Richard Guy and the Encyclopedia of Integer Sequences: A Fifty-Year Friendship

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Visiting Scholar, Rutgers; OEIS Foundation, Highland Park

Conference "Celebrating Richard Guy"
University of Calgary, October 2, 2020

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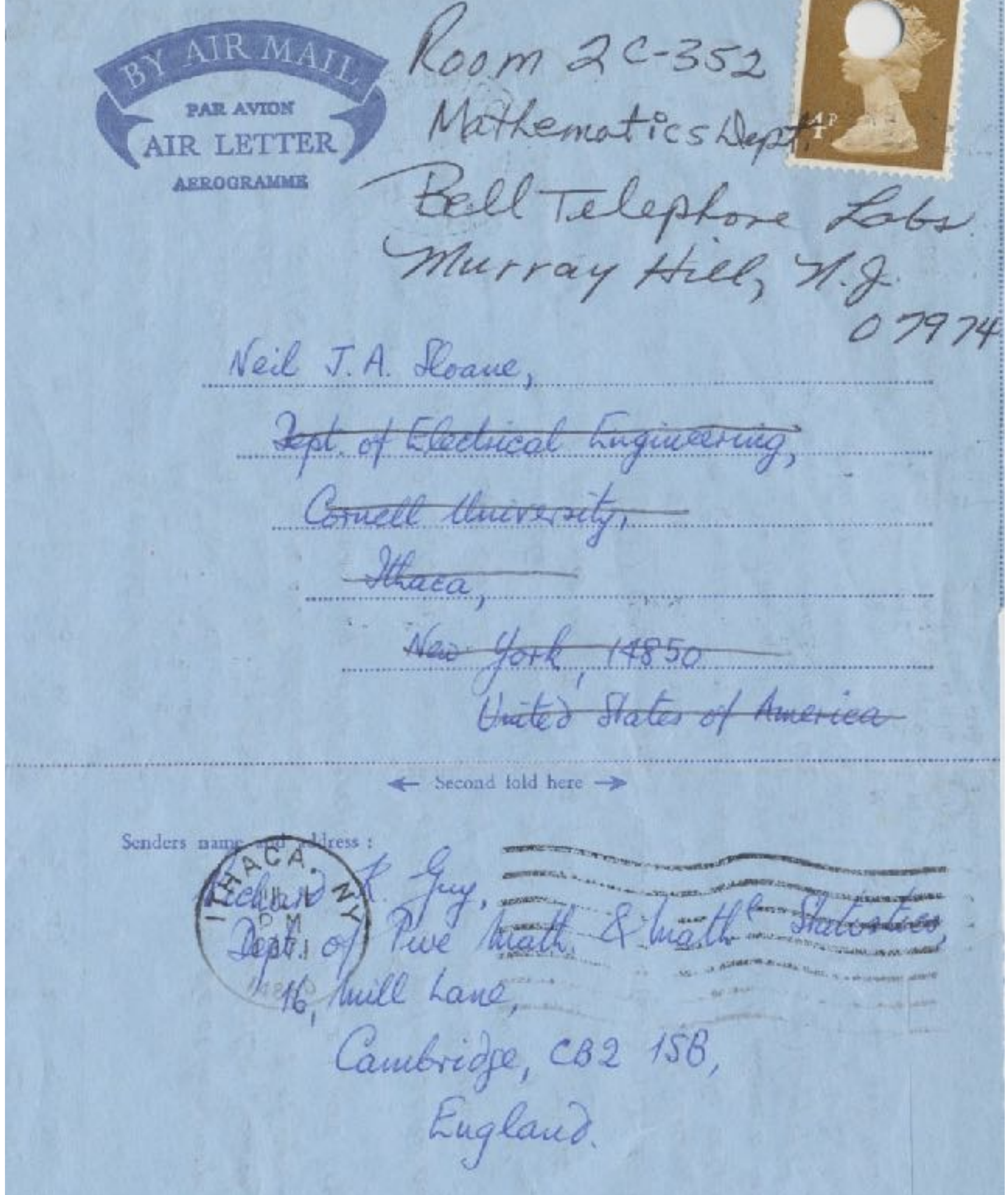
More Than 50 Years

- 1964: Start of integer sequence database to help with thesis
- 1967 onwards: many contributions from RKG
- 1973: *Handbook of Integer Sequences*, 2372 seqs
- 1988: RKG*: Strong Law of Small Numbers*
- 1995: (with Simon Plouffe) Encyclopedia of Integer Sequences, 5K seqs
- 1996: Online! OEIS = On-Line Encyc. of Int. Seqs., 10K seqs
- 2004: 100K E-party
- 2009: The OEIS Foundation Inc., Trustees: David Applegate, Ray Chandler, Russ Cox, Susanna Cuyler, Ron Graham, Richard Guy, David Johnson, Marc LeBrun, Tony Noe, Simon Plouffe, self.
- 2010: OEIS moved off my AT&T home page to commercial hosting site
- 2020: 337000 entries, 80 editors, 200 updates/day, half-million queries/day, 9000 citation in the literature

1971: Typical letter from RKG with new sequences



BY AIR MAIL
PAR AVION
AIR LETTER
AEROGRAMME

Room 2C-352
Mathematics Dept.
Bell Telephone Labs
Murray Hill, N.J.
07974

Neil J.A. Sloane,
~~Dept. of Electrical Engineering,~~
~~Cornell University,~~
~~Ithaca,~~
~~New York, 14850~~
~~United States of America~~

← Second fold here →

Senders name and address :
Richard K. Guy,
Dept. of Pure Math. & Math'l Statistics,
16, Mill Lane,
Cambridge, CB2 1SB,
England.

f

no longer at Oxford, now at Cambridge, but only until July 3.
address from July 4–9: Dept. of Math, Royal Holloway College, Englefield Green, Surrey, England. From July 12–22, c/o R L Graham, Bell Labs., 600 Mountain Ave., Murray Hill, NJ, 07974, U.S.A. From July 23 onwards, Dept of Math., Statistics & Computing Science, The Univ. of Calgary, Calgary 44, Alberta, Canada. 403 284 5202
−3 hrs

UNIVERSITY OF OXFORD

Mathematical Institute

24-29 St Giles

Oxford OX1 3LB

Telephone 0865 54295

June 24
1971

2
2
−2
(B)

Dear Neil,
Some sequences I have come across recently which you may not have (until recently I had access to a 1st edition; now no access; in Calgary I have editions 1, 2 & 4.

#	n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
2572 ✓	✓A	1	1	2	3	5	9	16	28	50	89	159	285					
2843	✓B	1	1	2	4	7	13	24	43	78	141	253	456					
2844	✓C }	1	1	2	5	13	36	102 {	296	871	2599							
	✓D }								295	864		A279196						
2845	/E	1	1	1	2	4	8	17	36	78	171	379						
2846	✓F	1	1	1	2	4	11	33	116	435	1832	8167	39700	201785	1099449	6237505		
	bound G	1	1	1	2	4	18	~~72~~ 54	~~288~~ 288	~~1440~~ 1152	7200	36000	311040					
2847	/H	1	2	3	7	15	43	131	468	1776	7559	34022	166749	853823	4682358			
	I	1	1	2	2	11	11	50?	A279197									
	J	0	0	0	2	7	52		A279198									
	K	1	1	2	6	25	115		A202705									
	L	0	1	3	9	30	117	512	A279199									
	M	1	2	5	15	55	232	A104429 (9)					(12)			(15)		
2848	N	0	0	1	1	2	2	3	7	15	12	30	8	32	162	21		
2849	P	0	0	1	2	4	6	3	10	25	12	42	8	40	202	21		

There are m[…] other numbe[…] but they are […] integer valu[…] polynomial[…]

e.g. $s(4\,3^n 2^m) = (n+1)\left\{\binom{m+2n+4}{n+2} - \right.$ […]

(A) (B)

✓A # of partitions of 1 into n+1 parts of size $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$

✓B — — — — — — — n " parts $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \dots$

(C) --- "non-isentropic binary trees" (Helen Alderson, JH Conway, etc at Cambr
with 2 branches at each stage and if A, B, C, D (v. fig. rt.) are further gro
equivalent to (AC)(BD) — otherwise one distinguishes left & right. # of c

·D # of polynomials $P(x,y)$ with non-neg. integer coeffs with $P(x,y) \equiv 1$, mod
(almost the same as C!)

(E) # of distinct values of $2^{2^{2^{\cdot^{\cdot^{2}}}}}$ with n 2's and the $n-1$ operations performed i
(have asked for a copy of note by Selfridge & self to be sent to you)

(F) # of sequences of "refinements" of partitions of n into 1^n e.g. 5
e.g. in the fig. ∃ 4 distinct paths of length $n-1 = 4$ from 5 to 1^5, so $s(5) =$
a paper on this.

·G an upper bound for F: $\dfrac{\left(\left[\frac{1}{2}n\right]!\right)^2 \left[\frac{1}{2}n\right]^{m-\varepsilon}}{2.5.\dots\frac{1}{2}(n-1)(n+2)}$ where $m(m+1) \leq n < (m+1)(m+2)$, and the numbers in the den

(H) Total # of paths from n towards 1^n of all lengths $0,1,2,\dots,n-1$. The c
$A(n) \geq s(n-1) + 2s(n-2) + 3s(n-3) + 7s(n-4) + 15s(n-5) + \dots$ used to obtain

I In the generalization of Sedláček's conjecture (Roger B. Eggleton & self) (copy of
"self-conjugate inseparable" solutions of $x+y=2z$ (integer, disjoint triples from {

J # of pairs of "conj. insep" " , e.g.

