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AMICABLE NUMBERS

By EDWARD BRIND ESCOTT

THE theory of numbers has been studied continuously from the time of Pythagoras (6th century B.C.) to the present time. Two subjects which were among the first to be studied were Perfect Numbers and Amicable Numbers.

A perfect number is a number which equals the sum of its aliquot divisors. By aliquot divisors is meant the divisors of the number (excluding the number itself). Example: 6 is the smallest perfect number. The aliquot divisors of 6 are 1, 2, 3 and since $1 + 2 + 3 = 6$ this is a perfect number.

Two numbers are called amicable if each equals the sum of the aliquot divisors of the other.

The smallest pair of amicable numbers is 220 and 284. The sum of the aliquot divisors of 220 is $1 + 2 + 4 + 5 + 10 + 11 + 20 + 22 + 44 + 55 + 110 = 284$. The sum of the aliquot divisors of 284 is $1 + 2 + 4 + 71 + 142 = 220$.

This pair was known to the Pythagoreans.

In the 9th century an Arabian mathematician, Thâbit ben Korrah, gave the following formulae for amicable numbers:

$2^n pq$ and $2^n r$ are amicable numbers if $p = 3 \cdot 2^{n-1} - 1$, $q = 3 \cdot 2^n - 1$ and $r = 9 \cdot 2^{2n-1} - 1$ are all primes and $n > 1$. This rule leads to amicable numbers for $n = 2$ (giving the above pair), $n = 4$, and $n = 7$ but for no further values of $n < 200$.

Fermat (1636) rediscovered this rule and gave the pair of amicable numbers for $n = 4$.

Descartes (1638) gave a rule the equivalent of the above and discovered the pair of amicable numbers for $n = 7$.

Euler (1750) was the next one to study these numbers and, treating the problem with his customary thoroughness, discovered 59 other pairs.

The complete list is given on page 65. The history of the subject up to 1919 is taken from Dickson.[1]

In studying this subject, the following formulae due to Euler will be

[1] L. E. Dickson, *History of the Theory of Numbers*, v. 1, Washington, Carnegie Institute of Washington (1919).

useful. . . . Denoting the sum of the divisors of a number (including the number itself) by $S(N)$ and let the number N be completely factored into its prime factors
$N = p^m q^n \ldots$ then $S(N) = [(p^{m+1} - 1)/(p - 1)]\cdot[(q^{n+1} - 1)/(q - 1)]\ldots$
If $N = p$, $S(p) = (p^2 - 1)/(p - 1) = p + 1$.

If M and N are relatively prime, $S(MN) = S(M)\cdot S(N)$.

The characteristic property of the amicable numbers m and n may be expressed by the two equations $S(m) - m = n$, $S(n) - n = m$.

These may be replaced by the two equations

$$S(m) = S(n) \tag{1}$$

$$S(m) = m + n \tag{2}$$

In this paper (excepting the Miscellaneous Forms) the common factor of the two numbers will be denoted by E (an integer, not usually prime) and by $p, q, r, s, \ldots$ distinct odd primes not dividing E.

Usually the pair of amicable numbers $Epq\ldots$ and $Ers\ldots$ will be written $E^{pq\ldots}_{rs\ldots}$.

The treatment of the possible solutions depends on the number of prime factors in the numbers m and n.

1st Form: *Epq*, *Er*.

In this case equations (1) and (2) become

$$S(E)\cdot(p + 1)(q + 1) = S(E)\cdot(r + 1) \tag{3}$$

$$S(E)\cdot(p + 1)(q + 1) = E(pq + r) \tag{4}$$

Equation (3) may be further simplified by removing the common factor $S(E)$ which gives

$$(p + 1)(q + 1) = r + 1 \tag{5}$$

Eliminating r between equations (5) and (4) we have

$$S(E)\cdot(p + 1)(q + 1) = E(2pq + p + q) \tag{6}$$

Then the problem reduces to the solution of equation (6) in the two unknowns p and q subject to their being unequal odd primes not dividing E.

Consider the case when $E = 2^n$. Since $S(2^n) = 2^{n+1} - 1$, equation (6), reduces to

$$pq - (2^n - 1)(p + q) = 2^{n+1} - 1$$

This equation may be factored in the form

$$[p - (2^n - 1)][q - (2^n - 1)] = 2^{2n} \tag{7}$$

Factoring the second member of this equation, the factors may be taken as 2^{n-m} and 2^{n+m}. This gives for the solution of equation (7)

$$p - (2^n - 1) = 2^{n-m}, \ q - (2^n - 1) = 2^{n+m}$$

whence

$$p = 2^{n-m}(2^m + 1) - 1, \ q = 2^n(2^m + 1) - 1, \ r = 2^{2n-m}(2^m + 1)^2 \qquad (8)$$

If $m = 1$ this solution takes the form

$$p = 3 \cdot 2^{n-1} - 1, \ q = 3 \cdot 2^n - 1, \ r = 9 \cdot 2^{2n-1} - 1 \qquad (9)$$

which is the formula of Thâbit ben Korrah.

If m is an even number, n is the difference of two squares and cannot be prime. If $m = 3$ either p or q would be the difference of two squares and could not both be prime. If $m = 5$ there are no known values of n which will make p, q, and r prime.

If $m = 7$ and $n = 8$ we have $p = 257$, $q = 33\,023$, $n = 8\,520\,191$, all prime numbers. This gives the solution discovered by Legendre.

These are all the examples known where $E = 2^n$.

On the general case, equation (6) may be written

$$\frac{S(E)}{E} = 2 - \frac{(p+1) + (q+1)}{(p+1)(q+1)}$$

$$= 2 - \frac{1}{g}$$

where g is a positive number (integral or fractional).

It is of advantage to make a table of values for which $S(E)/E$ is less than 2 or a table of values of g and the corresponding values of E.

Example: Take, for example, $E = 3^2 7 \cdot 13$, then $g = \frac{9}{2}$.

$S(E) = 13 \cdot 8 \cdot 14$. From equation (6) we have

$$2pq - 7(p + q) = 16$$

which may be factored

$$(2p - 7)(2q - 7) = 81$$

Equating $2p - 7$ and $2q - 7$ to all possible unequal factors of 81 we have

$$\begin{aligned} 2p - 7 &= 1,\ 3 \\ 2q - 7 &= 81,\ 27 \\ p &= 4,\ 5 \\ q &= 44,\ 17 \end{aligned}$$

The only prime values of p and q are $p = 5$, $q = 17$, whence $r = 107$. We have the solution

$$3^2 \cdot 7 \cdot 13 \begin{matrix} 5 \cdot 17 \\ 107 \end{matrix}$$

2nd Form: *Epq, Ers.*

In this case equations (1) and (2) become

$$S(E) \cdot (p + 1)(q + 1) = S(E) \cdot (r + 1)(s + 1) \qquad (10)$$

$$S(E) \cdot (p + 1)(q + 1) = E \cdot (pq + rs) \qquad (11)$$

In equation (10) the common factor $S(E)$ may be removed

$$(p + 1)(q + 1) = (r + 1)(s + 1) \qquad (12)$$

Solving equation (12) for one of the unknowns—say s—and substituting in equation (11) we have

$$(r - 15)pq - (15r + 31)(p + q) - 16r^2 - 31(r + 1) = 0 \quad (13)$$

In order to find solutions for p and q in integers, it is of advantage to have the coefficient of pq as small as possible.

