HOW CUTTING IS A CUT POINT?

Frank Harary

University of Michigan, Ann Arbor, MI, U.S.A.

and

Phillip A. Ostrand

University of California, Santa Barbara, CA, U.S.A.

The *cutting number* $c(v)$ of a point v of a connected graph G is the number of pairs of points $\{u,w\}$ of G such that $u, w \neq v$ and every $u - w$ path contains v. Obviously $c(v) > 0$ if and only if v is a cut point. For a graph G of connectivity 1 (see [1]* for definitions), let $c = c(G)$ be the maximum cutting number of a point. Then the *cutting center* of G is the set of all points v such that $c(v) = c(G)$.

It is easy to see that every tree T with $p \geq 3$ points has a cutting center which induces a connected subgraph, i.e., a subtree.

Let n be the number of points in the cutting center of T and call $c = c(T)$ the *cutting number* of T.

Theorem 1. *For any tree T with $p \geq 3$ points, the cutting center of T is a path.*

In order to prove this assertion, it is sufficient to verify numerically that no tree T contains a subtree of the form $K_{1,3}$ in which all 4 points u, v_1, v_2, v_3 have the same cutting number. For this purpose, it is convenient to include a modest lemma.

Lemma. Let u, v and w be three points of T with equal cutting numbers and uv, vw lines of T such that in $T - uv - vw$, the components containing u, v, w have a, b, c points respectively. Then $2b \leq a$ and $2b \leq c$.

* Harary, Graph Theory, Addison Wesley '69