243 264
576 375
1158 1158
9 13 11 9 11 10
10 14 12 12 14 13

K = I+2J, # of "insep" solut

M = K+L, # of solutions.

L # of "separable" solutions, e.g.

132
486
597

(N) # of solutions of $x+y=z$ ch
counting only " which induc

Best wishes,
Richard K. Guy.

Am. Math. Monthly 1988

S728 + many more

The Strong Law of Small Numbers

RICHARD K. GUY
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This article is in two parts, the first of which is a do-it-yourself operation, in which I'll show you 35 examples of patterns that *seem* to appear when we look at several small values of n, in various problems whose answers depend on n. The question will be, in each case: do you think that the pattern persists for all n, or do you believe that it is a figment of the smallness of the values of n that are worked out in the examples?

Caution: examples of both kinds appear; they are not all figments!

In the second part I'll give you the answers, insofar as I know them, together with references.

Try keeping a scorecard: for each example, enter your opinion as to whether the observed pattern is known to continue, known not to continue, or not known at all.

This first part contains no information; rather it contains a good deal of disinformation. The first part contains one theorem:

You can't tell by looking.

It has wide application, outside mathematics as well as within. It will be proved by intimidation.

Here are some well-known examples to get you started.

Example 1. The numbers $2^{2^0}+1=3$, $2^{2^1}+1=5$, $2^{2^2}+1=17$, $2^{2^3}+1=257$, $2^{2^4}+1=65537$, are primes.

A215 and A19434

Example 2. The number 2^n-1 can't be prime unless n is prime, but $2^2-1=3$, $2^3-1=7$, $2^5-1=31$, $2^7-1=127$, are primes.

A43 and A668

FORTUNATE NUMBERS

Example 11. When you use Euclid's method to show that there are unboundedly many primes:

$$2 + 1 = 3$$
$$(2 \times 3) + 1 = 7$$
$$(2 \times 3 \times 5) + 1 = 31$$
$$(2 \times 3 \times 5 \times 7) + 1 = 211$$
$$(2 \times 3 \times 5 \times 7 \times 11) + 1 = 2311$$

you don't always get primes:

$$(2 \times 3 \times 5 \times 7 \times 11 \times 13) + 1 = 30031 = 59 \times 509$$
$$(2 \times 3 \times 5 \times 7 \times 11 \times 13 \times 17) + 1 = 510511 = 19 \times 97 \times 277$$
$$(2 \times 3 \times 5 \times 7 \times 11 \times 13 \times 17 \times 19) + 1 = 9699691 = 347 \times 27953$$

but if you go to the *next* prime, its difference from the product is always a prime

$$5 - 2 = 3$$
$$11 - (2 \times 3) = 5$$
$$37 - (2 \times 3 \times 5) = 7$$
$$223 - (2 \times 3 \times 5 \times 7) = 13$$
$$2333 - (2 \times 3 \times 5 \times 7 \times 11) = 23$$
$$30047 - (2 \times 3 \times 5 \times 7 \times 11 \times 13) = 17$$
$$510529 - (2 \times 3 \times 5 \times 7 \times 11 \times 13 \times 17) = 19$$
$$9699713 - (2 \times 3 \times 5 \times 7 \times 11 \times 13 \times 17 \times 19) = 23$$

45235

A5235

11. R. F. Fortune conjectured that these differences are always prime: see [8], [9] and A2 in [12]. The next few are 37, 61, 67, 61, 71, 47, 107, 59, 61, 109, 89, 103, 79. There's a high probability that the conjecture is true, because the difference can't be divisible by any of the first k primes, so the smallest composite candidate for $P = \prod p_k$ is p_{k+1}^2, which is approximately $(k \ln k)^2$ in size. The product of the first k primes is about e^k: to find a counter example we need a gap in the primes near N of size at least $(\ln N \ln\ln N)^2$. Such gaps are believed not to exist, but it's beyond our present means to prove this.

A005235 Fortunate numbers: least m > 1 such that m + prime(n)# is prime, where p# denotes the product of all primes <= p.
(Formerly M2418)

3, 5, 7, 13, 23, 17, 19, 23, 37, 61, 67, 61, 71, 47, 107, 59, 61, 109, 89, 103, 79, 151, 197, 101, 103, 233, 223, 127, 223, 191, 163, 229, 643, 239, 157, 167, 439, 239, 199, 191, 199, 383, 233, 751, 313, 773, 607, 313, 383, 293, 443, 331, 283, 277, 271, 401, 307, 331 (list; graph; refs; listen; history; edit; text; internal format)

OFFSET 1,1

COMMENTS R. F. Fortune conjectured that a(n) is always prime.
The first 500 terms are primes. - Robert G. Wilson v [The first 2000 terms are prime. - Joerg Arndt, Apr 15 2013]
The strong form of Cramér's conjecture implies that a(n) is a prime for n > 1618, as previously noted by Golomb. - Charles R Greathouse IV, Jul 05 2011

(a very large entry)

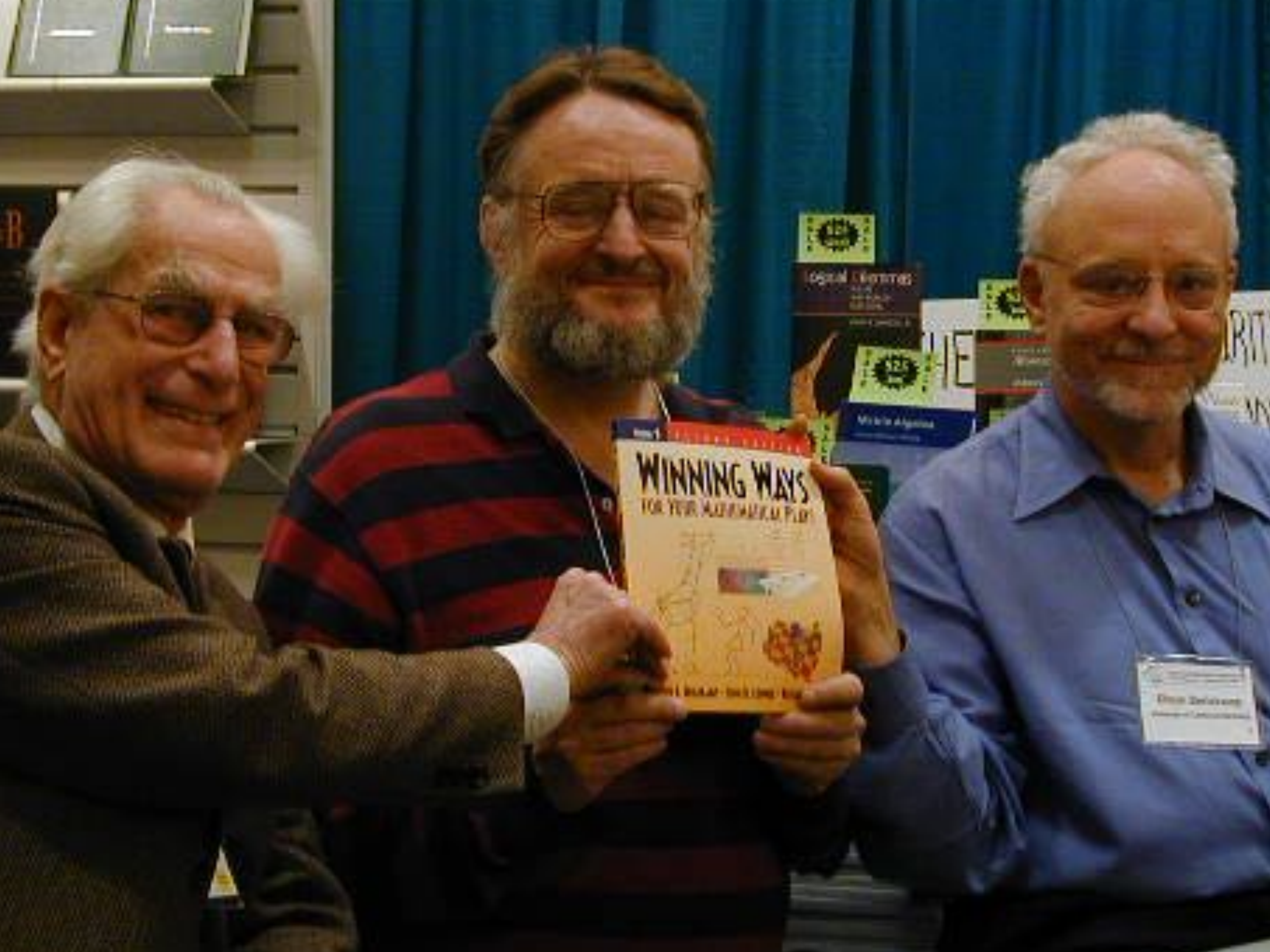
2004: OEIS REACHES 100,000 SEQUENCES

E-PARTY !

