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JRAM 221 (1966)

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Applying Lemma 5 to G'_7 with U^6 as Y, we get $R_\infty[G(K(7^\infty)/Q)] = GL_2(Z_7)$. As a simp… consequence, we notice that $G(K(7^n)/Q)$ is isomorphic to $GL_2(Z/7^nZ)$ for every posit… integer n. The verification for larger l's is left to the voluntary reader.

8. Concluding remarks

If the curve V_q is of genus > 1, we have to consider the Jacobian variety W… of V_q. Then the reciprocity law for the fields generated by the coordinates of the poin… of finite order on W_q can be described in terms of the eigen-values of Hecke operators[9]. We can prove an analogous result for algebraic curves uniformized by automorp… functions belonging to an indefinite quaternion algebra [16]. The eigen-values of He… operators can be obtained by the trace-formula of Eichler and Selberg. In any ca… the determination of Hasse zeta function of an algebraic curve includes, as a natu… consequence, a reciprocity law of certain algebraic extensions, though we have … characterization, other than the properties such as given in our theorem, of th… extensions, except for the case of complex multiplication (of dimension $\geqq 1$). T… subject is closely connected with the theory of automorphic forms with respect … an arithmetically defined discontinuous group. In the investigations [5], [15], [16], on… cusp forms of weight 2 came into the problem. Now it can be shown [12] that the aut… morphic forms of higher weight are also connected with the zeta function of an algebr… variety, the discontinuous group being a unit group in an indefinite division quatern… algebra. In this case, M. Kuga [11] has obtained an interesting result; namely, the eig… values of Hecke operators for such forms are again related to the decomposition of prim… in the number fields of our type.

Needless to say we should aim at putting all these results into one unified theor… Even though we are, at present, far from the completion of the task, it is quite cert… that here is a vast fertile plain in number theory, little of which has been brought und… cultivation.

2070 Table of c_p for $p < 2000$; $\sum_{m=1}^{\infty} c_m x^m = x \cdot \prod_{n=1}^{\infty} (1 - x^n)^2 (1 - x^{11n})^2$.[10]

ignore signs

p	c_p	p	c_p	p	c_p	p	c_p	p	c_p	p	c_p
2	—2	239	—30	563	4	887	—22	1259	—25	1619	-20
3	—1	241	—8	569	0	907	—12	1277	—47	1621	22
5	1	251	—23	571	—28	911	12	1279	—15	1627	78
7	—2	257	—2	577	33	919	10	1283	—36	1637	33
11	1	263	14	587	28	929	—30	1289	0	1657	-2
13	4	269	10	593	44	937	8	1291	—8	1663	4
17	—2	271	—28	599	40	941	42	1297	48	1667	48
19	0	277	—2	601	2	947	—27	1301	27	1669	50
23	—1	281	—18	607	—22	953	34	1303	39	1693	-6
29	0	283	4	613	—16	967	—32	1307	28	1697	-42
31	7	293	24	617	18	971	47	1319	—30	1699	40
37	3	307	8	619	—25	977	—27	1321	47	1709	-45
41	—8	311	12	631	7	983	39	1327	68	1721	-3

[9]) In certain cases, the Jacobian variety W_q turns out to be simple and of dimension > 1 (cf. [4]. [1… It will be interesting to determine the Galois groups of the fields analogous to $K(l)$ in these cases.

[10]) I wish to acknowledge my gratitude to H. F. Trotter for making the table by an electronic comp…

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	p	c_p	p	c_p	p	c_p	p	c_p	p	c_p
—6	313	—1	641	—33	991	—8	1361	12	1723	—46
8	317	13	643	29	997	38	1367	—72	1733	—6
6	331	7	647	—7	1009	—10	1373	39	1741	17
5	337	—22	653	—41	1013	39	1381	—68	1747	—57
12	347	28	659	10	1019	—10	1399	60	1753	34
7	349	30	661	37	1021	22	1409	—15	1759	—40
3	353	—21	673	14	1031	32	1423	29	1777	8
4	359	—20	677	—42	1033	—16	1427	—12	1783	59
10	367	—17	683	—16	1039	5	1429	—70	1787	—57
—6	373	—26	691	17	1049	—55	1433	54	1789	10
15	379	—5	701	2	1051	2	1439	0	1801	52
—7	383	—1	709	—25	1061	—13	1447	28	1811	12
2	389	—15	719	15	1063	44	1451	52	1823	—56
—16	397	—2	727	3	1069	—20	1453	—71	1831	—43
18	401	2	733	—36	1087	8	1459	—20	1847	—52
10	409	—30	739	50	1091	—58	1471	22	1861	62
9	419	20	743	4	1093	—51	1481	32	1867	28
8	421	22	751	—23	1097	—42	1483	49	1871	—3
—18	431	—18	757	—22	1103	—51	1487	58	1873	—6
—7	433	—11	761	12	1109	—30	1489	—15	1877	18
10	439	40	769	20	1117	48	1493	—36	1879	—35
—10	443	—11	773	—6	1123	24	1499	55	1889	70
2	449	35	787	—32	1129	50	1511	37	1901	77
—7	457	—12	797	53	1151	2	1523	—41	1907	—52
4	461	12	809	0	1153	—31	1531	32	1913	—36
—12	463	—11	811	—38	1163	34	1543	—36	1931	—18
—6	467	—27	821	22	1171	—3	1549	—15	1933	54
—15	479	20	823	39	1181	—18	1553	—56	1949	—40
7	487	23	827	—52	1187	—12	1559	—60	1951	—23
17	491	—8	829	25	1193	—21	1567	—52	1973	79
4	499	20	839	—5	1201	2	1571	—28	1979	30
—2	503	—26	853	14	1213	—41	1579	—30	1987	—22
0	509	15	857	8	1217	—42	1583	34	1993	—66
12	521	—3	859	—15	1223	14	1597	—32	1997	—72
19	523	—16	863	24	1229	60	1601	2	1999	—20
18	541	—8	877	—12	1231	—18	1607	33		
15	547	8	881	—43	1237	18	1609	—10		
24	557	—2	883	4	1249	40	1613	—6		

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