

Aug 18, 1972

Dear Dr. Sloane,

~~A~~2988

A81

A55

A220

A14

A2989

A2990

A2991

A2992

Enclosed you will find a preprint of the paper counting the number of unlabeled, nonisomorphic spanning trees of a wheel. It is opened to the table for your convenience. Note that the first three members of the sequence are arbitrary since there are no wheels for  $n=1, 2,$  and  $3$ . The values of  $1, 1,$  and  $1$  were chosen partly because they sounded plausible, but mainly because these are the numbers that fell out when we turned the crank on our generating function!

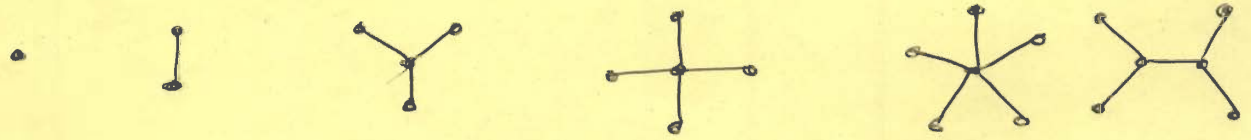
Knowing that you are interested in cataloging sequences, I have enclosed <sup>other</sup> two items which might interest you.

First is a table giving the number of trees, rooted trees, identity trees, and homeomorphically irreducible trees up to 39 points. As far as I know, these numbers had not been calculated beyond 26 for the first two columns, and 12 for the last two. I do not plan to publish this table, but it will be used to extend the existing table in the forthcoming 2nd edition of Graph Theory by Frank Harary.

The last Table gives trees with a forbidden limb. To give an example, the small trees not containing



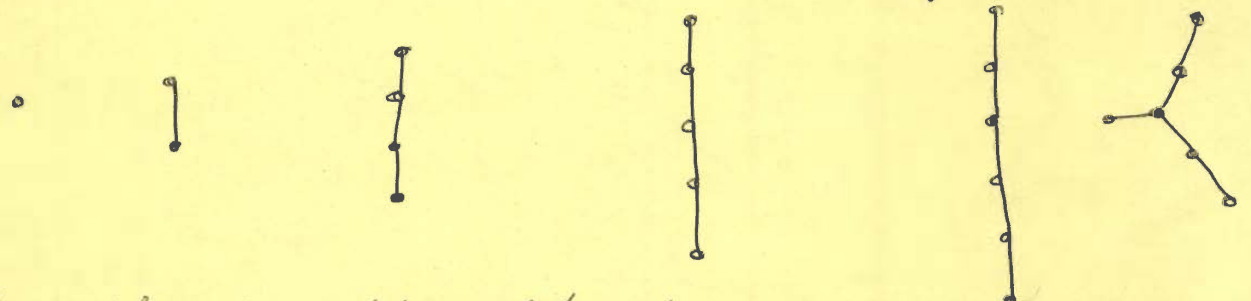
are



Similarly, the small trees avoiding



are



The idea is, that the forbidden rooted tree must not occur as a subtree in such a manner that the root is its only connection to the rest of the tree. (the limb)

An interesting lemma is that if  $T_1$  and  $T_2$  are two rooted trees of the same size, then the number of trees avoiding  $T_1$  as a limb is the same as the number of trees avoiding  $T_2$ !

One can conclude that given any tree  $T_1$ , then it is ~~at a limb~~, asymptotically, a limb in almost every sufficiently large tree.

All of this will appear in a forthcoming paper, "Trees with forbidden limbs". Now that I think about it, it seems a bit obscure for your catalog.