1st. Let $r = 17$, equation (13) becomes

$$pq - 143(p + q) = 2591$$

or, factored,

$$(p - 143)(q - 143) = 23{,}040 = 2^9 3^2 5$$

Put the factors $(p - 143)$ and $(q - 143)$ in the first member equal to all possible pairs of even factors of the second member (48 pairs altogether); neglecting all values of p and q which are not prime. From equation (12) find s and omitting values of s which are not prime, only two sets of values of p, q, r, and s remain, which give the following pairs of amicable numbers:

$$2^4 \begin{matrix} 17 \cdot 10\,303 \\ 167 \cdot 1103 \end{matrix} \qquad\qquad 2^4 \begin{matrix} 17 \cdot 5119 \\ 239 \cdot 383 \end{matrix}$$

2nd. Let $r = 19$. One solution is found.
3rd. Let $r = 23$. Three solutions are found.
4th. Let $r = 47$. One solution is found.

Historical Notes:* This list contains 390 pairs of amicable numbers discovered during the past 2500 years. Iamblichus attributes to Pythagoras (*c.* 540 B.C.) the discovery of the first pair of amicable numbers 220 ($= 2^2 \cdot 5 \cdot 11$) and 284 ($= 2^2 \cdot 71$). The next two pairs were discovered by Fermat and Descartes. Euler added 59 pairs in the next century but even to the end of the nineteenth century only 66 pairs had been discovered. Details in this regard may be found in L. E. Dickson, *History of the Theory of Numbers*, v. 1, Washington, 1919, p. 38–50.

The complete record of discoveries, with approximate dates, is as follows:

Pythagoras	1 (540 B.C.)	T. E. Mason[1]	14 (1921)
Fermat	1 (1636)	P. Poulet[2]	65 (1929)
Descartes	1 (1638)	A. Gerardin[3]	5 (1929?)
Euler	59 (1747–50)	E. B. Escott[4]	233 (1934)
Legendre	1 (1830)	B. H. Brown[5]	1 (1939)
R. N. I. Paganini	1 (1867)	Poulet and Gerardin	4 (1929)
P. Seelhoff	2 (1884)		
L. E. Dickson	2 (1911)	TOTAL (May 1943)	390

Form E_r^{pq}

(1) $2^2 \, \frac{5 \cdot 11}{71}$ (Pythagoras) (2) $2^4 \, \frac{23 \cdot 47}{1151}$ (Fermat) (3) $2^7 \, \frac{191 \cdot 383}{73727}$ (Descartes)

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* From *Mathematical Tables and Other Aids to Computation*, v. 1, 1943, p. 95–96.

[1] T. E. Mason, "On Amicable Numbers and Their Generalizations," *Amer. Math. Mo.*, v. 28 (1921), p. 195–200.

[2] P. Poulet, *La Chasse Aux Nombres. Fascicule I. Parfait, Amiables et Extensions*, Brussels, 1929, p. 28–51. The 156 pairs of amicable numbers, known at this time, are here classified, and include the 68 new pairs found by Poulet.

[3] All of these pairs are in Poulet's list of 1929. It is possible that their discovery was announced earlier in Gerardin's periodical *Sphinx Oedipe*.

[4] P. Poulet, "De nouveaux amiables," *Sphinx*, v. 4 (1934), p. 134–135. Poulet here states that Mr. Escott had sent him 322 pairs of amicable numbers; he prints 21 pairs discovered by Mr. Escott, and all but one of the 42 numbers are less than 10^9. The other 214 pairs had not been published.

[5] B. H. Brown, "A New Pair of Amicable Numbers," *Amer. Math. Mo.*, v. 46 (1939), p. 345.

Otto Gmelin. *Ueber volkommene und befreundete Zahlen.* (Inaugural-Diss., Heidelberg, 1917.)
The author of this doctor's dissertation informed the writer that it was written under trying circumstances. World War I was in progress and most of the students were in service. No new examples are given.

(Euler)

(4) $2^{2}23\ \begin{matrix}5\cdot137\\827\end{matrix}$
(5) $2^{2}13\cdot17\ \begin{matrix}389\cdot509\\198899\end{matrix}$
(6) $3^{2}5\cdot7\ \begin{matrix}53\cdot1889\\102059\end{matrix}$
(7) $3^{2}5\cdot13\ \begin{matrix}11\cdot19\\239\end{matrix}$
(8) $3^{2}5\cdot13\cdot19\ \begin{matrix}29\cdot569\\17099\end{matrix}$
(9) $3^{2}5\cdot19\cdot37\ \begin{matrix}7\cdot887\\7103\end{matrix}$
(10) $3^{2}7\cdot13\ \begin{matrix}5\cdot17\\107\end{matrix}$
(11) $3^{2}7\cdot13\cdot41\cdot163\ \begin{matrix}5\cdot977\\5867\end{matrix}$
(12) $3^{2}7^{2}11\cdot13\ \begin{matrix}41\cdot461\\19403\end{matrix}$
(13) $3^{2}7^{2}13\ \begin{matrix}5\cdot41\\251\end{matrix}$
(14) $3^{2}7^{2}13\cdot97\ \begin{matrix}5\cdot193\\1163\end{matrix}$
(15) $3^{4}5\cdot11\ \begin{matrix}29\cdot89\\2699\end{matrix}$
(16) $3^{5}7^{2}13\cdot53\ \begin{matrix}11\cdot211\\2543\end{matrix}$

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(17) $2^{5}\ \begin{matrix}257\cdot33023\\8520191\end{matrix}$ (Legendre)
(18) $2^{7}263\ \begin{matrix}4271\cdot280883\\1199936447\end{matrix}$ (Poulet)
(19) $2^{7}467\ \begin{matrix}281\cdot2107103\\594203327\end{matrix}$ (Poulet)
(20) $3^{2}7^{2}13\cdot19\cdot23\ \begin{matrix}83\cdot1931\\162287\end{matrix}$ (Seelhoff)
(21) $3^{2}5^{2}13\cdot31\ \begin{matrix}149\cdot449\\67499\end{matrix}$ (Poulet and Gerardin)
(22) $3^{3}5^{3}13\ \begin{matrix}149\cdot449\\67499\end{matrix}$ (Poulet and Gerardin)
(23) $3^{4}5\cdot11\cdot59\ \begin{matrix}89\cdot5309\\477899\end{matrix}$ (Mason)

(Escott)

(24) $2\cdot5\cdot11\cdot61\ \begin{matrix}239\cdot161039\\38649599\end{matrix}$
(25) $2\cdot5^{2}31\cdot79\ \begin{matrix}17\cdot7109\\127979\end{matrix}$
(26) $3^{2}5\cdot7\cdot107\ \begin{matrix}3209\cdot4493\\14425739\end{matrix}$
(27) $3^{2}7^{2}13\cdot17\cdot23\ \begin{matrix}1931\cdot4691\\9064943\end{matrix}$
(28) $3^{2}7^{2}13\cdot19\cdot23\ \begin{matrix}83\cdot1931\\162287\end{matrix}$
(29) $3^{4}7\cdot11^{2}17\cdot271\ \begin{matrix}179\cdot5419\\975599\end{matrix}$
(30) $3^{5}7^{2}13\cdot17\cdot271\ \begin{matrix}179\cdot5419\\975599\end{matrix}$
(31) $3^{4}5\cdot11^{2}71\ \begin{matrix}709\cdot2129\\1512299\end{matrix}$
(32) $3^{4}5\cdot11\cdot149\ \begin{matrix}31\cdot14303\\443423\end{matrix}$
(33) $3^{2}7^{2}13\cdot19\cdot29\ \begin{matrix}41\cdot173\\7307\end{matrix}$