Also 40th Anniversary of start of database

Celebrates with E-party
130 guests from 28 countries

Richard Guy: “I told you 40 years ago not to start this, but you wouldn’t listen”


WINNING WAYS
FOR YOUR MATHEMATICAL PLAYS

PART 2

Unsolved problems I never got to tell Richard about

- **Operations on numbers and sequences**
- **The Enots Wolley Sequence and other LES sequences**
- **Three Cousins of Recamán’s Sequence**
- **Graphical enumeration and stained glass windows**

Some operations on numbers and sequences

1 2 4 8 16 32 64 128 256 512

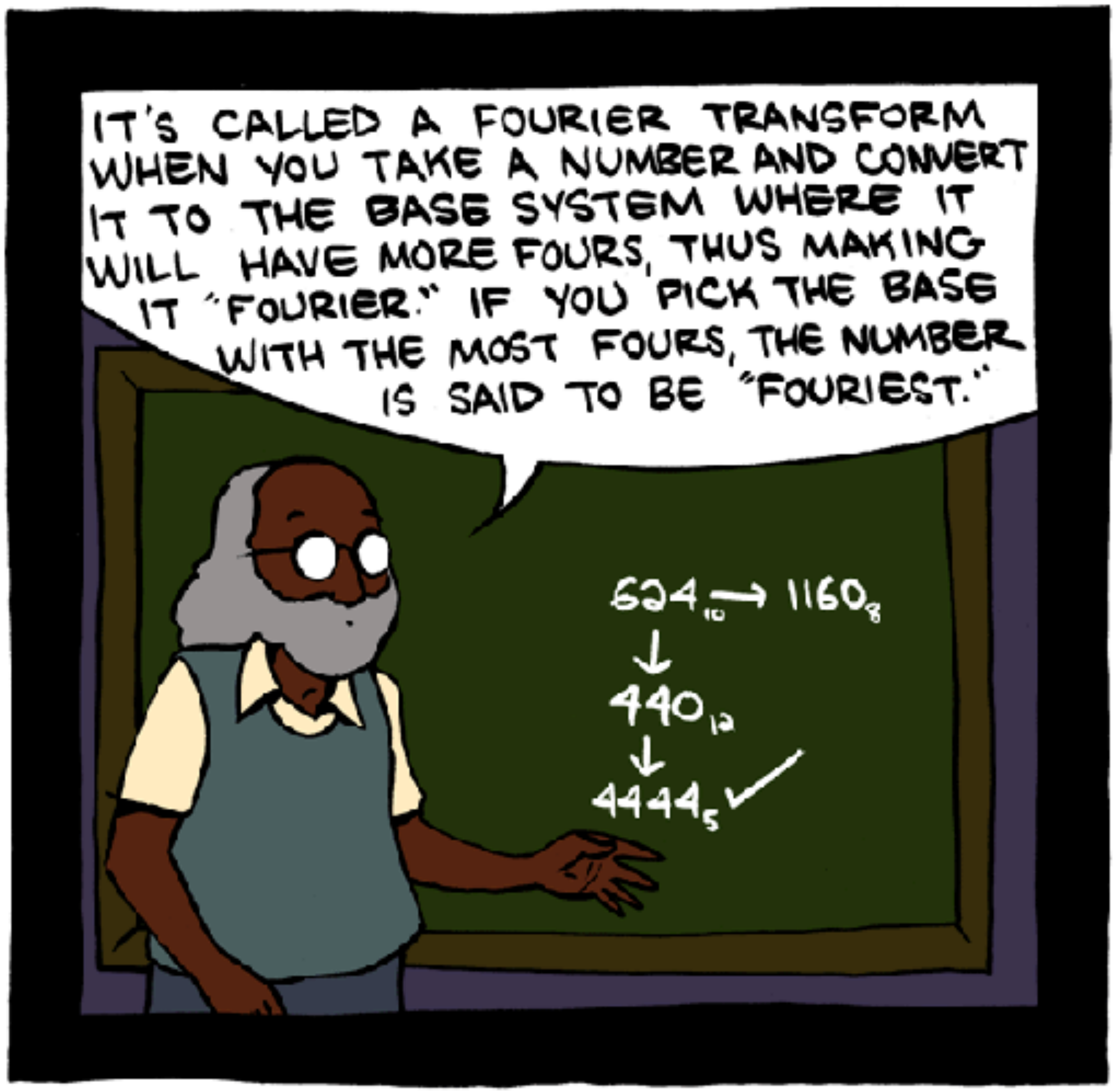
1024 2048 4096 8192 16384

32768 3 6 12 24 48 96 192

384 768 1536 3072 61 1 2 4 8

Periodic, easy - explain!

The Fouriest Transform of n



Teaching math was way more fun after tenure.

Zach Weinersmith, Saturday Morning Breakfast Cereal

Write n in that base b >= 4 where you get the most 4's

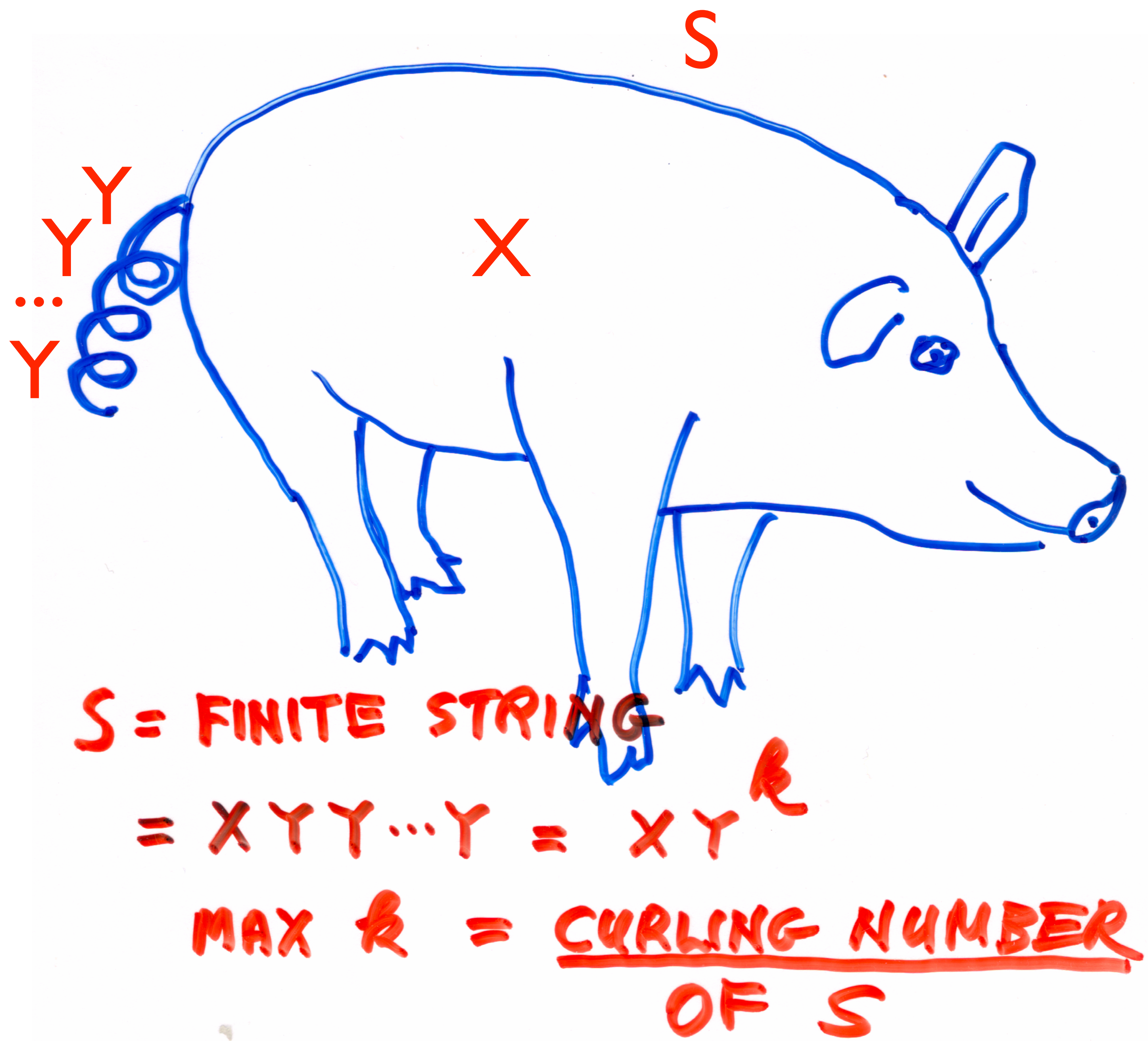
a(10)=14 (use base 6)

A268236

0, 1, 2, 3, 4, 11, 12, 13, 20, 14, 14, 14, 14, 14, 24, 14, 24,..

The Curling Number of a Sequence

Definition
of
Curling
Number



S = 7 5 2 2 5 2 2 5 2 2, k = 3

Gijswijt's Sequence

Fokko v. d. Bult, Dion Gijswijt, John Linderman,
N. J. A. Sloane, Allan Wilks (J. Integer Seqs., 2007)

Start with 1, always append curling number

1 1 <u>2</u>
1 1 2 <u>2</u> <u>2</u> <u>3</u>
1 1 2
1 1 2 2 2 3 <u>2</u>
1 1 2
1 1 2 2 2 3
1 1 2
1 1 2 2 2 3 2 <u>2</u> <u>2</u> <u>3</u> <u>2</u> <u>2</u> <u>2</u> <u>3</u> <u>3</u> <u>2</u>
1 1 2
. . . . . .

$a(220) = 4$

(A090822)

Is there a 5?

