/15, 3, 5/3, 3...in which a new term of the form

$$\frac{2^{2^n} + 1}{2^{2^n} - 1}$$

$2^{2^n}+1$ (A Fermat number)

$2^{2^n}-1$ (The Mersenne number closest to the Fermat number in the numerator).

...is introduced every 2^nth term.

Here, n starts with 0.

The numbers in this series are 3/1, 5/3, 17/15, 257/255, 65537/65535...

which we write out in reverse to use in our algorithm:

...65537/65535, 257/255, 17/15, 5/3, 3/1

= $\ldots a_5 \quad a_4 \quad a_3 \quad a_2 \quad a_1$

RULES: Given n, find the decimal number in the series 1,3,5,15,17,51,85,... and then convert the result to Binary. This will give the nth row of PT, MOD 2.

(1) Take n, convert to Binary, say decimal 6 = 110. Starting at the left place a 1 over the 1, then for each successive 0 you come to, double the previous number, and for each successive 1 you come to, double the previous number and add 1.

	1	3	6	Note: each 0 corresponds to an even
Example: 6 =	1	1	0	decimal number, 1 to odd.

(2) Take differences in a row above the decimals derived in rule #1.

1	0	1	Gray Code
1	2	3	Diff.
1	3	6	Dec.
1	1	0	Binary

The difference row is 1 2 3 which has a parity of 1 0 1. This is the Gray Code number corresponding to Binary 110 = decimal 6.

RULES, continued...

(3) Take the difference row and apply the numbers in order to members of:

$$\ldots a_5,\quad a_4,\quad a_3,\quad a_2,\quad a_1$$

which in this case would be $a_3^1 \quad a_2^2 \quad a_1^3$

and then take the product:

$$(17/15)^1 \text{ X } (5/3)^2 \text{ X } (3)^3 = 85.$$

(4) Last, convert the decimal number derived in Rule 3 to Binary,

= 1 0 1 0 1 0 1

= the 6th row of PT, MOD 2, since the 6th Row in PT is

1 6 15 20 15 6 1

THE CONSTANT 5.711285429...

Only certain exponents allowed by the Gray Code generate integers, for Rule #3. Pascal's Triangle, MOD 2, is the infinite set of those sequences of bits generated by Rule 3, and Rule 4. If the exponents are all 1's, we have the interesting number 5.711285429... =

...65537/65535 X 257/255 X 17/15 X 5/3 X 3/1

Algebraically, this =

$$5 + \frac{2}{2^2-1} + \frac{2}{(2^2-1)(2^4-1)} + \frac{2}{(2^2-1)\ (2^4-1)\ (2^8-1)} \ldots$$

or $$5 + 2/3 + \frac{2}{(3)(15} + \frac{2}{(3)(15)(255)} + \frac{2}{(3)\ (15)\ (255)\ (65535)} \ldots$$

THE TOWER OF HANOI WHEEL:

This gives the Binary & Gray Code equivalents for any decimal number, where the Gray Code series (see examples for 123: 1 2 4 8 15 31 62 = 123, and for 89: 1 1 3 6 11 22 45) gives the exponents for the a's: For example, to find the 89th row of PT, MOD 2, our Gray Code series is 1 1 3 6 11 22 45, and we take these as exponents, applying them to a number of "a" terms equal to 7:

$$(a_7)^1\ (a_6)^1\ (a_5)^3\ (a_4)^6 (a_3)^{11}\ (a_2)^{22}\ (a_1)^{45}$$

The product of the terms will give a decimal number, which converted to the Binary, = the 89th row in PT, MOD 2. The Gray Code numbers also give the number of moves per disk through move (sum of exponents, = 89), with disks numbered the same as the "a" subscripts: Disk #1 moved 45 times, #2: 22, etc.

THE TOWER OF HANOI WHEEL

Each Compartment has a larger number and smaller (except for 1). The larger number is the decimal counterpart to the Binary, or total number of moves per disks as indicated by typed numbers in the vertical line. Draw a straight line from the center to a given larger number, and record larger numbers and smaller in each compartment the line goes through. These then are the Binary & Gray Code sequence of terms, as discussed previously. Example, pick 123, larger number of outer rim compartment about 11:00. Draw a line from center to that compartment, noting that the line goes through compartments with:

1	2	4	8	15	31	62	(smaller) = Gray Code =	1 0 0 0 1 1 0 = 123
1	3	7	15	30	61	123	(larger) = Binary	
1	1	1	1	0	1	1	= Binary for 123	

