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THE SELF-AVOIDING WALK

[Ch. 7]

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n	C_n	$h_n = U_n/(2n)$
1	6	0
2	30	0
3	138	6.2
4	618	8.3
5	2730	10.6
6	11946	12.15
7	51882	14.42
8	224130	16.123
9	964134	18.380
10	4133166	20.1212
11	17668938	22.3966
12	75355206	24.13265
13	320734686	26.45144
14	1362791250	28.155955
15	5781765582	30.545690
16	24497330322	32.1930635
17	103673967882	34.6897210
18	438296739594	36.24852576
19	1851231376374	38.90237582
20	7812439620678	40.329896569
21	138825972053046	42.1213528736

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Table 1. Triangular lattice data.

- Number of self-avoiding walks (C_n)

C_1 to C_{11} are from Sykes (1961) and Hiley and Sykes (1961). Martin *et al.* (1967) reported C_{12} to C_{17} . Errors in their C_{16} and C_{17} were found by Grassberger (1982). His C_{16} was confirmed by Guttman (1984). C_{17} , C_{18} were given by Majid *et al.* (1983a) and Guttman (1984), C_{19} by Rapaport (1985a), C_{20} by Guttman (1989b); C_{21} by Guttman & Wang (1991).

- Number of self-avoiding polygons ($U_n = 2nh_n$)

Guttman (personal communication) has computed h_n for $n \leq 32$ and his values for 19, 20, 21 are given here. He confirms earlier data: $n \leq 17$, Martin *et al.* (1967); $n = 18$, Sykes, McKenzie, Watts and Martin (1972) and Guttman (1984).

n	C_n	U_n
1	12	0
2	132	0
3	1404	48
4	14700	264
5	152532	1680
6	1573716	11640
7	16172148	86352
8	165697044	673104
9	1693773924	5424768
10	17281929564	44828400
11	176064704412	377810928
12	1791455071068	3235366752
13	18208650297396	28074857616
14	184907370618612	246353214240

Table 2. Exact enumeration data

for the face-centred cubic lattice:
 numbers of self-avoiding walks
 (C_n) and polygons (U_n): C_1 to
 C_{12} and U_1 to U_{13} from Martin *et*
al. (1967), C_1 to C_7 having been
 given earlier by Sykes (1961);
 U_{14} from Sykes, McKenzie,
 Watts and Martin (1972); C_{13} and
 C_{14} from McKenzie (1979b).

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NUMERICAL METHODS

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n	C_n	n	$h_n = U_n/(2n)$
1	4	4	1 - 8 = 8
2	12	6	2 - 12 = 24
3	36	8	7 - 16 = 12
4	100	10	28 - 20 = 560
5	284	12	124 - 24
6	780	14	588
7	2 172	16	2 938
8	5 916	18	15 268
9	16 268	20	81 826
10	44 100	22	449 572
11	120 292	24	2 521 270
12	324 932	26	14 385 376
13	881 500	28	83 290 424
14	2 374 444	30	488 384 528
15	6 416 596	32	2 895 432 660
16	17 245 332	34	17 332 874 364
17	46 466 676	36	104 653 427 012
18	124 658 732	38	636 737 003 384
19	335 116 620	40	3 900 770 002 646
20	897 697 164	42	24 045 500 114 388
21	2 408 806 028	44	149 059 814 328 236
22	6 444 560 484	46	928 782 423 033 008
23	17 266 613 812	48	5 814 401 613 289 290
24	46 146 397 316	50	36 556 766 640 745 936
25	123 481 354 908	52	230 757 492 737 449 636
26	329 712 786 220	54	1 461 972 662 850 874 916
27	881 317 491 628	56	9 293 993 428 791 901 042
28	2 351 378 582 244		
29	6 279 396 229 332		

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Table 3. Square lattice self-avoiding walks and self-avoiding polygons.
 C_1 to C_{18} from Sykes (1961) and Hiley & Sykes (1961); C_{19} to C_{24} from Sykes, Guttmann, Watts & Roberts (1972); C_{25} from Rapaport (1985a) [C_{19} to C_{22} recalculated by Grassberger (1982) and C_{23}, C_{24} by Rapaport (1985a)]; C_{26}, C_{27} from Guttmann (1987); C_{28}, C_{29} from Guttmann & Wang (1991); $h_n = U_n/(2n)$ for $n \leq 18$ from Rushbrooke & Eve (1959), for $n \leq 38$ from Enting (1980), for $n \leq 46$ from Enting & Guttmann (1985), confirming U_n for $n \leq 26$ from Sykes, McKenzie, Watts & Martin (1972) and U_{28} from Privman & Rudnick (1985); for $n \leq 56$ from Guttmann & Enting (1988a).

