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Dear Neil:

At last I get around to some notes on the Handbook (probably too late!)

1. H 1184 is $q(n,n)$ of Stanton and Cowan, but it is also $Q_n(1) = P_n(3)$, Comb. Id. 78 (Legendre polynomials) and

$$Q_n(1) = Q_n = n^{-1}[(6n-3)Q_{n-1} - (n-1)Q_{n-2}]$$

Also, if $d(n,r) = q(n-r,r)$

$$d_n(x) = \sum d(n,r)x^r = (1+x)d_{n-1}(x) + x\, d_{n-2}(x)$$

$$d_n = d_n(1) = 2d_{n-1} + d_{n-2}$$, the Pell nos of H552

The numbers $d(n,r)$ appear in Irving Kaplansky, The Asymptotic Distribution of Runs of Consecutive Elements, Annals of Math. Stat. 16 (1945), 200-203 as

the number of ways of picking r of $2n-1$ objects: $A_1 \ldots A_{n-1}$; $B_1 \ldots B_n$ without pairs: $A_{j-1}B_j$, A_jB_j, $A_{j+1}B_j$

2. Carlitz, Fibonacci Quarterly, 4 (1976), 327-342, looks at (a disguise for) $Y_n(x,0,x,0,\ldots)$ and $Y_n(0,x,0,x,\ldots)$ enumerators by no. of parts ~~specification~~ size of set partitions with all parts odd or even. I enclose tables of $U_n(x) = Y_n(x,0,x,0,\ldots)$ and $V_{2n}(x) = Y_{2n}(0,x,0,x,\ldots)$ $[V_{2n+1}(x) = 0]$, and also of and auxiliary polynomial $v_n(x)$. Because of the congruences:

$U_p(x) \equiv x + x^p$, mod p (p prime) $U_{p+n}(x) \equiv U_{n+1}(x) + x^p U_n(x)$, mod p

$V_{p+n}(x) \equiv V_{n+1}(x)$ (p) $V_{np}(x) \equiv V_n(x)$ (p)

I feel a little surer of my hand calculations;

Over

3 Here are some additions to 1174 $\frac{1}{2n+1}\binom{3n}{n}$
1430715, 8414640, 500067108, 3005305 76 111247 55664

4. #1240 is $y_n(2)$, $y_n(x)$ - Bessel polynomial $y_n(2) = (4n-2)y_{n-1}(2) + y_{n-2}(2)$

5. #1468 I have corrected 37782 to 38232 and added
5?3859, 1040172, 202601898

6 #1625 is; $|s_{n+1,2}| - n!$ Stirling number of first kind

That's all for now. Regards - love to Ann.

John.

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$U_n(x) = Y_n(x, 0, x, 0, \dots) =$ enumerator by no. parts of set-partitions with all parts odd

$$U_{n+1}(x) = x \sum \binom{n}{2j} U_{n-2j}(x)$$

k\n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(total)	1	1	1	2	5	12	37	128	457	1872	8169	37600	188685	990784	5497741	32333824
0	1															
1		1		1		1		1		1		1		1		1
2			1		4		16		64		256		1024		4096	
3				1		10		91		820		7381		66430		597871
4					1		20		336		5440		87296		1397760	
5						1		35		966		24970		631631		15857205
6							1		56		2352		90112		3331328	
7								1		84		5082		273988		14057043
8									1		120		10032		732160	
9										1		165		18447		1768195
10											1		220		32032	
11												1		286		53053
12													1		364	
13														1		455
															1	
																1

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$V_n(x) = Y_n(0, x, \ldots, 0, x)$: enumerator by no. parts of set partitions with all parts even
$= \sum V_{nk} x^{k/2}$

$V_{np}(x) \equiv V_n(x) \ (p)$
$V_{p+n}(x) \equiv V_{n+1}(x) \ (p)$

$V_{2p+1}(x) = 0 \qquad V_{2n}(x) = \sum a_k V_{nk} x^k = V_n(ax) \qquad a_n = 1 \cdot 1 \cdot 3 \cdots (2n-1) \quad a_0 = a_1 = 1$

$V(n,k)$: $V(n+1,k) = V(n,k-1) + k^2 \wedge$

Not in H		1	1	4	31	379	6556	150349	4373461	156297964	6698486371
$V_{2n}(x)$	$k \backslash n/2$	0	1	2	3	4	5	6	7	8	9
	0	1									
	1		1	1	1	1	1	1	1	1	1
$4^{n-1}-1$	2			3	15	63	255	1023	4095	16383	65535
	3				15	210	2205	21120	195195	1777230	16076985
	4					105	3150	65835	1201200	20585565	341809656
	5						945	51975	1891890	58108050	1637971335
	6							10395	945945	54864810	2614321710
	7								135135	18918900	1640268630
	8									2027025	413513100
	9										34459425
	10										

$V_{n+1}(x) = [x + xD + x^2D^2]\, V_n(x), \quad D = d/dx, \quad = [x + (xD)^2]\, V_n$

	k	0	1	2	3	4	5	6	7	8	9
	0	1									
$V_n(x)$	1		1	1	1	1	1	1	1	1	1
$\frac{1}{3}(4^{n-1}-1)$	2			1	5	21	85	341	1365	5461	21845
	3				1	14	147	1408	13013	118482	1071799
$V_{nn} = 1$	4					1	30	627	11440	196053	3255330
$V_{n,n-1} = \binom{n}{3} + \binom{n+1}{3}$	5						1	55	2002	61490	1733303
	6							1	91	5278	251498
	7								1	140	12138
	8									1	204
	9										1
	10										
Not in H	Sum	1	1	2	7	37	264	2433	27913	386906	6346119

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