Form $E\ \begin{matrix}pqr\\s\end{matrix}$

(Escott)

(34) $2^{3}31\ \begin{matrix}17\cdot107\cdot4339\\8436959\end{matrix}$
(35) $3^{2}7^{2}13\ \begin{matrix}5\cdot43\cdot167\\44351\end{matrix}$
(36) $2^{4}\ \begin{matrix}17\cdot151\cdot1283\\3513023\end{matrix}$
(37) $3^{2}7^{2}13\ \begin{matrix}5\cdot53\cdot97\\31751\end{matrix}$
(38) $2^{4}\ \begin{matrix}17\cdot167\cdot709\\2147039\end{matrix}$
(39) $3^{2}7^{2}13\cdot19\ \begin{matrix}11\cdot79\cdot2029\\1948799\end{matrix}$
(40) $2^{4}\ \begin{matrix}19\cdot137\cdot167\\463679\end{matrix}$

Form $E\ \begin{matrix}pq\\rs\end{matrix}$

(Euler)

(41) $2^{2}\ \begin{matrix}5\cdot131\\17\cdot43\end{matrix}$
(42) $2^{2}\ \begin{matrix}5\cdot251\\13\cdot107\end{matrix}$
(43) $2^{3}\ \begin{matrix}17\cdot79\\23\cdot59\end{matrix}$
(44) $2^{4}\ \begin{matrix}17\cdot5119\\239\cdot383\end{matrix}$
(45) $2^{4}\ \begin{matrix}19\cdot1439\\149\cdot191\end{matrix}$
(46) $2^{4}\ \begin{matrix}23\cdot1367\\53\cdot607\end{matrix}$
(47) $2^{4}\ \begin{matrix}23\cdot467\\103\cdot107\end{matrix}$
(48) $2^{4}\ \begin{matrix}23\cdot479\\89\cdot127\end{matrix}$
(49) $2^{4}\ \begin{matrix}47\cdot89\\53\cdot79\end{matrix}$
(50) $2^{5}\ \begin{matrix}37\cdot12671\\227\cdot2111\end{matrix}$
(51) $2^{5}\ \begin{matrix}59\cdot1103\\79\cdot827\end{matrix}$
(52) $2^{6}\ \begin{matrix}79\cdot11087\\383\cdot9203\end{matrix}$
(53) $2^{5}\ \begin{matrix}383\cdot9203\\1151\cdot3067\end{matrix}$
(54) $2^{4}67\ \begin{matrix}37\cdot2411\\227\cdot401\end{matrix}$
(55) $2^{5}\ \begin{matrix}53\cdot10559\\79\cdot7127\end{matrix}$
(56) $3^{2}5\cdot13\ \begin{matrix}19\cdot47\\29\cdot31\end{matrix}$
(57) $3^{2}5\cdot13\cdot19\ \begin{matrix}37\cdot1583\\227\cdot263\end{matrix}$
(58) $2^{2}11\ \begin{matrix}17\cdot263\\43\cdot107\end{matrix}$
(59) $2^{4}\ \begin{matrix}17\cdot10303\\167\cdot1103\end{matrix}$
(60) $3^{4}7\cdot11^{2}19\ \begin{matrix}47\cdot7019\\389\cdot863\end{matrix}$
(61) $3^{5}7^{2}13\cdot19\ \begin{matrix}47\cdot7019\\389\cdot863\end{matrix}$
(62) $3^{2}5\cdot13\ \begin{matrix}11\cdot199\\29\cdot79\end{matrix}$
(63) $3^{2}5\cdot19\ \begin{matrix}7\cdot227\\37\cdot47\end{matrix}$
(64) $3^{3}5\ \begin{matrix}7\cdot71\\17\cdot31\end{matrix}$
(65) $3^{4}7\cdot11^{2}19\ \begin{matrix}53\cdot6959\\179\cdot2087\end{matrix}$
(66) $3^{5}7^{2}13\cdot19\ \begin{matrix}53\cdot6959\\179\cdot2087\end{matrix}$

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(67) $2^{6}\ \begin{matrix}139\cdot863\\167\cdot719\end{matrix}$ (Seelhoff)
(68) $2\cdot5^{2}31\ \begin{matrix}19\cdot359\\47\cdot149\end{matrix}$ (Mason)
(69) $3^{2}7^{2}13\cdot19\ \begin{matrix}11\cdot10499\\89\cdot1399\end{matrix}$ (Mason)
(70) $3^{4}5\cdot11\ \begin{matrix}41\cdot599\\59\cdot419\end{matrix}$ (Mason)

EDW

(71) $2^{6}\ \begin{matrix}73\cdot264959\\479\cdot40847\end{matrix}$
(72) $2^{7}\ \begin{matrix}137\cdot99839\\2879\cdot4783\end{matrix}$
(73) $2^{7}\ \begin{matrix}179\cdot736447\\443\cdot298559\end{matrix}$
(74) $2^{8}\ \begin{matrix}263\cdot109919\\16487\cdot17599\end{matrix}$
(75) $2^{8}\ \begin{matrix}263\cdot36227327\\8513\cdot1123327\end{matrix}$
(76) $2^{8}\ \begin{matrix}269\cdot4755967\\5039\cdot254783\end{matrix}$
(77) $2^{8}\ \begin{matrix}293\cdot58367\\3583\cdot4787\end{matrix}$
(78) $2^{8}\ \begin{matrix}311\cdot3062399\\1429\cdot668159\end{matrix}$
(79) $2^{8}\ \begin{matrix}383\cdot7643\\1567\cdot1871\end{matrix}$
(80) $2^{3}19\ \begin{matrix}67\cdot1367\\101\cdot911\end{matrix}$ (Gerardin)
(81) $2^{3}29\ \begin{matrix}19\cdot2087\\173\cdot239\end{matrix}$ (Poulet and Gerardin)
(82) $2^{3}37\ \begin{matrix}101\cdot348628799\\3019\cdot11774879\end{matrix}$

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(107) $2\cdot5\cdot11\ \begin{matrix}53\cdot1759\\59\cdot1583\end{matrix}$
(108) $2\cdot5\cdot31\ \begin{matrix}7\cdot30689\\59\cdot4091\end{matrix}$
(109) 2[illegible]349 $\begin{matrix}17\cdot150767\\971\cdot2791\end{matrix}$
(110) $2^{5}79\ \begin{matrix}227\cdot10427\\631\cdot3761\end{matrix}$
(111) $2^{5}3593\ \begin{matrix}37\cdot22765247\\227\cdot3794207\end{matrix}$
(112) $2^{6}131\ \begin{matrix}2357\cdot6436223\\19387\cdot782783\end{matrix}$
(113) $2^{6}131\ \begin{matrix}3373\cdot132047\\6287\cdot70853\end{matrix}$
(114) $2^{7}337\ \begin{matrix}673\cdot9104399\\2699\cdot2272727\end{matrix}$
(115) $2^{9}1087\ \begin{matrix}13043\cdot536423\\31247\cdot223921\end{matrix}$
(116) $2^{9}1087\ \begin{matrix}15217\cdot2647943\\20663\cdot1950077\end{matrix}$
(117) $2^{9}1279\ \begin{matrix}5867\cdot25579\\7673\cdot19559\end{matrix}$
(118) $2^{10}\ \begin{matrix}1279\cdot4725863\\5147\cdot1175039\end{matrix}$
(119) $2^{10}\ \begin{matrix}1279\cdot126359\\6911\cdot23399\end{matrix}$
(120) $2^{10}\ \begin{matrix}1279\cdot125063\\6947\cdot23039\end{matrix}$
(121) $2^{12}\ \begin{matrix}5119\cdot1013687\\23039\cdot225263\end{matrix}$
(122) $2^{12}\ \begin{matrix}6143\cdot187067\\16127\cdot71263\end{matrix}$
(123) $2^{12}\ \begin{matrix}6143\cdot7610483\\12347\cdot3786751\end{matrix}$
(124) $3^{2}5\cdot13\cdot19\ \begin{matrix}29\cdot44687\\1063\cdot1259\end{matrix}$

