

591

2851

TECHNOLOGICAL UNIVERSITY EINDHOVEN - Department of Mathematics

PO BOX 513 - EINDHOVEN
THE NETHERLANDS

Dr. N.J.A. Sloane
Bell Laboratories
600 Mountain Avenue
MURRAY HILL, New Jersey 07974
U.S.A.

EINDHOVEN,

October 30, 1975

Dear Neil,

The authors of the scheduled report [1] have a correction and an addition to page 72 (sequence 595) of your book: A handbook of integer sequences. We claim that the number $N(n)$ of nonisomorphic connected cubic graphs on n vertices is:

$n =$	4	6	8	10	12	14
$N(n) =$	1	2	5	19	85	509

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The number $N(12) = 87$ appearing in your book is false. As a matter of fact, your reference HA5, K01 seems not to be very reliable. We do not know, and would like to know, whether R.W. Robinson has published anything, as was announced in [2]. The number $N(12) = 86$ appears in [3], but the enumeration contains 2 graphs which are isomorphic. For our $N(n) = 85$ we have 3 independent checks, namely by the first, and by the second author of [1], and by [3] if one of the 2 isomorphic graphs is deleted. Our claim $N(14) = 509$ will be published in [1], and is based on a computer search. Appended is an outline of the scheduled [1]; we are open to your additional suggestions.

By separate mail I send you some preprints. In addition, Delsarte, Goethals and I have done work on Spherical codes and designs, along the lines of [Bounds], but with the additional notion of spherical designs, a refinement of the notion of eutactic stars due to good old Schläfli. We shall send you a preprint when it is available.

I hope everyone is well, especially Ann and Jessie. Please give regards,

sincerely yours,

Jaap

J.J. Seidel.

Abstract for the planned T.H.-Report

Computer investigation of cubic graphs

F.C. Bussemaker, D. Cvetković, S. Čobeljić, J.J. Seidel.

All cubic graphs with up to 14 vertices were generated and their characteristic polynomials and spectra calculated. We intend to represent all this material with some comments. We correct a mistake in a recent Russian paper about cubic graphs on 12 vertices. We have found 3 pairs of cospectral graphs on 14 vertices. We shall quote the numbers of cubic graphs with up to 14 vertices with certain properties (bipartite, without triangles, without quadrangles etc.). We shall illustrate the relation between the second eigenvalue and the connectivity of a graph.

The proposed table of graphs will be unique in the literature in the sense that it represents a non-trivial set of graphs (with several interesting structural properties of its members) but with a reasonably big cardinality.

The report will have 60 - 70 pages.

DEPARTMENT OF
COMPUTER SCIENCE



SOUTHERN METHODIST UNIVERSITY
INSTITUTE OF TECHNOLOGY
DALLAS, TEXAS 75275
214/692-3083

November 3, 1975

N.J.A. Sloane
Mathematics Research Center
Bell Telephone Laboratories, Inc.
Murray Hill, New Jersey

Dear Mr. Sloane:

Several years ago I wrote to you with information on the number of cubic graphs, for your Handbook of Integer Sequences. I subsequently forgot about the book until a student of mine brought it to my attention this fall, whereupon I purchased a copy. I note that you plan to issue supplements to the book. I would appreciate receiving any supplements that have been issued, and being put on the list for future ones.

Incidentally, I have a new sequence counting certain polynomials related to graph coloring, namely $1, 1, 2, 4, 7, 14, 30, 60, \dots$. This does not yet appear in the literature, but when it does I will let you know.

Sincerely yours,

A handwritten signature in black ink, appearing to read "RRK/kth".

Robert R. Korfhage
Professor

RRK/kth

November 6, 1975

Professor J. J. Seidel
Department of Mathematics
Technological University of Eindhoven
P.O. Box 513
Eindhoven
THE NETHERLANDS

Dear Jaap:

Thank you very much for your letter of October 30 and also for the reprints. Some of mine are enclosed in exchange.

