Proof of Lemma. Let a^* be the cutting number of u in its component, and let b^*, c^* be defined similarly.

Note that $0 \leq a^* \leq \binom{a-1}{2}$ so that $2(a^*+a) < (a+1)^2$. Without loss of generality, suppose $c \leq a$. Obviously in T we find the cutting numbers

$$c(u) = a^* + (a-1)(b+c) \tag{1}$$

$$c(v) = b^* + (b-1)(a+c) + ac \tag{2}$$

$$c(w) = c^* + (c-1)(a+b). \tag{3}$$

By hypothesis $c(u) = c(v)$ and $c(v) = c(w)$, so that

$$a^* + a = b^* + (c+1)b \tag{4}$$

$$c^* + c = b^* + (a+1)b. \tag{5}$$

Since $b^* \geq 0$, we see by (4) that

$$2(c+1)b < (a+1)^2 \tag{6}$$

and similarly by (5) that

$$2(a+1)b < (c+1)^2, \tag{7}$$

so that

$$4b^2 < (a+1)(c+1). \tag{8}$$

Thus our supposition that $c \leq a$ yields

$$4b^2 < (a+1)^2 \tag{9}$$

and we have

$$2b \leq a. \tag{10}$$

By (7) and (10) we obtain at once

$$4b^2 < 2b(a+1) < (c+1)^2, \text{ whence } 2b \leq c.$$

Proof of theorem. Consider a tree T containing a subtree $K_{1,3}$ (see Figure 1) in which the four points have equal cutting numbers in T.

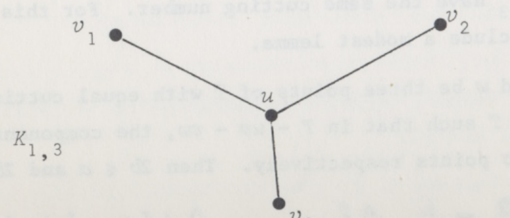

Figure 1. An impossible cutting center

In the graph obtained from T by removing the lines of $K_{1,3}$ each of the four points v_1, v_2, v_3, u lies in a different component; let these contain a, b, c, d points respectively.

Again without loss of generality, let $a \leq b \leq c$. Upon applying the lemma to T and regarding v_1, u, v_2 as u, v, w, we have $2(c+d) \leq b \leq c$. But this contradicts $c > 0$ and completes the proof of the theorem.

The strongest possible assertion subject to the restriction imposed by Theorem 1 can now be made!

Theorem 2. *For every positive integer n, there exists a tree T with a cutting center consisting of n points on a path.*

The proof is too long for inclusion here, and will eventually appear elsewhere.

The trees which we believe to have the smallest number p of points with cutting center containing $n \leq 5$ points are shown in Figure 2, while Table 1 contains the information p, n, and $c(T)$ for each tree in the figure. In general, it is an unsolved problem to determine this minimum p, given n.

Size of cutting center   pts in graph   max cutting no

Table 1

n	p	c
1	3	1
2	4	2
3	7	9
4	10	20
5	50	670

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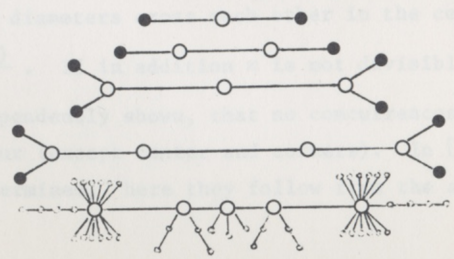

Figure 2