(Poulet)

(71)	2^{6}	73·264959 / 479·40847
(72)	2^{7}	137·99839 / 2879·4783
(73)	2^{7}	179·736447 / 443·298559
(74)	2^{8}	263·109919 / 16487·17599
(75)	2^{8}	263·36227327 / 8513·1123327
(76)	2^{8}	269·4755967 / 5039·254783
(77)	2^{8}	293·58367 / 3583·4787
(78)	2^{8}	311·3062399 / 1429·668159
(79)	2^{8}	383·7643 / 1567·1871
(80)	$2^{3}19$	67·1367 / 101·911 (Gerardin)
(81)	$2^{3}29$	19·2087 / 173·239 (Poulet and Gerardin)
(82)	$2^{3}37$	101·348628799 / 3019·11774879
(83)	$2^{4}37$	151·281717 / 281·151847
(84)	$2^{4}37$	179·5623 / 239·4217
(85)	$2^{4}41$	71·2707631 / 761·25583
(86)	$2^{4}43$	61·98899 / 859·7129
(87)	$2^{4}43$	67·104059 / 373·18919
(88)	$2^{4}43$	89·4987 / 173·2579
(89)	$2^{4}67$	29·2215823 / 6967·69529
(90)	$2^{4}131$	23·5501 / 251·523
(91)	$2^{4}157$	19·2392403 / 6907·69259
(92)	$2^{4}281$	17·102387407 / 30347·60727
(93)	$2^{4}281$	17·25626356999 / 20249·2277898
(94)	$2^{4}331$	19·6619 / 199·661
(95)	$2^{4}683$	29·155723 / 37·122939
(96)	$2^{4}997$	17·10767599 / 199·969083
(97)	$2^{4}2137$	17·119671 / 167·12821
(98)	$2^{4}7639$	17·381949 / 149·45833
(99)	$3^{2}5^{2}13\cdot31$	191·589049 / 271·415799
(100)	$3^{3}5^{3}13$	191·589049 / 271·415799
(101)	$3^{2}7\cdot11\cdot13$	23·7523 / 53·3343
(102)	$3^{3}5\cdot11$	23·7523 / 53·3343
(103)	$3^{2}7\cdot11\cdot13$	23·659 / 79·197
(104)	$3^{3}5\cdot11$	23·659 / 79·197
(105)	$3^{4}7\cdot11^{2}19$	89·503 / 107·419
(106)	$3^{5}7^{2}13\cdot19$	89·503 / 107·419

(Escott)

(107)	$2\cdot5\cdot11$	53·1759 / 59·1583
(108)	$2\cdot5\cdot31$	7·30689 / 59·4091
(109)	$2^{4}349$	17·150767 / 971·2791
(110)	$2^{5}79$	227·10427 / 631·3761
(111)	$2^{5}3593$	37·22765247 / 227·3794207
(112)	$2^{6}131$	2357·6436223 / 19387·782783
(113)	$2^{6}131$	3373·132047 / 6287·70853
(114)	$2^{7}337$	673·9104399 / 2699·2272727
(115)	$2^{9}1087$	13043·536423 / 31247·223921
(116)	$2^{9}1087$	15217·2647943 / 20663·1950077
(117)	$2^{9}1279$	5867·25579 / 7673·19559
(118)	2^{10}	1279·4725863 / 5147·1175039
(119)	2^{10}	1279·126359 / 6911·23399
(120)	2^{10}	1279·125063 / 6947·23039
(121)	2^{12}	5119·1013687 / 23039·225263
(122)	2^{12}	6143·187067 / 16127·71263
(123)	2^{12}	6143·7610483 / 12347·3786751
(124)	$3^{2}5\cdot13\cdot19$	29·44687 / 1063·1259
(125)	$3^{2}5\cdot13\cdot41$	11·2686319 / 223·143909
(126)	$3^{2}5\cdot31$	7·929 / 11·619
(127)	$3^{2}5^{2}11\cdot43$	2579·133979 / 4729·73079
(128)	$3^{2}5^{4}11\cdot71$	499·280979 / 839·167249
(129)	$3^{2}7\cdot11\cdot13\cdot43$	67·874619 / 1289·46103
(130)	$3^{3}5\cdot11\cdot43$	67·874619 / 1289·46103
(131)	$3^{2}7^{2}11\cdot13$	41·1286107 / 463·116423
(132)	$3^{4}5\cdot11\cdot47$	563·9859 / 1409·3943
(133)	$3^{4}5\cdot11\cdot59$	89·1691647 / 7079·21503
(134)	$3^{4}7\cdot11^{2}17$	101·64271 / 311·21011
(135)	$3^{5}5^{2}13\cdot17$	101·64271 / 311·21011
(136)	$3^{4}7\cdot11^{2}17$	101·4799 / 479·1019
(137)	$3^{5}5^{2}13\cdot17$	101·4799 / 479·1019
(138)	$3^{4}7\cdot11^{2}19\cdot83$	1493·163199 / 5099·47807
(139)	$3^{5}7^{2}13\cdot19\cdot83$	1493·163199 / 5099·47807
(140)	$3^{4}7\cdot11^{2}19\cdot3229$	53·774959 / 179·232487
(141)	$3^{5}7^{2}13\cdot19\cdot3229$	53·774959 / 179·232487

Form E $\frac{pqr}{st}$

(Euler)

No.	E	pqr	st
(142)	2^2	5·13·1187	43·2267
(143)	2^3	11·23·1619	647·719
(144)	2^3	11·23·1871	467·1151
(145)	2^3	11·23·2543	383·1907
(146)	2^3	11·29·239	191·449
(147)	2^3	11·163·191	31·11807
(148)	2^3	11·59·173	47·2609
(149)	2^3	29·47·59	17·4799
(150)	2^4	17·167·13679	809·51071
(151)	2^4	23·47·9767	1583·7103
(152)	$2 \cdot 5$	23·29·673	7·60659
(153)	$2 \cdot 5$	7·19·107	47·359
(154)	$3^2 5$	7·11·29	31·89
(155)	$3^2 7 \cdot 13$	5·17·1187	131·971
(156)	$3^3 7 \cdot 13 \cdot 23$	11·19·367	79·1103
(157)	$3^3 5 \cdot 23$	11·19·367	79·1103
(158)	$3^3 5$	17·23·397	7·21491

(Mason)

No.	E	pqr	st
(159)	2^3	11·31·233	127·701
(160)	2^3	11·31·2099	79·10079
(161)	2^3	11·41·173	71·1217
(162)	2^3	13·23·149	199·251
(163)	2^3	13·23·251	97·863
(164)	2^3	13·23·1109	71·5179
(165)	2^3	17·19·281	53·1879
(166)	$3^2 7^2 13 \cdot 19$	17·23·1335949	3079·187379

(Dickson)

No.	E	pqr	st
(167)	2^4	17·137·262079	12959·50231
(168)	2^4	17·137·2990783	10103·735263

(Poulet)