n	square	cubic	n	square	cubic
1	1.0000000	1.000000	16	51.9925007	29.626838
2	2.6666667	2.400000	17	56.7164116	31.849341
3	4.5555556	3.880000	18	61.7664657	34.134753
4	7.0400000	5.553719	19	66.7657827	36.409358
5	9.5633803	7.234295	20	72.0765498	38.741770
6	12.5743590	9.070542	21	77.3367445	41.063808
7	15.5561694	10.897236	22	82.8958189	
8	19.0128465	12.845084	23	88.4044404	
9	22.4113597	14.780576	24	94.2010351	
10	26.2425397	16.817245	25	99.9472905	
11	30.0176570	18.841373	26	105.9719098	
12	34.1869930	20.952845	27	111.9463138	
13	38.3043403	23.052131	28	118.1905224	
14	42.7864376	25.228468	29	124.3846486	
15	47.2177466	27.393070			

Table 6. Mean-square displacement $\langle R_n^2 \rangle$ for square and simple cubic lattices (shown here correct to the number of decimal places exhibited). In the cited sources [except for the simple cubic data of Domb (1963)] exact integer values of $C_n \langle R_n^2 \rangle$ are given.

Square lattice. Guttmann (1987) $n \leq 27$; Guttmann & Wang (1991) $n = 28, 29$. Also Grassberger (1982) $14 \leq n \leq 22$; Domb (1963) $n \leq 16$ [replace his $C_5 \langle R_5^2 \rangle = 5432$ by 2716]; Martin & Watts (1971) $n = 19, 20$.

Simple cubic lattice. Guttmann (1987), $n \leq 20$; Guttmann (1989b), $n = 21$. See also Martin and Watts (1971) $n = 14, 15$; Domb (1963) $n \leq 10$.

Triangular. Martin & Watts (1971) $7 \leq n \leq 14$; Grassberger (1982) $n \leq 16$; Djordjevic *et al.* (1983) $n \leq 18$; Rapaport (1985a) $n = 19$; Guttmann (1989b) $n = 20$; Guttmann and Wang (1991) $n = 21, 22$.

Diamond (tetrahedral). Domb (1963) $n \leq 14$; Wall & Hioe (1970a) $n \leq 20$ [for errors in Hioe's approach see Chay (1970)]; Guttmann (1989b) $n \leq 27$.

Body-centred cubic. Domb (1963) $n \leq 8$; Martin & Watts (1971) $n \leq 12$, Guttmann (1989b) $n \leq 16$.

Face-centred cubic. Domb (1963) $n \leq 7$; Martin & Watts (1971) $n \leq 10$; Majid *et al.* (1983b) $n \leq 12$ (see errata for $n = 11$); Rapaport (1985b), $n \leq 12$.

Table 7: Some sources of data on $\langle R_n^2 \rangle$ for lattices not covered in Table 6.

n	C_n	U_n
1	6	0
2	30	0
3	150	0
4	726	24
5	3534	0
6	16926	264
7	81390	0
8	387966	3312
9	1853886	0
10	8809878	48240
11	41934150	0
12	198842742	762096
13	943974510	0
14	4468911678	12673920
15	21175146054	0
16	100121875974	218904768
17	473730252102	0
18	2237723684094	3891176352
19	10576033219614	0
20	49917327838734	70742410800
21	235710090502158	

Table 4. *Exact enumeration data for simple cubic lattice.*

• *Number of self-avoiding walks (C_n).* Here C_1 to C_{10} are from Sykes (1961) and Hiley and Sykes (1961). Their erroneous value (41933286) of C_{11} has been corrected by later authors; C_{11} to C_{16} are from Sykes (1963); C_{17} to C_{19} from Sykes, Guttmann, Watts and Roberts (1972); C_{20} is from Guttmann (1987) and C_{21} from Guttmann (1989b).

• *Number of self-avoiding polygons (U_n).* Here U_4 to U_{16} are from Rushbrooke and Eve (1959, 1962) [who give $h_n = U_n/(2n)$] and note some errors in earlier related data of Wakefield (1951)]; U_{18} to U_{20} are from Sykes, McKenzie, Watts and Martin (1972).

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honeycomb – walks to C_{24} in Sykes (1961) and to C_{34} in Sykes, Guttmann, Watts and Roberts (1972) and in Guttmann and Sykes (1973); polygons to U_{82} in Enting and Guttmann (1989).

kagomé – walks to C_{13} and polygons to U_{13} in Fisher and Sykes (1959).

hydrogen peroxide – walks to C_{30} in Leu (1969).

diamond – walks to C_{14} in Martin (1962) and to C_{27} in Guttmann (1989b); replace C_{16} in Essam and Sykes (1963) by 42922452 [Domb (1974), p. 92];

hypertriangular – walks to C_{14} in Leu (1969).

body-centred cubic – walks to C_9 in Sykes (1961), to C_{15} in Sykes, Guttmann, Watts and Roberts (1972), to C_{16} in Guttmann (1989b); polygons to U_{16} in Sykes, McKenzie, Watts and Martin (1972).

Table 5. *Sources of enumeration data for lattices not listed in Tables 1-4.*