300,000 terms: no 5

$2 \cdot 10^6$ terms: no 5

10^{120} terms: no 5

NJAS, FvdB: first 5 at about term $10^{10^{23}}$

RUNS

H H H T H T T T H H T

RUNS transform = 3 1 1 3 2 …

RUNS Transformation of a sequence:

HHHTTHTTH… becomes 3212…

Kolakoski A2 = 1,2,2,1,1,2,1,2,2,… is fixed **(A mystery)**

Golomb A1462 = 1,2,2,3,3,4,4,4,5,5,5,6,6,6,6,7,… is fixed

$$a(n) = Cn^{\phi-1} + \epsilon$$

Are the two hybrids A156253 and A321020 analyzable?

RUNS, RUNS, RUNS

A306211, from a high-school student, Jan 29 2019

Start with S = 1
Append RUNS(S) to S
Repeat

1
1 1
1 1 2
1 1 2 2 1
1 1 2 2 1 2 2 1
1 1 2 2 1 2 2 1 2 2 1 2 1
1 1 2 2 1 2 2 1 2 2 1 2 1 2 2 1 2 1 2 1 1 1

When first see

1	**2**	**3**	**4**	**5**	**6**
1	3	37	60	225	??

Conj. 1: 5 is max term
Conj. 2: Every n appears

After 65 generations (10^13 terms), still no 6 (Ben Chaffin)

The Enots Wolley Sequence

Suggested by Scott Shannon (Melbourne) in August 2020

The Australian politician Enots ("Snotty") Wolley?

“LES” Sequences

Lexicographically Earliest Infinite Sequence of distinct positive numbers with property that ******

No other condition: 1, 2, 3, 4, 5, 6, 7, … **A27**

(The earliest of them all!)

LES examples

EKG sequence: gcd(a(n), a(n-1)) >1 for n>2:

1, 2, 4, 6, 3, 9, 12, 8, 10, 5, 15, ... **A64413**

The Yellowstone Permutation: gcd(a(n), a(n-2)) > 1 and gcd(a(n), a(n-1)) = 1 for n>3: **A98550**

1, 2, 3, 4, 9, 8, 15, 14, 5, 6, 25, 12, 35, 16, 7, 10, 21, ...

The Enots Wolley Sequence: gcd(a(n), a(n-1)) > 1 and gcd(a(n), a(n-2)) = 1 for n>2: **A336957**

1, 2, 6, 15, 35, 14, 12, 33, 55, 10, 18, 21, 77,

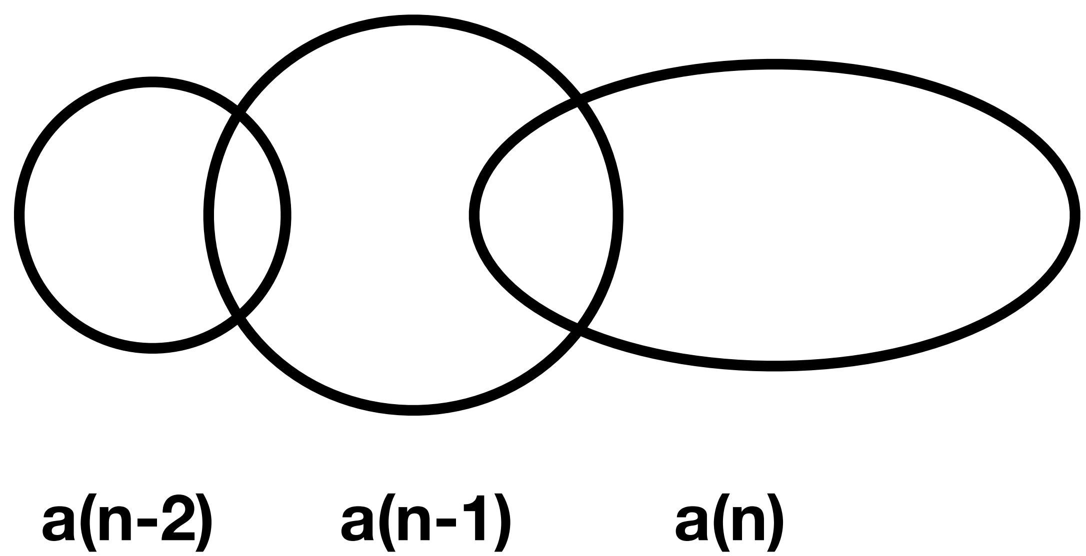
↑ **Not 4 !**

(Two other unsolved LES sequences dear to my heart, Sigrist's A280864, and the set theory version of Yellowstone, A252867)

The Enots Wolley Sequence

A336957

n	1	2	3	4	5	6	7	8	9	10
a(n)	1	2	6	15	35	14	12	33	55	10
2 ?		✓	✓			✓	✓			✓
3 ?			✓	✓			✓	✓		
5 ?				✓	✓				✓	✓
7 ?					✓	✓				
11 ?								✓	✓	



The Enots Wolley Sequence (cont.)

Ker(n) := set of primes dividing n.

Theorem 1: For n>2, a(n) = smallest m not in sequence such that:

(i) $\mathrm{Ker}(m) \cap \mathrm{Ker}(a(n-1)) \neq \emptyset$

(ii) $\mathrm{Ker}(m) \cap \mathrm{Ker}(a(n-2)) = \emptyset$

(iii) $\mathrm{Ker}(m) \setminus \mathrm{Ker}(a(n-1)) \neq \emptyset$

Proof: m exists, is unique, and a(n) can't be less than m.

The Enots Wolley Sequence (cont.)

Theorem 2: For n>2, a(n) is divisible by at least two different primes.

So not a permutation of pos. integers.

No primes or prime powers except 1 and 2.

Conjecture 1: Sequence consists of 1, 2, and all numbers with at least 2 prime factors.

Theorem 3: (a) Sequence is infinite.
(b) For any prime p, p divides some term.
(c) For any prime p, p divides infinitely many terms.
(d) There are infinitely many even terms.
(e) There are infinitely many odd terms.

The Enots Wolley Sequence (cont.)

Theorem 4: When an odd prime p first divides a(n),
a(n) = qp
where q is a prime < p.

What is q ?

Conjectures: q = 5 iff p = 7
q = 3 for exactly 34 values of p (2, 5, 11, 13, 17, …, 233, 367)
q = 2 for p = 3, 7, …, and all primes >367

Conjecture: For any odd prime p, there is a term 2p.

Conjecture : All even numbers (except 2^k, k>1) appear.

The Yellowstone Permutation Theorem

A98550

a(n) = smallest number not yet in seq. such that
gcd(a(n-2), a(n)) >1, gcd(a(n-1), a(n) = 1.

1, 2, 3, 4, 9, 8, 15, 14, 5, 6, 25, 12, 35, 16, 7, 10, 21, 20, 27

Theorem 5(*): Every positive number appears

Proof: 1. Sequence is infinite
2. Given B, exists n_0 s.t. n > n_0 implies a(n) > B.
3. Every prime divides some term.
4. Any p divides oo many terms.
5. Every prime p appears naked in sequence.
6. All numbers appear.
QED

(*) Applegate, Havermann, Selcoe, Shevelev, NJAS, Zumkeller, 2015

The EKG sequence (cont)

A64413

Theorem 6: Every positive number appears

Proof:

There are several steps. (i) Sequence is infinite (easy).
(ii) Let $T(m) =$ n such that $a(n)=m$, or -1 if m is missing from sequence.
Let $W(m) = \max T(i)$, $i \le m$. Then if $n > W(m)$, $a(n) > m$.

(iii) Let p = prime. Exists n such that $p \mid a(n)$. If not, no prime $q>p$ can divide any term either, because if $a(n) = qk$ then pk would be a smaller choice.
So all terms are products just of primes $< p$.
Choose $n>W(p^2)$, say $a(n) = qk$, for prime $q<p$, so $qk > p^2$.
Then $pk < p^2 < qk$ was a smaller candidate for a(n), contradiction.

(iv) When p first divides a(n), say $a(n) = kp$, then k is a prime $< p$.
If $k = 2$ we have $a(n)=2p$, $a(n+1)=p$. Otherwise we have $a(n)=kp$, $a(n)=p$, $a(n+1)=2p$. Either way we see adjacent terms p and 2p.

Proof (continued)

(v) If for some prime p there are infinitely many multiples of p, then all multiples of p are in the sequence. If not, let kp = smallest missing multiple of p. Find n >W(kp) with a(n) = mp. Then kp < mp was a smaller candidate for a(n), a contradiction.

(vi) If for some prime p all multiples of p are in the sequence then all numbers appear. For suppose k is smallest missing number. Find n > W(k) such that a(n) is multiple of kp. Then k was smaller candidate for a(n), contradiction.

(vii) By (iii) and (iv) we see infinitely many multiples of 2, and by (v) and (vi) we see all numbers.

QED

Three Cousins of Recamán’s Sequence

**Max Alekseyev, Joseph Meyers, Richard Schroeppel,
Scott Shannon, NJAS, and Paul Zimmermann(*)**

(on the arXiv; Fib. Quart. to appear)

(*) P.Z. announced in February 2020 that he and five others had factored the 250-digit RSA challenge number RSA-250, taking 2700 physical core-years.