No.	E	pqr	st
(169)	$2 \cdot 7 \cdot 11$	13·19·10889	83·36299
(170)	$2 \cdot 7 \cdot 11$	13·71·241	23·10163
(171)	2^4	17·139·336247	5279·160481
(172)	2^4	17·149·8887	2221·10799
(173)	2^4	17·191·26249	503·179999
(174)	2^4	17·379·13339	227·400199
(175)	2^4	17·599·5927	191·333449
(176)	2^4	17·727·6269	181·451439
(177)	2^4	17·811·27919	173·2345279
(178)	2^4	19·79·10691	2591·6599
(179)	2^4	19·83·3719	1399·4463
(180)	2^4	19·131·3359	199·44351
(181)	2^4	19·163·233	491·1559
(182)	2^4	23·47·13999	1398·11519
(183)	2^4	23·59·571	359·2287
(184)	2^4	23·107·17807	83·549503
(185)	2^4	29·59·139	179·1399
(186)	2^4	29·191·293	47·35279
(187)	2^4	37·43·107	263·683
(188)	2^4	37·79·4787	41·346559
(189)	2^4	43·47·1097	53·42943
(190)	2^4	23·61·3299	197·24799
(191)	2^4	47·167·389	29·104831
(192)	2^4	59·359·683	23·615599
(193)	2^4	101·367·185429	19·348015023
(194)	2^4	179·743·19001	17·141374879
(195)	2^4	179·797·6959	17·55540799

(Escott)

No.	E	pqr	st
(196)	$2 \cdot 5$	7·11·11369	757·1439
(197)	$2 \cdot 5$	11·13·809	19·6803
(198)	$2 \cdot 5$	19·47·179	7·21599
(199)	$2 \cdot 5^2 19$	29·113·151637	3581·144779
(200)	$2 \cdot 5^2 19$	29·113·77417	3761·70379
(201)	$2 \cdot 5^2 19$	37·61·3693689	12011·724469
(202)	$2 \cdot 5^2 19$	37·61·404189	14969·63611
(203)	$2 \cdot 5^2 31$	13·109·319679	9239·53279
(204)	$2 \cdot 5^2 31$	13·149·1097	853·2699
(205)	$2 \cdot 5^2 31$	19·59·599	79·8999
(206)	$2^2 11$	13·47·1407449	7919·119419
(207)	$2^2 13$	19·97·7019	17·764399
(208)	$2^2 23$	5·137·17327	911·15731
(209)	$2^2 23$	5·137·11177	971·9521

No.	E	pqr	st
(210)	$2^3 17$	71·1223·1172663	10547·980423
(211)	$2^3 17$	71·1223·5025239	91367·4847039
(212)	$2^3 17$	71·1223·8663003	89963·8486207
(213)	$2^3 19$	47·179·1883051	24623·660719
(214)	$2^3 23$	29·137·2887	359·33211
(215)	$2^3 31$	23·61·449	199·3347
(216)	2^4	17·167·114299	761·453599
(217)	$3^2 5 \cdot 7$	53·1889·886463	139967·646379
(218)	$3^2 5 \cdot 7$	53·1889·1411829	121013·1190699
(219)	$3^2 5 \cdot 7$	59·419·325939	30319·270899
(220)	$3^2 5 \cdot 7$	59·419·5316959	25439·5266799
(221)	$3^2 5 \cdot 7$	59·419·636473	27449·584303
(222)	$3^2 5 \cdot 7$	59·419·147377	68207·54449
(223)	$3^2 5 \cdot 7$	59·419·2011129	25849·1960559
(224)	$3^2 5 \cdot 7$	59·419·1274249	26249·1223279
(225)	$3^2 5 \cdot 7$	59·419·233279	33599·174959
(226)	$3^2 5 \cdot 7$	59·419·170741	40949·105071
(227)	$3^2 5 \cdot 7$	59·419·182159	38639·118799
(228)	$3^2 5 \cdot 7$	59·419·244199	32999·186479
(229)	$3^2 5 \cdot 7$	83·139·5742623	11807·5719279
(230)	$3^2 5 \cdot 7$	83·139·93683	16879·65267
(231)	$3^2 5 \cdot 7$	83·139·78539	19403·47599
(232)	$3^2 5 \cdot 7$	83·139·108863	15679·81647
(233)	$3^2 5 \cdot 7$	59·461·9337	8819·29347
(234)	$3^2 5 \cdot 7$	53·2099·49633	26891·209299
(235)	$3^2 5 \cdot 7$	83·149·5807	2879·25409
(236)	$3^2 5 \cdot 7$	97·113·25849	12539·23029
(237)	$3^2 5 \cdot 7$	83·149·42767	1889·285119
(238)	$3^2 5 \cdot 7$	53·1931·198769	81971·252979
(239)	$3^2 5 \cdot 7$	53·1931·211319	77279·285281
(240)	$3^2 5 \cdot 13$	11·19·1409	449·751

(210)	$2^3 17$	71·1223·1172663 10547·980423
(211)	$2^3 17$	71·1223·5025239 91367·4847039
(212)	$2^3 17$	71·1223·8663003 89963·8486207
(213)	$2^3 19$	47·179·1883051 24623·660719
(214)	$2^3 23$	29·137·2887 359·33211
(215)	$2^3 31$	23·61·449 199·3347
(216)	2^4	17·167·114299 761·453599
(217)	$3^2 5\cdot 7$	53·1889·886463 139967·646379
(218)	$3^2 5\cdot 7$	53·1889·1411829 121013·1190699
(219)	$3^2 5\cdot 7$	59·419·325939 30319·270899
(220)	$3^2 5\cdot 7$	59·419·5316959 25439·5266799
(221)	$3^2 5\cdot 7$	59·419·636473 27449·584303
(222)	$3^2 5\cdot 7$	59·419·147377 68207·54449
(223)	$3^2 5\cdot 7$	59·419·2011129 25849·1960559
(224)	$3^2 5\cdot 7$	59·419·1274249 26249·1223279
(225)	$3^2 5\cdot 7$	59·419·233279 33599·174959
(226)	$3^2 5\cdot 7$	59·419·170741 40949·105071
(227)	$3^2 5\cdot 7$	59·419·182159 38639·118799
(228)	$3^2 5\cdot 7$	59·419·244199 32999·186479
(229)	$3^2 5\cdot 7$	83·139·5742623 11807·5719279
(230)	$3^2 5\cdot 7$	83·139·93683 16879·65267
(231)	$3^2 5\cdot 7$	83·139·78539 19403·47599
(232)	$3^2 5\cdot 7$	83·139·108863 15679·81647
(233)	$3^2 5\cdot 7$	59·461·9337 8819·29347
(234)	$3^2 5\cdot 7$	53·2099·49633 26891·209299
(235)	$3^2 5\cdot 7$	83·149·5807 2879·25409
(236)	$3^2 5\cdot 7$	97·113·25849 12539·23029
(237)	$3^2 5\cdot 7$	83·149·42767 1889·285119
(238)	$3^2 5\cdot 7$	53·1931·198769 81971·252979
(239)	$3^2 5\cdot 7$	53·1931·211319 77279·285281
(240)	$3^2 5\cdot 13$	11·19·1409 449·751
(241)	$3^2 5\cdot 13$	7·1949·12239 127·2983499
(242)	$3^2 5\cdot 13\cdot 19$	29·569·152459 23099·112859
(243)	$3^2 5\cdot 13\cdot 19$	29·569·113021 28349·68171
(244)	$3^2 5\cdot 13\cdot 19$	29·569·117779 27179·74099
(245)	$3^2 5\cdot 13\cdot 19$	29·569·125113 25849·82763
(246)	$3^2 5\cdot 13\cdot 19$	29·569·289381 19531·253349
(247)	$3^2 5\cdot 13\cdot 19$	37·113·1165187 4363·1156643
(248)	$3^2 5\cdot 13\cdot 19$	37·113·255587 4483·246923
(249)	$3^2 5\cdot 13\cdot 19$	37·113·28499 7219·17099
(250)	$3^4 5\cdot 11$	23·359·161783 9719·143807
(251)	$3^4 5\cdot 11$	23·359·52223 16319·27647
(252)	$3^4 5\cdot 11^2$	53·89·660887 4931·651239
(253)	$3^4 5\cdot 11^2$	53·89·445463 4967·435779
(254)	$3^4 5\cdot 11^2$	53·89·137933 5237·127979
(255)	$3^4 5\cdot 11^2$	53·89·73553 5657·63179
(256)	$3^4 5\cdot 11^2$	53·89·30293 8747·16829
(257)	$3^4 5\cdot 11^2$	53·89·28109 11243·12149
(258)	$3^2 5^2 13\cdot 31$	149·449·6203411 69011·6067499
(259)	$3^3 5^3 13$	149·449·6203411 69011·6067499
(260)	$3^2 5^2 13\cdot 31$	139·569·6594659 287279·1831849
(261)	$3^3 5^3 13$	139·569·6594659 287279·1831849
(262)	$3^2 5^2 13\cdot 31$	149·449·2366099 71699·2227499
(263)	$3^3 5^3 13$	149·449·2366099 71699·2227499
(264)	$3^2 5^2 13\cdot 31$	149·449·1707947 73547·1567499
(265)	$3^3 5^3 13$	149·449·1707947 73547·1567499
(266)	$3^2 5^2 13\cdot 31$	149·449·1558619 74219·1417499
(267)	$3^3 5^3 13$	149·449·1558619 74219·1417499
(268)	$3^2 5^2 13\cdot 31$	149·449·1076399 77999·931499
(269)	$3^3 5^3 13$	149·449·1076399 77999·931499
(270)	$3^2 5^2 13\cdot 31$	149·449·603899 91499·445499
(271)	$3^3 5^3 13$	149·449·603899 91499·445499

