

Formula for Central Terms in Triangle A027907

Center-justified Triangle of Coefficients in Expansion of $(1 + x + x^2)^n$

	Row Sum	
n	S_n	3^n
0	1	3^0
1	3	3^1
2	9	3^2
3	27	3^3
4	81	3^4
5	243	3^5
6	729	3^6
7	2187	3^7
8	6561	3^8

								1																
								1	1	1														
								1	2	3	2	1												
								1	3	6	7	6	3	1										
								1	4	10	16	19	16	10	4	1								
								1	5	15	30	45	51	45	30	15	5	1						
								1	6	21	50	90	126	141	126	90	50	21	6	1				
								1	7	28	77	161	266	357	393	357	266	161	77	28	7	1		
								1	8	36	112	266	504	784	1016	1107	1016	784	504	266	112	36	8	1

The central terms in expansions of $(1 + x + x^2)^n$ are 1, 1, 3, 7, 19, 51, 141, 393, 1107, ... (highlighted above).

The central terms are given by the following formula

$$a_n = \sum_{i=0}^{n/2} \frac{n!}{(n-2i)! i! i!}$$

REFERENCE:

Shara Lalo and Zagros Lalo, Polynomial Expansion Theorems and Number Triangles, Zana Publishing, 2018, ISBN: 978-1-9995914-0-3.