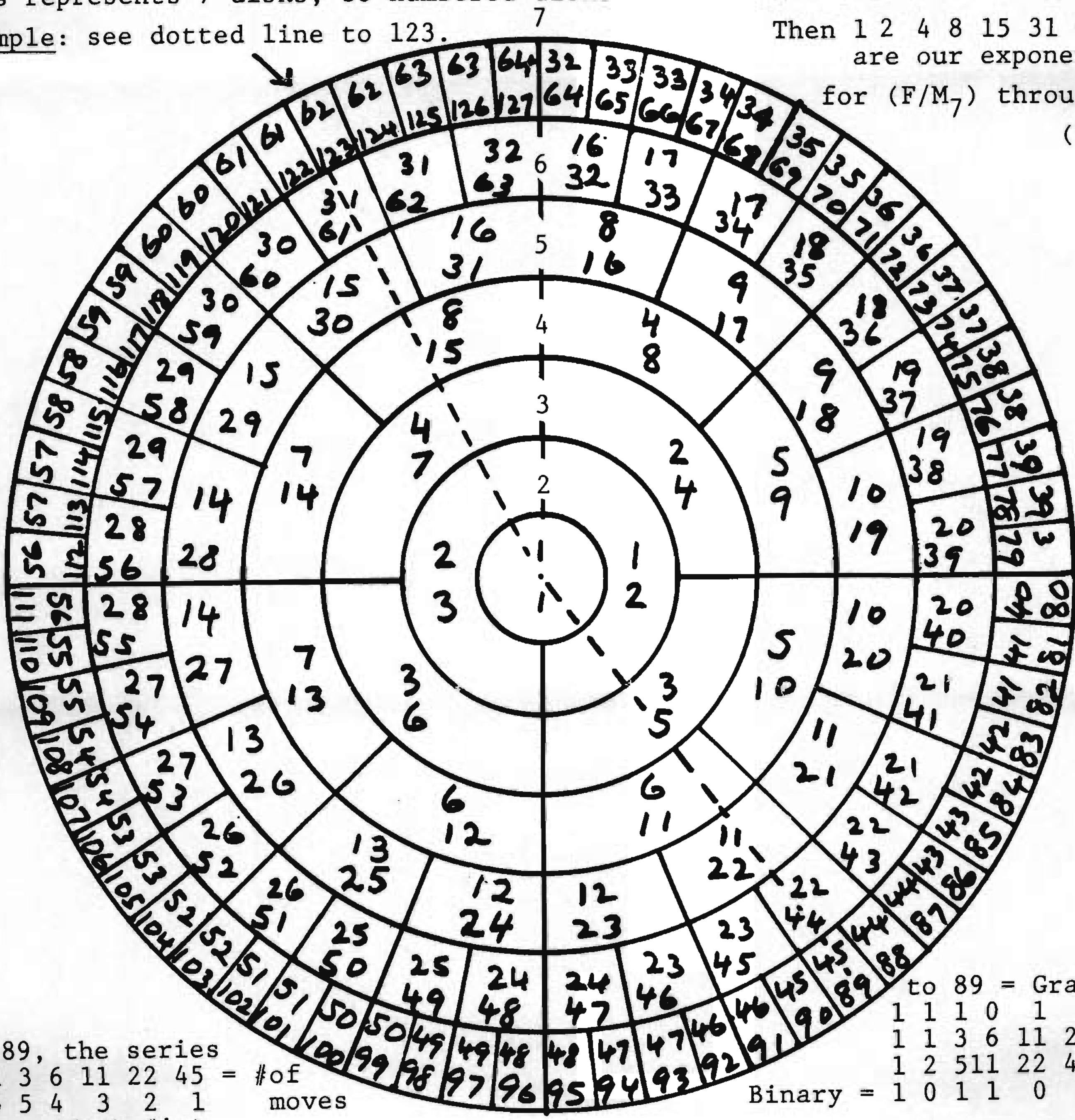
(moves per disk)

1	2	4	8	15	31	62	= 123
7	6	5	4	3	2	1	Disk#

This represents 7 disks, so numbered disks have moved: (see above)

Example: see dotted line to 123.

Then 1 2 4 8 15 31 62, are our exponents for (F/M_7) through (F/M_1)



To 89, the series

1	1	3	6	11	22	45	= #of moves
7	6	5	4	3	2	1	(disk #'s)

to 89 = Gray Code:

	1	1	1	0	1	0	1
	1	1	3	6	11	22	45
	1	2	5	11	22	44	89
Binary =	1	0	1	1	0	0	1