(272)	$3^2 5^2 13 \cdot 31$	$149 \cdot 449 \cdot 521399$	$98999 \cdot 355499$
(273)	$3^3 5^3 13$	$149 \cdot 449 \cdot 521399$	$98999 \cdot 355499$
(274)	$3^2 5^2 13 \cdot 31$	$149 \cdot 449 \cdot 438899$	$115499 \cdot 256499$
(275)	$3^3 5^3 13$	$149 \cdot 449 \cdot 438899$	$115499 \cdot 256499$
(276)	$3^2 5^2 13 \cdot 31$	$149 \cdot 449 \cdot 395039$	$148139 \cdot 179999$
(277)	$3^3 5^3 13$	$149 \cdot 449 \cdot 395039$	$148139 \cdot 179999$
(278)	$3^2 5^2 13 \cdot 31$	$149 \cdot 461 \cdot 291857$	$14699 \cdot 1375901$
(279)	$3^3 5^3 13$	$149 \cdot 461 \cdot 291857$	$14699 \cdot 1375901$
(280)	$3^3 5$	$7 \cdot 17 \cdot 59$	$31 \cdot 269$
(281)	$3^2 7 \cdot 11 \cdot 13$	$17 \cdot 197 \cdot 1379069$	$3581 \cdot 1372139$
(282)	$3^3 5 \cdot 11$	$17 \cdot 197 \cdot 137069$	$3581 \cdot 1372139$
(283)	$3^2 7 \cdot 11 \cdot 13$	$17 \cdot 197 \cdot 135089$	$3761 \cdot 127979$
(284)	$3^3 5 \cdot 11$	$17 \cdot 197 \cdot 135089$	$3761 \cdot 127979$
(285)	$3^2 7 \cdot 11 \cdot 13$	$17 \cdot 197 \cdot 49139$	$4211 \cdot 41579$
(286)	$3^3 5 \cdot 11$	$17 \cdot 197 \cdot 49139$	$4211 \cdot 41579$
(287)	$3^2 7 \cdot 11 \cdot 13$	$17 \cdot 197 \cdot 21059$	$7019 \cdot 10691$
(288)	$3^3 5 \cdot 11$	$17 \cdot 197 \cdot 21059$	$7019 \cdot 10691$
(289)	$3^2 7 \cdot 11 \cdot 13$	$19 \cdot 89 \cdot 1590467$	$24097 \cdot 118799$
(290)	$3^3 5 \cdot 11$	$19 \cdot 89 \cdot 1590467$	$24097 \cdot 118799$
(291)	$3^2 7 \cdot 11 \cdot 13$	$19 \cdot 89 \cdot 1205819$	$26729 \cdot 81199$
(292)	$3^3 5 \cdot 11$	$19 \cdot 89 \cdot 1205819$	$26729 \cdot 81199$
(293)	$3^2 7 \cdot 11 \cdot 13$	$19 \cdot 89 \cdot 910909$	$38219 \cdot 42899$
(294)	$3^3 5 \cdot 11$	$19 \cdot 89 \cdot 910909$	$38219 \cdot 42899$
(295)	$3^2 7^2 13 \cdot 19$	$11 \cdot 83 \cdot 83591$	$1031 \cdot 81647$
(296)	$3^2 7^2 13 \cdot 19$	$11 \cdot 83 \cdot 63439$	$1039 \cdot 61487$
(297)	$3^2 7^2 13 \cdot 19$	$11 \cdot 83 \cdot 38821$	$1061 \cdot 36847$
(298)	$3^2 7^2 13 \cdot 19$	$11 \cdot 83 \cdot 5711$	$2351 \cdot 2447$
(299)	$3^3 5^2 31$	$17 \cdot 29 \cdot 193$	$89 \cdot 1163$
(300)	$3^6 5 \cdot 23 \cdot 137 \cdot 547 \cdot 1093$	$17 \cdot 29 \cdot 193$	$89 \cdot 1163$
(301)	$3^{10} 5 \cdot 23 \cdot 107 \cdot 3851$	$17 \cdot 29 \cdot 193$	$89 \cdot 1163$
(302)	$3^3 5^2 31$	$17 \cdot 29 \cdot 223$	$83 \cdot 1439$
(303)	$3^6 5 \cdot 23 \cdot 137 \cdot 547 \cdot 1093$	$17 \cdot 29 \cdot 223$	$83 \cdot 1439$
(304)	$3^{10} 5 \cdot 23 \cdot 107 \cdot 3851$	$17 \cdot 29 \cdot 223$	$83 \cdot 1439$
(305)	$3^4 5 \cdot 11$	$23 \cdot 347 \cdot 2645189$	$46559 \cdot 474497$
(306)	$3^4 5 \cdot 11$	$23 \cdot 349 \cdot 1115759$	$27893 \cdot 335999$
(307)	$3^4 5 \cdot 11$	$23 \cdot 349 \cdot 1377713$	$27299 \cdot 423911$
(308)	$3^4 5 \cdot 11$	$23 \cdot 349 \cdot 982151$	$28349 \cdot 291007$
(309)	$3^4 5 \cdot 11$	$23 \cdot 349 \cdot 572417$	$31859 \cdot 150919$
(310)	$3^4 5 \cdot 11$	$23 \cdot 359 \cdot 52223$	$16319 \cdot 27647$
(311)	$3^4 5 \cdot 11$	$23 \cdot 359 \cdot 161783$	$9719 \cdot 143807$
(312)	$3^4 5 \cdot 11$	$29 \cdot 89 \cdot 1195991$	$2711 \cdot 1190699$
(313)	$3^4 5 \cdot 11$	$29 \cdot 89 \cdot 483209$	$2729 \cdot 477899$
(314)	$3^4 5 \cdot 11$	$29 \cdot 89 \cdot 87359$	$2879 \cdot 81899$
(315)	$3^4 5 \cdot 11$	$29 \cdot 89 \cdot 80177$	$2897 \cdot 74699$
(316)	$3^4 5 \cdot 11$	$29 \cdot 89 \cdot 67619$	$2939 \cdot 62099$
(317)	$3^4 5 \cdot 11$	$29 \cdot 89 \cdot 19889$	$4049 \cdot 13259$
(318)	$3^4 5 \cdot 11$	$29 \cdot 89 \cdot 15913$	$5449 \cdot 7883$
(319)	$3^4 5 \cdot 11$	$29 \cdot 89 \cdot 15749$	$5669 \cdot 7499$
(320)	$3^4 5 \cdot 11$	$37 \cdot 53 \cdot 312799$	$13679 \cdot 46919$
(321)	$3^4 5 \cdot 11$	$37 \cdot 53 \cdot 269749$	$14939 \cdot 37049$
(322)	$3^4 7 \cdot 11^2 19$	$71 \cdot 179 \cdot 239$	$1151 \cdot 2699$
(323)	$3^5 7^2 13 \cdot 19$	$71 \cdot 179 \cdot 239$	$1151 \cdot 2699$
(324)	$3^4 7 \cdot 11^2 19$	$53 \cdot 167 \cdot 9931$	$6047 \cdot 14897$
(325)	$3^5 7^2 13 \cdot 19$	$53 \cdot 167 \cdot 9931$	$6047 \cdot 14897$
(326)	$3^3 5^2 19 \cdot 31$	$138053 \cdot 167039$	$359 \cdot 911 \cdot 70237$
(327)	$3^4 7 \cdot 11^2 19^2 127$	$138053 \cdot 167039$	$359 \cdot 911 \cdot 70237$
(328)	$3^5 7^2 13 \cdot 19^2 127$	$138053 \cdot 167039$	$359 \cdot 911 \cdot 70237$
(329)	$3^6 5 \cdot 19 \cdot 23 \cdot 137 \cdot 547 \cdot 1093$	$138053 \cdot 167039$	$359 \cdot 911 \cdot 70237$
(330)	$3^{10} 5 \cdot 19 \cdot 23 \cdot 107 \cdot 3851$	$138053 \cdot 167039$	$359 \cdot 911 \cdot 70237$
(331)	$3^3 5^2 19 \cdot 31$	$61559 \cdot 565247$	$359 \cdot 911 \cdot 105983$
(332)	$3^4 7 \cdot 11^2 19^2 127$	$61559 \cdot 565247$	$359 \cdot 911 \cdot 105983$
(333)	$3^5 7^2 13 \cdot 19^2 127$	$61559 \cdot 565247$	$359 \cdot 911 \cdot 105983$