Recamán's Sequence

0	1	2	3	4	5	6	7	8	9	...
0	1	3	6	2	7	13	20	12	21	...

$$a_n = a_{n-1} - n$$

(A5132)

if positive and new, otherwise

$$a_n = a_{n-1} + n$$

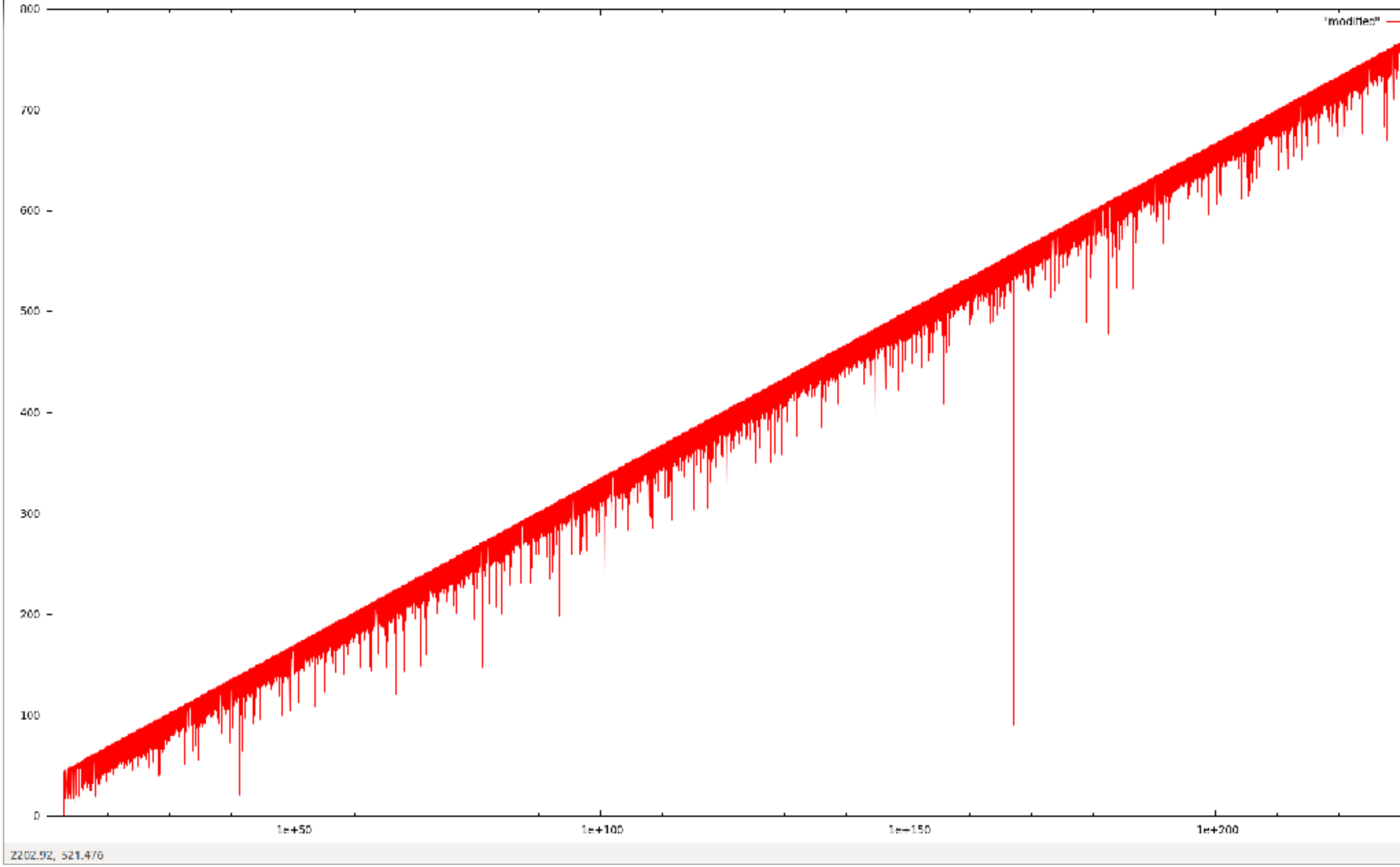
- from Bernardo Recamán Santos (Colombia), circa 1992

Numbers that take a record number of steps to appear:

1	1
2	4
4	131
19	99,734
61	181,653
879	328,002
1355	325,374,625,245
2406	394,178,473,633,984
852655	$> 10^{230}$

(Benjamin Chaffin)

(A64228) (A64227)



(Benjamin Chaffin)

Source:
https://oeis.org/A005132

The First Cousin, A(n), n >= 3

To find A(n), start with n, and add n+1, n+2, …, n+k, and stop when d = n+k+1 divides the sum

BINGO!
2

n = 3: 3 7 12 (2 STEPS)
4 5 6

BINGO! 5

n=4: 4 9 15 22 30 39 49 60
5 6 7 8 9 10 11 12
(7 STEPS)

n	k	d	sum	sum/d
3	2	6	12	2
4	7	12	60	5
5	14	20	180	9
6	3	10	30	3
.	.	.	.	.

IN OEIS!
A82183

The First Cousin, A(n), n >= 3 (cont.)

Our A(n) = minimum k > 0 such that

n + k + 1 divides (k+1)n + k(k+1)/2

A82183(n) = minimum s > 0 such that

T(n) + T(s) = T(m)

for some m, where T(i) = i(i+1)/2

Which led to the solution:

Theorem 1: Look at odd divisors d of n(n+1), different from n and n+1, and minimize | d - n(n+1)/d |

Then the minimum s = s(n) is (| d - n(n+1)/d | - 1) / 2

Theorem 2: Solve for m from T(n-1) + T(s(n-1)) = T(m)
then A(n) = s(n-1) + m - n

The Third Cousin C(n)

A332580

To find C(n), start with n, and successively concatenate
n+1, n+2, ..., n+k, and stop when
n || n+1 || n+2 || ... ||n+k
is divisible by n+k+1. Set C(n) = k.
Or C(n) = -1 if no such k exists!

n=1: 1|| 2 = 12 is divisible by 3. Took one step, so C(1) = 1.

|| means concatenate

n=8: 8||9 is not divisible by 10, so we get 8||9||10.
8910 IS divisible by 11, two steps, so C(8) = 2.

n=7: 7||8||9||10||11||12||13||14||15||16||17||18||19||20 is divisible by 21,
7891011121314151617181920 divided by 21 = 375762434348292934151520
13 steps, so C(7) = 13

C(2) = 80: the concatenation 2 || 3 || ... || 82 is
23456789101112131415161718192021222324252627282930313233343536373839\
4041424344454647484950515253545556575859606162636465666768697071727374
576777879808182, which is divisible by 83.

n	$C(n)$	n	$C(n)$	n	$C(n)$	n	$C(n)$
1	1	26	33172	51	2249	76	320
2	80	27	9	52	21326	77	59
3	1885	28	14	53	53	78	248
4	6838	29	317	54	98	79	31511
5	1	30	708	55	43	80	20
6	44	31	1501	56	20	81	5
7	13	32	214	57	71	82	220
8	2	33	37	58	218	83	49
9	1311	34	34	59	91	84	12
10	18	35	67	60	1282	85	25
11	197	36	270	61	277	86	22
12	20	37	19	62	56	87	105
13	53	38	20188	63	47	88	34
14	134	39	78277	64	106	89	4151
15	993	40	10738	65	1	90	1648
16	44	41	287	66	890	91	2221
17	175	42	2390	67	75	92	218128159460
18	124518	43	695	68	280	93	13
19	263	44	2783191412912	69	19619	94	376
20	26	45	3	70	148	95	23965
21	107	46	700	71	15077	96	234
22	10	47	8303	72	64	97	321
23	5	48	350	73	313	98	259110640
24	62	49	21	74	34	99	109
25	15	50	100	75	557	100	346

A332580

All C(n) known exactly for n <= 1000, except two values:

10^14 <= C(539) <= 887969738466613

and

C(158) = -1 or > 10^14.

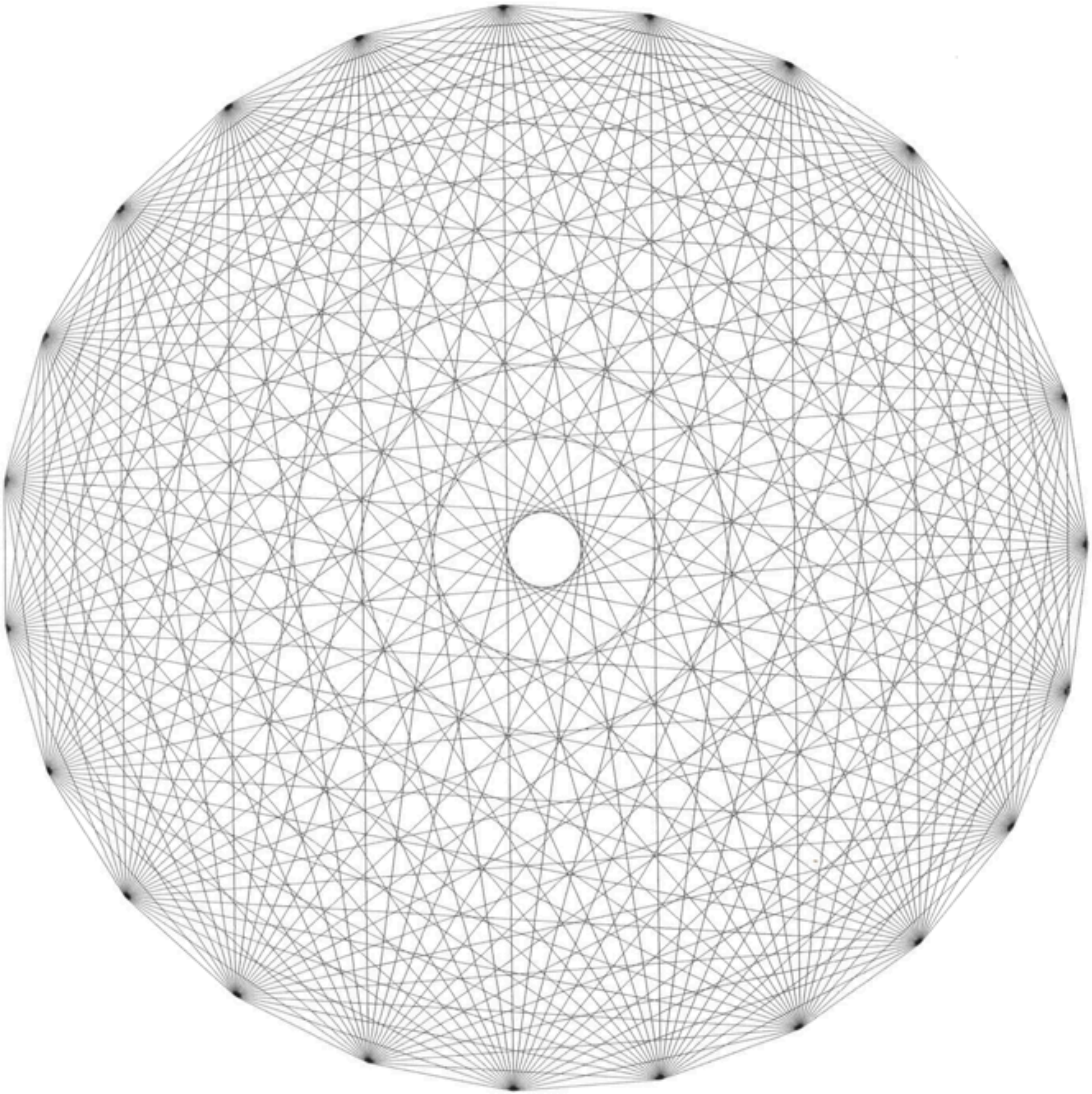
Conjecture 3: C(n) is never -1, k always exists.