Concerning Sequence 595. I have heard nothing about this sequence since the Handbook was published. $N(12) = 87$ was stated in a letter to me, dated September 17, 1971, from R. R. Korfhage, Director, Computer Science and Operations Research Center, Southern Methodist University, Dallas, Texas 75222. In a letter to me dated April 15, 1970, R. W. Robinson said "'Cubic Graphs" may never get written up - the solution available now, to which Harary has referred, is so involuted that I consider it unsatisfactory (through doubtless better in the limit than Parthasarathy's "method")." Since these letters I have heard nothing until your letter which arrived today. I am very happy to have the corrected terms of the sequence, and will put them into the next supplement to the book (long overdue). Couldn't you include the terms of the sequence in your abstract (for concreteness)? Please send me a copy when it is ready, and also of your new paper on spherical codes.

It is fascinating to see root-systems appearing in the "Line graphs ..." paper. A superficial reading produced a few minor comments. Page 1.1, line -9, no comma after "therefore". Line -8, "two thirds" needs a hyphen. Line -8 "The final part is involved since" might better read "The connection with elliptic geometry is that". Page 1.2, line -1, change "in connection" to "related".

Best regards,

MH-1216-NJAS-mv

N. J. A. Sloane

Enc.
As above



Bell Laboratories

600 Mountain Avenue
Murray Hill, New Jersey 07974
Phone (201) 582-3000

November 7, 1975

Professor R. R. Korfhage
Department of Computer Science
Southern Methodist University
Dallas, Texas 75275

Dear Professor Korfhage:

Thank you for your letter of November 3; I should like to hear more about your new graph-coloring sequence 1,1,2,4,7,14,... when it is available. A copy of Supplement I to the Handbook is enclosed, another is long overdue. I also enclose a recent offprint which may amuse you, as it arose out of a coincidence in graph theory.

By another coincidence your name came up yesterday - I received a letter from J. J. Seidel, Math. Dept., Technological University of Eindhoven, Netherlands, claiming that your enumeration of cubic graphs is in error, and giving the values

n:	4	6	8	10	12	14
A_n :	1	2	5	19	85	509,

where A_n is the number of nonisomorphic connected cubic graphs on n vertices. I sent him your address so he will no doubt send you a copy of the paper when it is finished.

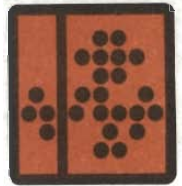
With best regards,

MH-1216-NJAS-mv

N. J. A. Sloane

Enc.
As above

DEPARTMENT OF
COMPUTER SCIENCE AND OPERATIONS RESEARCH



SOUTHERN METHODIST UNIVERSITY
INSTITUTE OF TECHNOLOGY
DALLAS, TEXAS 75275
214/692-3083

December 4, 1975

Dr. N.J.A. Sloane
Bell Laboratories
600 Mountain Avenue
Murray Hill, New Jersey 07974

Dear Dr. Sloane:

Another note on cubic graphs. I just rechecked my catalog and found that indeed, there are not 87 cubic graphs on 12 vertices. However, I believe that Seidel's figure of 85 is also wrong, since I seem to have 86 distinct ones. I am writing directly to Seidel, so this should be straightened out shortly.

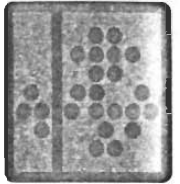
Sincerely yours,

A handwritten signature in blue ink, appearing to read 'RRK Korfhage'.

Robert R. Korfhage
Professor

RRK/ktb

DEPARTMENT OF
COMPUTER SCIENCE AND OPERATIONS RESEARCH



SOUTHERN METHODIST UNIVERSITY
INSTITUTE OF TECHNOLOGY
DALLAS, TEXAS 75275
214/692-3083

January 2, 1975

↑
1976?