(334) $3^6 5\cdot 19\cdot 23\cdot 137\cdot 547\cdot 1093\,\genfrac{}{}{0pt}{}{61559\cdot 565247}{359\cdot 911\cdot 105983}$

(335) $3^{10} 5\cdot 19\cdot 23\cdot 107\cdot 3851\,\genfrac{}{}{0pt}{}{61559\cdot 565247}{359\cdot 911\cdot 105983}$

Form $E\,\genfrac{}{}{0pt}{}{pqr}{stu}$

(Escott)

(336) $2\cdot 5\,\genfrac{}{}{0pt}{}{7\cdot 863\cdot 2579}{23\cdot 29\cdot 24767}$

(337) $2\cdot 5\,\genfrac{}{}{0pt}{}{11\cdot 19\cdot 115877}{17\cdot 61\cdot 24919}$

(338) $2\cdot 7\cdot 11\,\genfrac{}{}{0pt}{}{13\cdot 43\cdot 13499}{29\cdot 359\cdot 769}$

(339) $2^2\,\genfrac{}{}{0pt}{}{5\cdot 71\cdot 2580241}{13\cdot 3163\cdot 25163}$

(340) $2^2\,\genfrac{}{}{0pt}{}{5\cdot 71\cdot 2162641}{13\cdot 3083\cdot 31643}$

(341) $2^2 11\,\genfrac{}{}{0pt}{}{13\cdot 35591\cdot 890999}{53\cdot 1049\cdot 7830239}$

(342) $2^3\,\genfrac{}{}{0pt}{}{17\cdot 19\cdot 270143}{27\cdot 1259\cdot 1607}$

(343) $2^3\,\genfrac{}{}{0pt}{}{11\cdot 113\cdot 4113059}{29\cdot 3527\cdot 53161}$

(344) $2^3\,\genfrac{}{}{0pt}{}{11\cdot 911\cdot 42499}{37\cdot 67\cdot 179999}$

(345) $2^3\,\genfrac{}{}{0pt}{}{17\cdot 19\cdot 291199}{57\cdot 1091\cdot 1999}$

(346) $2^3\,\genfrac{}{}{0pt}{}{17\cdot 19\cdot 591623}{47\cdot 809\cdot 5477}$

(347) $2^4\,\genfrac{}{}{0pt}{}{29\cdot 59\cdot 737279}{79\cdot 2399\cdot 6911}$

(348) $2^4\,\genfrac{}{}{0pt}{}{43\cdot 53\cdot 662591}{47\cdot 1217\cdot 26927}$

(349) $3^3 5^2 31\,\genfrac{}{}{0pt}{}{19\cdot 23\cdot 8438201}{79\cdot 613\cdot 82457}$

(350) $3^3 5^2 31\,\genfrac{}{}{0pt}{}{11\cdot 107\cdot 491299}{89\cdot 2039\cdot 3467}$

(351) $3^6 5\cdot 23\cdot 137\cdot 547\cdot 1093\,\genfrac{}{}{0pt}{}{11\cdot 107\cdot 491299}{89\cdot 2039\cdot 3467}$

(352) $3^3 5^2 31\,\genfrac{}{}{0pt}{}{19\cdot 4643\cdot 35831}{29\cdot 47\cdot 2311163}$

(353) $3^6 5\cdot 23\cdot 137\cdot 547\cdot 1093\,\genfrac{}{}{0pt}{}{19\cdot 4643\cdot 35831}{29\cdot 47\cdot 2311163}$

(354) $3^{10} 5\cdot 23\cdot 107\cdot 3851\,\genfrac{}{}{0pt}{}{19\cdot 4643\cdot 35831}{29\cdot 47\cdot 2311163}$

(355) $3^5 7\cdot 13\,\genfrac{}{}{0pt}{}{11\cdot 29\cdot 535679}{23\cdot 1487\cdot 5399}$

(356) $3^7 5\cdot 41\,\genfrac{}{}{0pt}{}{11\cdot 29\cdot 535679}{23\cdot 1487\cdot 5399}$

(Euler)

(357) $3^2 5^2\,\genfrac{}{}{0pt}{}{11\cdot 59\cdot 179}{17\cdot 19\cdot 359}$

Form $E\,\genfrac{}{}{0pt}{}{pqrs}{tu}$

(Escott)

(358) $2^3\,\genfrac{}{}{0pt}{}{11\cdot 29\cdot 79\cdot 264599}{37799\cdot 201599}$