Graphical Enumeration and Stained Glass Windows

Lars Blomberg, Scott Shannon, and NJAS

Part 1 is on the arXiv (#2009.07918, Sep 16 2020)

Complete graph K_23



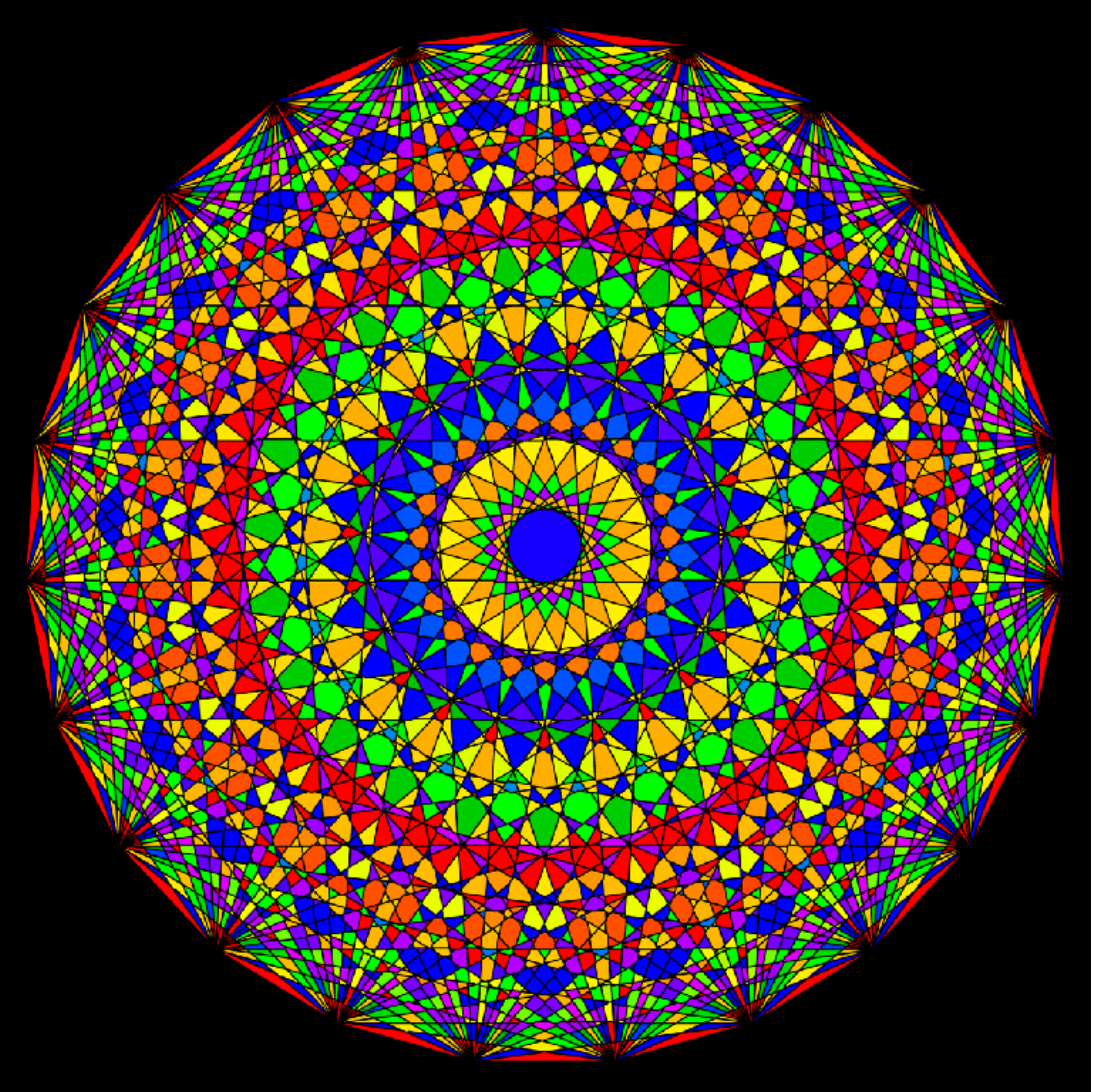
9086 cells (R)
8878 nodes (V)
17963 edges (E)

Solved by Poonen
and Rubinstein 1998

Euler says
E = R+V-1.

R and V about equal
tells us most
crossings
are simple.

Source:
https://oeis.org/A007678



Complete graph
K_23
with 9086 cells.
Colored by our
special algorithm.

Source:
https://oeis.org/A007678

Motivation

1. Extend work of Poonen-Rubinstein, Legendre-Griffiths to other families of graphs

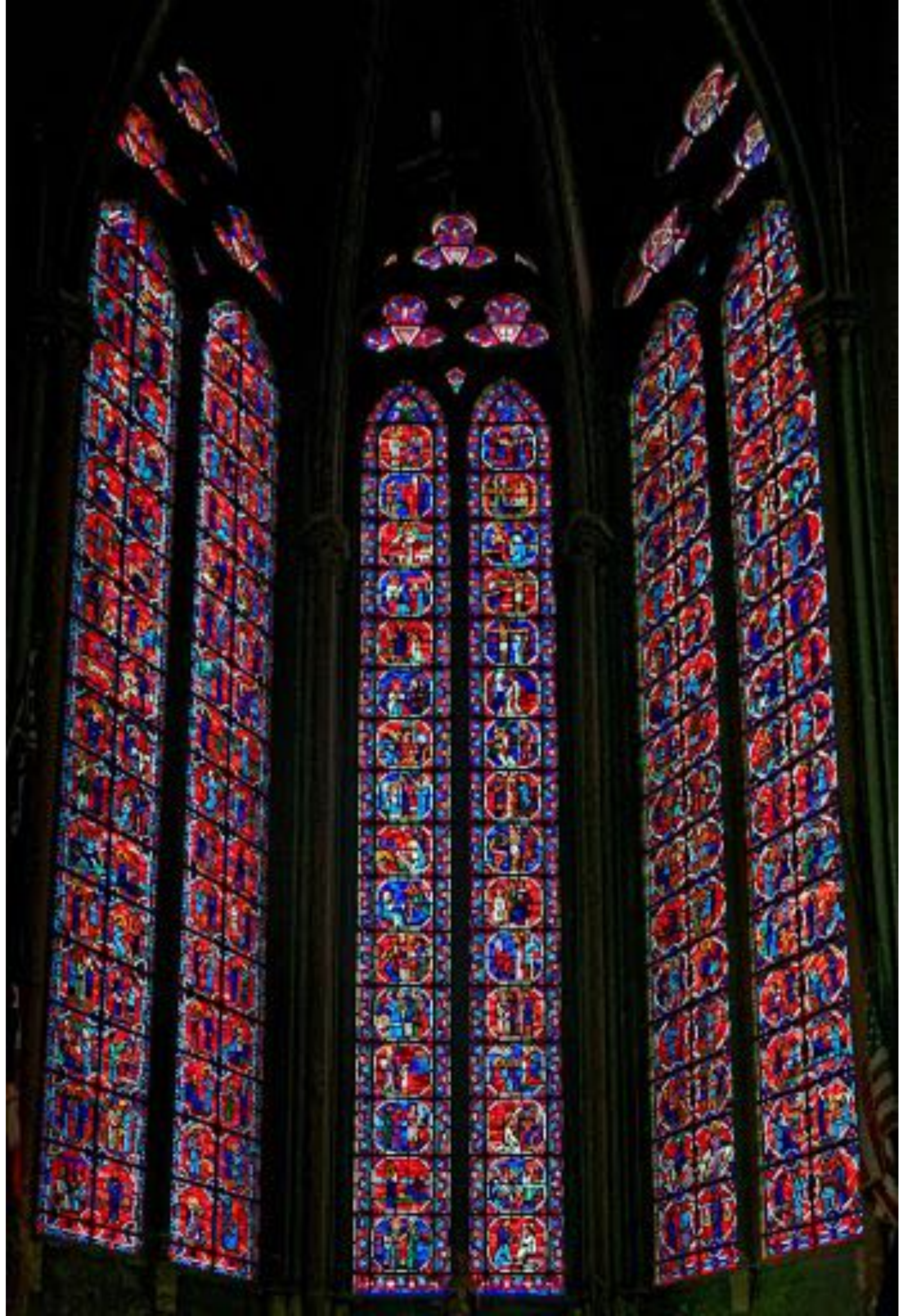
2. Desire to create our own stained glass windows, in homage to Amiens, Sainte-Chapelle, Chartres, Strasbourg.