J.J. Seidel
Department of Mathematics
P. O. Box 513
Eindhoven
The Netherlands

Dear Professor Seidel:

Thank you for your letter of 11 December. I have verified that your data are correct and mine are wrong, so there are indeed only 85 12-vertex cubic graphs. I look forward to receiving a copy of your report and, if possible, your data on the 14-vertex graphs. I am curious about your motivation for this research. Mine was a byproduct of work on the bandwidth of cubic graphs.

Sincerely yours,

Robert R. Korfhage
Professor

RRK/ktb

xc: N.J.A. Sloane

UNIVERSITY OF CALIFORNIA
LOS ALAMOS SCIENTIFIC LABORATORY
(CONTRACT W-7405-ENG-36)
P. O. Box 1663
Los Alamos, New Mexico 87544
(505-667-4567)

IN REPLY
REFER TO: C-7

July 16, 1971

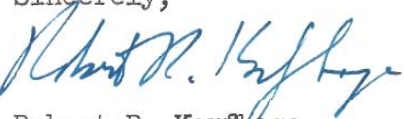
Dr. N. J. A. Sloane
Room 26-363
Bell Telephone Laboratory
Murray Hill, NJ 07971

Dear Dr. Sloane:

87

Paul Stein has shown me your catalog of combinatorial sequences, and indicated that you might be interested in the following sequence: 1, 2, 5, 19, ~~86~~. These are the numbers of cubic graphs on 4, 6, 8, 10, and 12 points respectively, and were determined by me through direct computation of the graphs.

In his latest book on graph theory, Harary refers (in a problem) to the 20 cubic graphs on 10 points. This is in error: the reference which he cites contains two isomorphic graphs. In his earlier book, Graph Theory and Theoretical Physics (Academic, 1967) Harary states (p. 19), "[R. W.] Robinson (unpublished work) has also obtained counting formulas for the number of cubic graphs with a given number of points." As far as I know, Robinson's work has not been published to this day. Hence I suspect that he may have found some error in his formulas. Thus at present no further numbers in this sequence are known, except possibly to Robinson.

Sincerely,

Robert R. Korfhage

RRK/dc

(Visiting Staff Member at LASL; permanent address: Computer Dept., SMU, Dallas)

July 28, 1971

Dr. Robert R. Korfhage
University of California
Los Alamos Scientific Laboratory
P.O. Box 1663
Los Alamos, New Mexico 87544

Dear Dr. Korfhage:

Thank you very much for sending the sequence of cubic graphs, which has been added to the catalog. In a letter to me dated April 15, 1970, R. W. Robinson said: " "Cubic graphs" may never get written up - the solution available now, to which Harary has referred, is so involuted that I consider it unsatisfactory (though doubtless better in the limit than Parthasarathy's "method")."

Thank you again.

With best wishes,

MH-1216-NJAS-bk

N. J. A. Sloane

UNIVERSITY OF CALIFORNIA
LOS ALAMOS SCIENTIFIC LABORATORY
LOS ALAMOS, NEW MEXICO 87544

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SOUTHERN METHODIST UNIVERSITY

INSTITUTE OF TECHNOLOGY
COMPUTER SCIENCE/OPERATIONS RESEARCH CENTER
DALLAS, TEXAS 75222

September 17, 1971

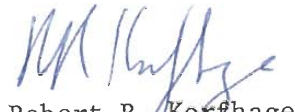
Dr. N. J. A. Sloane
Room 26-363
Bell Telephone Laboratory
Murray Hill, N. J. 07971

Dear Dr. Sloane:

I find that when I wrote you this summer from Los Alamos on the number of cubic graphs, I stumbled over my fingers. A recount of my graph catalog shows that there are 87, rather than 86, cubic graphs on 12 points.

Thank you, incidentally, for the information about the status of Robinson's work.

Sincerely,


Robert R. Korfhage
Director

RRK:mk

1, 2, 5, 19, 87
4 6 8 10 12