(359) $2^3\,\genfrac{}{}{0pt}{}{11\cdot 29\cdot 79\cdot 211499}{42299\cdot 143999}$

(360) $2^3\,\genfrac{}{}{0pt}{}{11\cdot 29\cdot 79\cdot 182239}{48239\cdot 108799}$

(361) $2^3\,\genfrac{}{}{0pt}{}{11\cdot 29\cdot 79\cdot 292979}{36479\cdot 231299}$

(362) $2^3\,\genfrac{}{}{0pt}{}{13\cdot 23\cdot 59\cdot 1117079}{20879\cdot 1078559}$

(363) $2^3\,\genfrac{}{}{0pt}{}{13\cdot 23\cdot 59\cdot 865261}{21059\cdot 826561}$

(364) $2^3\,\genfrac{}{}{0pt}{}{13\cdot 23\cdot 59\cdot 325439}{23039\cdot 284759}$

(365) $2^3\,\genfrac{}{}{0pt}{}{13\cdot 23\cdot 59\cdot 354551}{22751\cdot 314159}$

(366) $2^3\,\genfrac{}{}{0pt}{}{13\cdot 23\cdot 59\cdot 210143}{25343\cdot 167159}$

(367) $2^3\,\genfrac{}{}{0pt}{}{13\cdot 23\cdot 59\cdot 114859}{41759\cdot 55439}$

(368) $2^3\,\genfrac{}{}{0pt}{}{13\cdot 23\cdot 59\cdot 148367}{29567\cdot 101159}$

(369) $2^3\,\genfrac{}{}{0pt}{}{13\cdot 17\cdot 443\cdot 3434129}{119447\cdot 3216779}$

(370) $2^3\,\genfrac{}{}{0pt}{}{13\cdot 17\cdot 449\cdot 79159}{55411\cdot 161999}$

(371) $2^3\,\genfrac{}{}{0pt}{}{11\cdot 23\cdot 257\cdot 44893}{24767\cdot 134681}$

Form $E\,\genfrac{}{}{0pt}{}{pqrs}{tuv}$

(Mason)

(372) $3^2 5^2 31\,\genfrac{}{}{0pt}{}{29\cdot 41\cdot 43\cdot 59}{19\cdot 131\cdot 1259}$

(373) $3^3 5^3\,\genfrac{}{}{0pt}{}{29\cdot 41\cdot 43\cdot 59}{19\cdot 131\cdot 1259}$

Miscellaneous Forms

(374) $\genfrac{}{}{0pt}{}{2^5 37}{2\cdot 5\cdot 11^2}$ (Paganini)

(375) $\genfrac{}{}{0pt}{}{2^3 19\cdot 41}{2^5 199}$ (Euler)

(376) $\genfrac{}{}{0pt}{}{2^3 41\cdot 467}{2^5 19\cdot 233}$ (Euler)

(377) $\genfrac{}{}{0pt}{}{2^3 41\cdot 3923}{2^5 17\cdot 2179}$ (Gerardin)

(378) $\genfrac{}{}{0pt}{}{2^3 17\cdot 58211}{2^5 43\cdot 5669}$ (Gerardin)

(379) $\genfrac{}{}{0pt}{}{2^3 107\cdot 15581}{2^5 13\cdot 28619}$ (Gerardin)

(380) $\begin{matrix} 2^5 67 \cdot 11959 \\ 2^7 311 \cdot 643 \end{matrix}$ (Gerardin)

(381) $\begin{matrix} 2^5 349 \cdot 10607 \\ 2^7 59 \cdot 15287 \end{matrix}$ (Poulet and Gerardin)

(382) $3^2 5 \begin{matrix} 7 \cdot 769 \cdot 860813 \\ 7^2 9239 \cdot 10067 \end{matrix}$ (Poulet)

(383) $3^2 5 \begin{matrix} 7 \cdot 769 \cdot 2117663 \\ 7^2 5417 \cdot 42239 \end{matrix}$ (Poulet)

(384) $\begin{matrix} 2^3 13 \cdot 173 \cdot 29021 \\ 2^5 2029 \cdot 8291 \end{matrix}$ (Escott)

(385) $\begin{matrix} 2^3 13 \cdot 157 \cdot 3277869 \\ 2^5 14051 \cdot 130349 \end{matrix}$ (Escott)

(386) $\begin{matrix} 2^3 3037 \cdot 4751627 \\ 2^5 13 \cdot 97 \cdot 2505109 \end{matrix}$ (Escott)

(387) $3^2 5 \begin{matrix} 7 \cdot 797 \cdot 4019 \\ 7^2 450239 \end{matrix}$ (Escott)

(388) $3^2 5 \begin{matrix} 7 \cdot 1091 \cdot 1709 \\ 7^2 262079 \end{matrix}$ (Escott)

(389) $3^4 11^2 19 \begin{matrix} 7 \cdot 50599 \cdot 120041 \\ 7^2 137 \cdot 6177599 \end{matrix}$ (Escott)

(390) $\begin{matrix} 3^3 5 \cdot 7 \cdot 13 \\ 3 \cdot 5 \cdot 7 \cdot 139 \end{matrix}$ (B. H. Brown)

CURIOSA

117. Martin's Problem. The solutions of Martin's problem discussed by J. Ginsburg. SCRIPTA MATHEMATICA, v. XI, 1945, p. 191, are of two types: that illustrated by Martin, in which the hypotenuses of three Pythagorean triangles form a new right triangle and that illustrated by Ginsburg, in which not only the hypotenuses but also corresponding legs form right triangles.

The Martin problem seems to be more prolific than originally supposed. For example, for the great hypotenuse 65 there are the following solutions:

Ginsburg	Martin	Martin	Martin
15, 20, 25	15, 20, 25	15, 20, 25	15, 20, 25
36, 48, 60	36, 48, 60	36, 48, 60	36, 48, 60
39, 52, 65	33, 56, 65	16, 63, 65	25, 60, 65

Martin	Martin	Martin	Martin
7, 24, 25	7, 24, 25	7, 24, 25	7, 24, 25
36, 48, 60	36, 48, 60	36, 48, 60	36, 48, 60
39, 52, 65	16, 63, 65	33, 56, 65	25, 60, 65

Martin	Martin	Martin	Martin
15, 36, 39	15, 36, 39	15, 36, 39	15, 36, 39
20, 48, 52	20, 48, 52	20, 48, 52	20, 48, 52
39, 52, 65	36, 63, 65	33, 56, 65	25, 60, 65

If the great hypotenuse is a product of two equal prime numbers, there is one solution of each type. For example:

Ginsburg	Martin
9, 12, 15	9, 12, 15
12, 16, 20	12, 16, 20
15, 20, 25	7, 24, 25

HINGHAM, MASS. GEORGE S. TERRY

SEQUENCES

BY PAUL ERI

SUPPOSE n one's and
ranged in a series.
For example, when n =
possible:

$1 + 1 - 1$
$1 - 1 + 1$
$1 - 1 - 1$

The sum of any of these s
by breaking off a series a
in any case it lies betwee
gation being made by one
in how many of the arrang
Of the 6 arrangements
Similarly, of the 20 arra
ceptable:

1 +
1 +
1 +
1 −
1 −

and of the 70 arrangemen
good ones. It is now e
${}_{2n}C_n/(n + 1)$ of the ${}_{2n}C_n$
It is a curious fact tha
to be wise to generalize
one's and let it be requir
Let us denote by $f(m, n)$
condition. If $m > n +$
sum cannot fulfill the con

$f(m,$

If $m = n$ or $n + 1$, we