Our motto: "If you can't solve it, make art"

Rose window

Amiens, France

Sainte-Chapelle, Paris

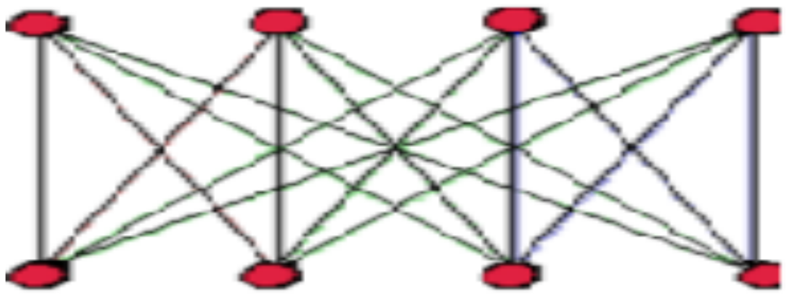


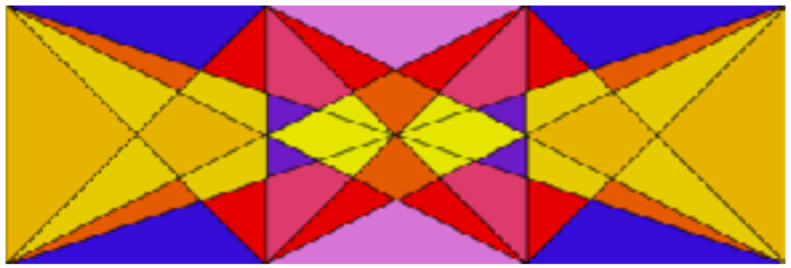
The Two Known Results

1. Poonen and Rubinstein, 1998: Number of nodes and cells in K_n :

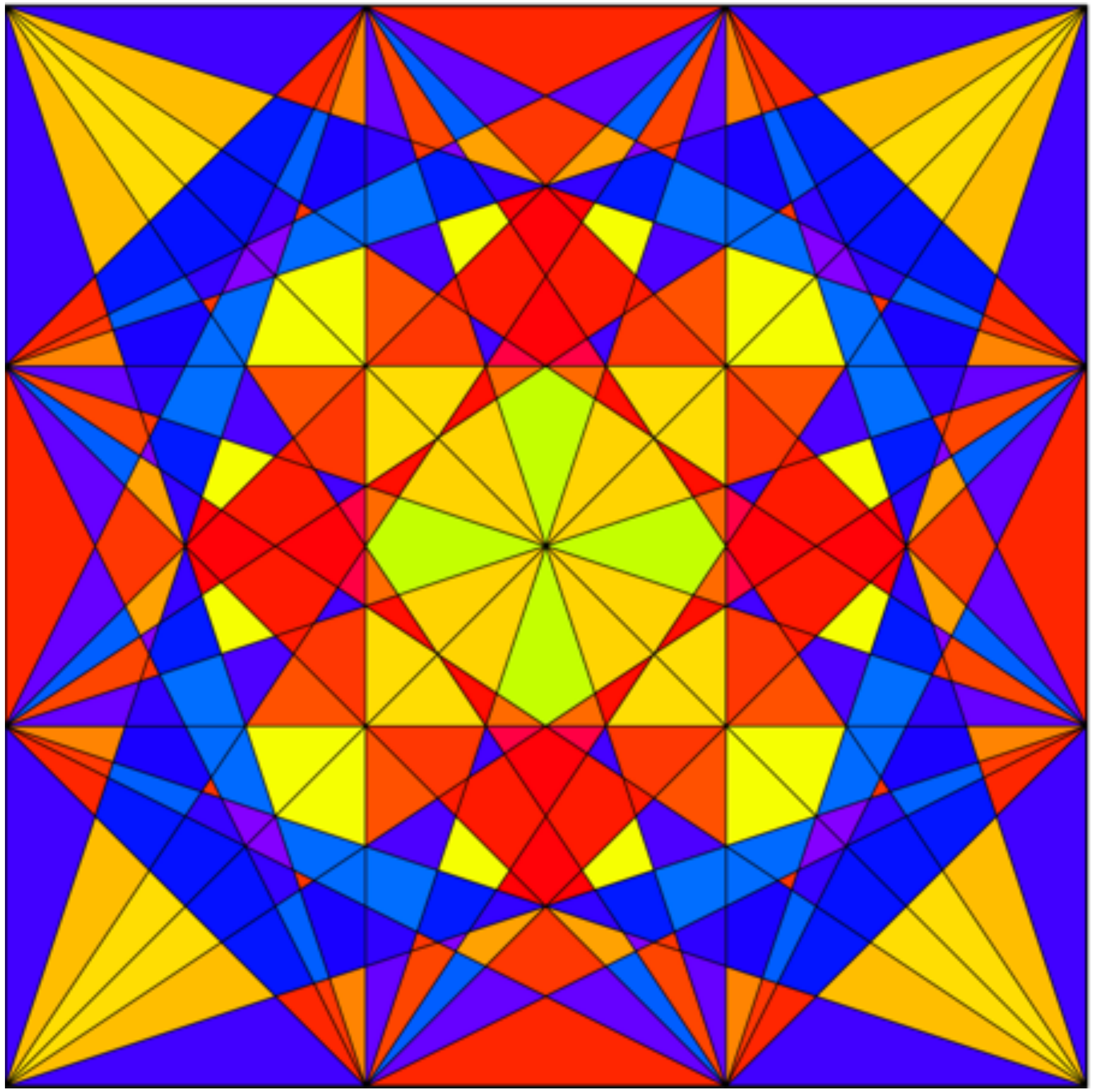
   Basically $\binom{n}{4}$ minus complicated correction terms.

2. Legendre (2009), Griffiths (2010), ditto for K_{n,n}.



or equivalently  = BC(1,3)

Source: https://oeis.org/A331452

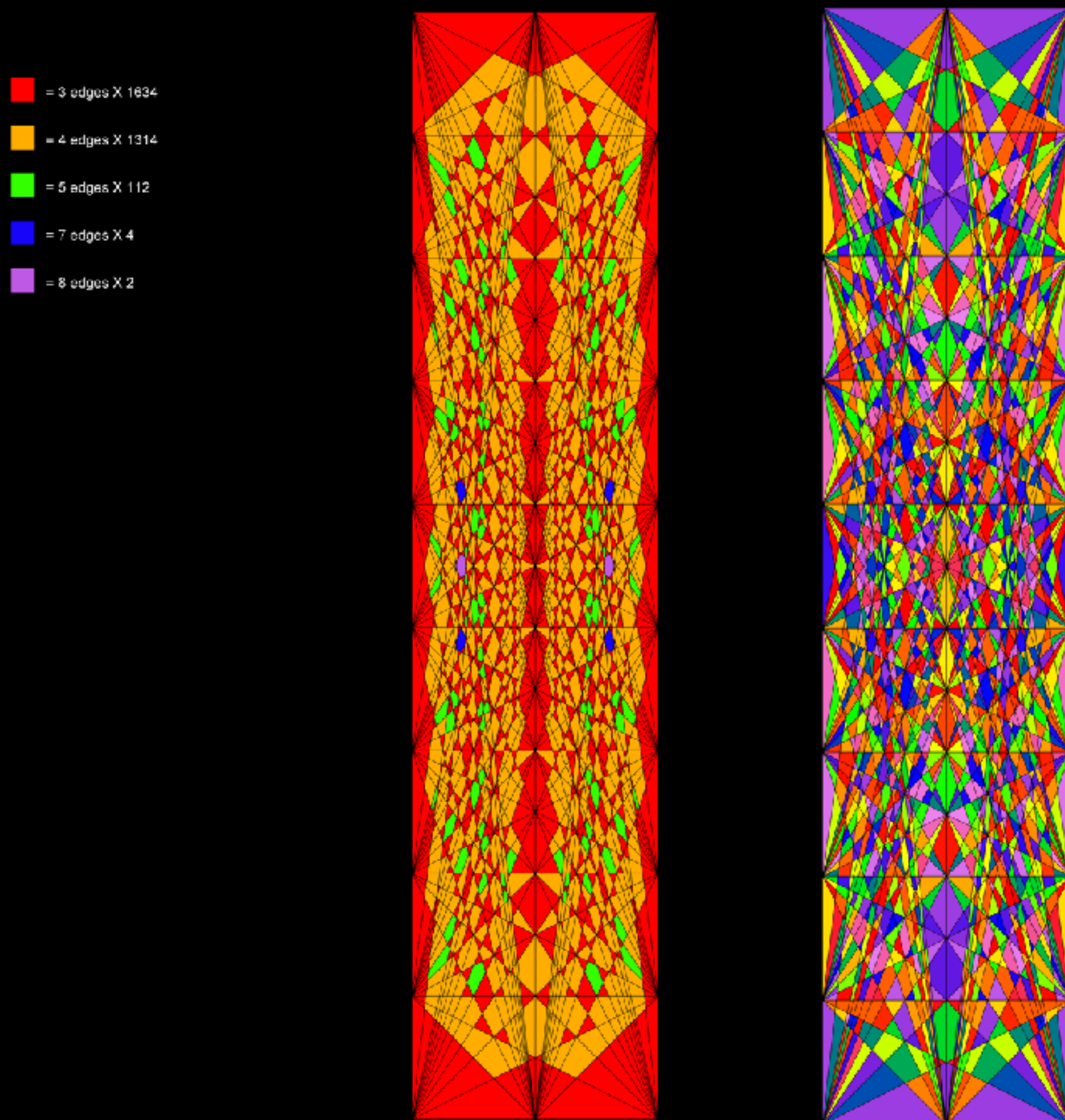


BC(3,3)

BC(m,n) = m X n grid of squares with every pair of boundary points joined by a line

BC = “Boundary Chords”

Source: Lars Blomberg, Scott R. Shannon, and N. J. A. Sloane, Graphical Enumeration and Stained Glass Windows, 1: Rectangular Grids, http://arxiv.org/abs/2009.07918, September 2020.



BC(9,2)

Left: Color-coded to show number of sides: 3 (red), 4 (orange), 5 (green), 7 (blue), 8 (purple)

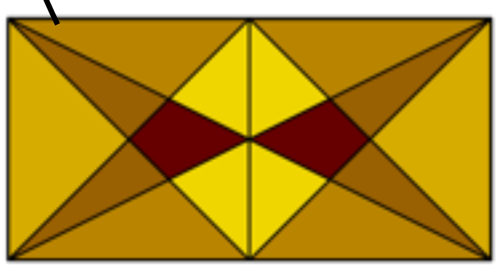
Right: Same graph, colored using our special algorithm.

Source:
https://oeis.org/A331452

Numbers of nodes & cells in BC(m,n)

$m \backslash n$	1	2	3	4	5	6	7
1	5, 4	13, 16	35, 46	75, 104	159, 214	275, 380	477, 648
2	13, 16	37, 56	99, 142	213, 296	401, 544	657, 892	1085, 1436
3	35, 46	99, 142	257, 340	421, 608	881, 1124	1305, 1714	2131, 2678
4	75, 104	213, 296	421, 608	817, 1120	1489, 1916	2143, 2820	3431, 4304
5	159, 214	401, 544	881, 1124	1489, 1916	2757, 3264	3555, 4510	5821, 6888
6	275, 380	657, 892	1305, 1714	2143, 2820	3555, 4510	4825, 6264	7663, 9360
7	477, 648	1085, 1436	2131, 2678	3431, 4304	5821, 6888	7663, 9360	12293, 13968

BC(1,1)
(5,4)



BC(1,2)
(13,16)

Open Problem 1:
Explain these numbers.

This is the main problem of this section.

For 37 rows and cols see A331453, A331452

Answers are known for BC(1,n)

Theorem 1 **(Stéphane Legendre (2009) and Martin Griffiths (2010))**

Define $$V(m, n, q) = \sum_{a=1..m} \sum_{\substack{b = 1..n \\ \gcd\{a, b\} = q}} (m+1-a)(n+1-b)$$

Nodes in BC(1,n): $$2(n+1) + V(n, n, 1) - V(n, n, 2)$$

Cells in BC(1,n): $$n^2 + 2n + V(n, n, 1)$$

Max Alekseyev pointed out that the Legendre-Griffiths results are equivalent to results in enumerating training sets for threshold functions found by him and coauthors (M.A., 2010; M.A., Basova, & Zolotykh, 2015).

Furthermore, their work implies:
Theorem 8: All cells in BC(1,n) are either triangles or quadrilaterals.

Open Problem 2: Find a purely geometrical proof!

Interior Nodes in BC(1,n)

It appears that most interior nodes in BC(1,n) are "simple", i.e. are where just two chords cross.

For n = 1, 2, 3, … the numbers of simple interior nodes are

1, 6, 24, 54, 124, 214, 382, 598, 950, 1334, …

A334701 has first 500 terms!

Open Problem 3: Find a formula.

This is a frequent problem: we have hundreds of terms of a sequence with a simple definition; the OEIS has 340,000 entries: need a smarter guessing program.

BC(2,n)

Conjecture 5
In BC(2,n) cells have at most 8 sides,
and if n>18, at most 6 sides

BC(m,n)

Theorem 2

The number of nodes in BC(m,n) is at most

$$\frac{1}{4}\{(m+n)(m+n-1)^2(m+n-4) \; + \; 2mn(2m+n-1)(m+2n-1)\} + 2(m+n)$$

and there is a similar bound for the number of cells.

(These are pretty good upper bounds)

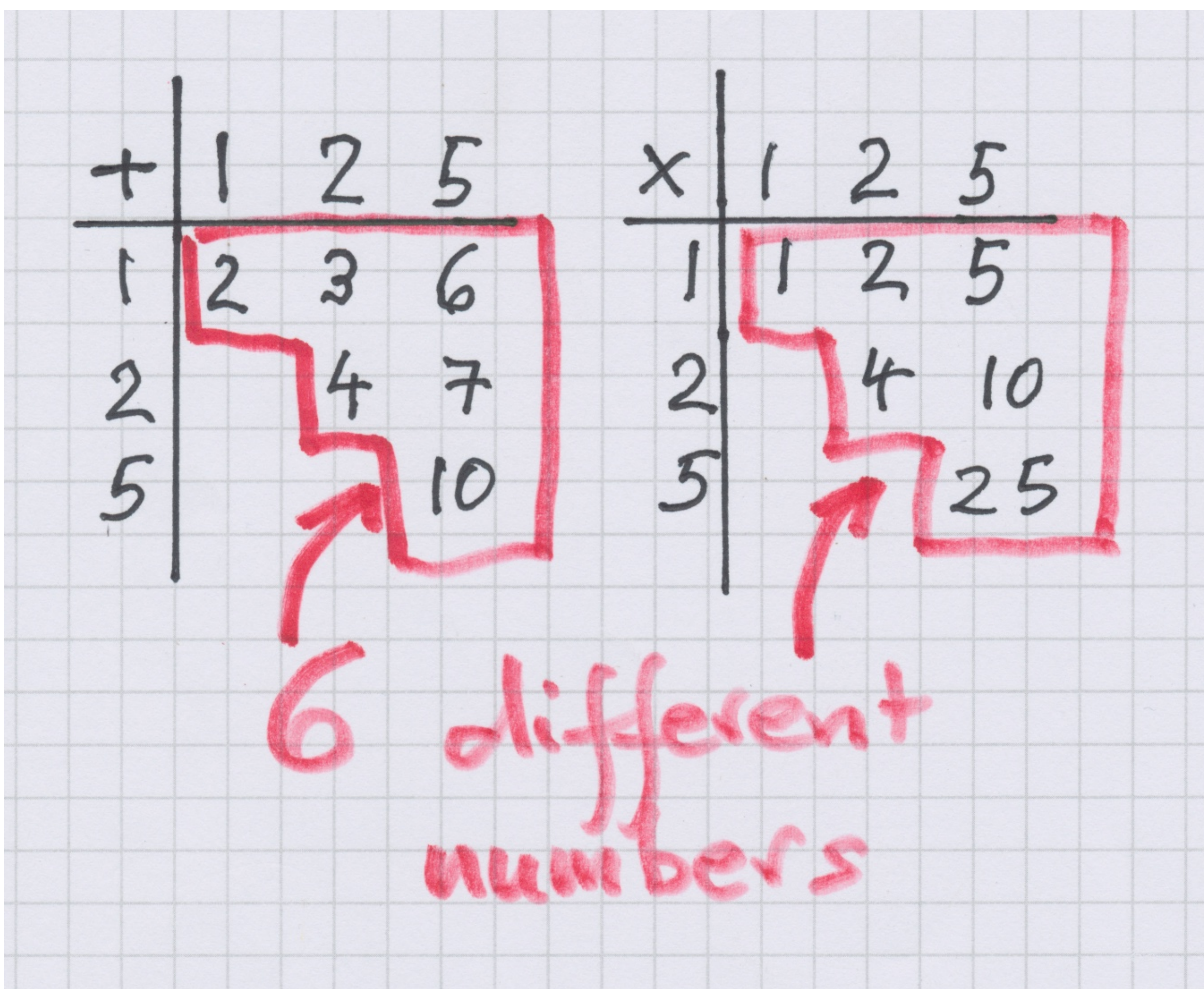
San Diego, 1998:

Clockwise: Doron Zeilberger, RKG, Susanna Cuyler, me, Max Alekseyev, Mohammad Azarian, Christian Bower (Photo: Christopher Hanusa)

Two Days Ago!

Jean-Paul Delahaye

1, 2, 5, 7, 15, 22, 31, 50, … A337655



Is there a formula?

Are these numbers related to some other problem?

We need more editors

We are swamped with submissions

No pay, but lots of fun

Work as much or as little as you like

Contact   njasloane@gmail.com

Requirements: Familiarity with Math, English, and the OEIS