Sincerely,  
Allen J. Schwenk

THE NUMBER OF TREES OF VARIOUS SPECIES

A81  
N 454

55  
N 299

220  
N 1022

2988  
14

POINTS ROOTED TREES ORDINARY TREES IDENTITY TREES HOMEO IRRED

POINTS	ROOTED TREES	ORDINARY TREES	IDENTITY TREES	HOMEO	IRRED
1	A81 1.	A55 1.	A220 1.	A14 1.	
2	1.	1.	-0.	1.	
3	2.	1.	-0.	-0.	
4	4.	2.	-0.	1.	
5	9.	3.	-0.	1.	
6	20.	6.	-0.	2.	
7	48.	11.	1.	2.	
8	115.	23.	1.	4.	
9	286.	47.	3.	5.	
10	719.	106.	6.	10.	
11	1842.	235.	15.	14.	
12	4766.	551.	29.	26.	
13	12486.	1301.	67.	42.	
14	32973.	3159.	139.	78.	
15	87811.	7741.	310.	132.	
16	235381.	19320.	667.	249.	
17	634847.	48629.	1480.	445.	
18	1721159.	123867.	3244.	842.	
19	4688676.	317955.	7241.	1561.	
20	12826228.	823065.	16104.	2988.	
21	35221832.	2144505.	36192.	5671.	
22	97055181.	5623756.	81435.	10981.	
23	268282855.	14828074.	184452.	21209.	
24	743724984.	39299897.	418870.	41472.	
25	2067174645.	104636890.	955860.	81181.	
26	5759636510.	279793450.	2187664.	160176.	
27	16033734329.	751065460.	5025990.	316749.	
28	45007066269.	2023443032.	11580130.	623933.	
29	126136554308.	5469566585.	26765230.	1256070.	
30	354426847597.	14830871802.	62027433.	2515169.	
31	997171512998.	40330829030.	144133676.	5049316.	
32	2839934352700.	109972410221.	335731381.	10172638.	
33	7929819784355.	300628862480.	783859852.	20543579.	
34	22409533673568.	823779631721.	1834104934.	41602425.	
35	63411730258053.	2262366343746.	4300433063.	34440886.	
36	179655930440464.	6226306037178.	12102854473.	171794492.	
37	509588049810620.	17169677490714.	23778351222.	350238175.	
38	1447023384581028.	47436313524262.	56063415268.	715497037.	
39	1113254119923149.	131299543779126.	132404269770.	1464407113.	

TABLE GENERATED BY ALLEN J. SCHWENK, UNIVERSITY OF MICHIGAN

↑  
Entered

N295.2  
new

N410.5  
new

N131.5  
new

N270.5  
new

THE NUMBERS OF TREES WITH A FORBIDDEN LIMB OF K LINES

2988

2989

2990

2991

PTS	K=2	K=3	K=4	K=5
1	1	1	1	1
2	1	1	1	1
3	-0	1	1	1
4	1	1	2	2
5	1	2	2	3
6	2	4	5	5
7	3	7	9	10
8	6	14	19	21
9	10	28	38	43
10	21	61	86	97
11	39	131	188	215
12	82	297	430	503
13	167	678	1026	1187
14	360	1592	2472	2876
15	766	3770	5997	7033
16	1692	9096	14835	17510
17	3726	22121	36964	43961
18	8370	54451	93246	111664
19	18866	135021	236922	285809
20	43029	337651	607111	737632
21	98581	849698	1565478	1915993
22	227678	2152048	4062797	5008652
23	528196	5479408	10599853	13163785
24	1232541	14022947	27797420	34774873
25	2888142	36048514	73224806	92282214
26	6798293	93061268	193709710	245930746
27	16061348	241160180	514456793	657931603
28	38086682	627179689	1370937140	1766481135
29	90607902	1636448181	3665714528	4758553683
30	216230205	4282964600	9931901555	12858286083
31	517482053	11241488953	26445886506	34844908142
32	1241778985	29584389474	71325268179	94681272368
33	2987268628	78052340510	192949235240	257918440679
34	7203242490	206409057990	522656230515	704251982382
35	17407585553	547053894451	1419635589561	1927257770333
36	42155516333	1452399097840	3964072295001	5235210142606
37	102287480657	3866287546989	10539295915634	14522600748253
38	248655424863	10307661280114	29794263037745	39979707209464
39	605531960590	27529021600841	79815047905374	110256666505366

TABLE GENERATED BY ALLEN J. SCHWENK, UNIVERSITY OF MICHIGAN

new  
W295.3

55

2992

K=6	K=7	K=8	K=INFINITY (ORDINARY TREES)
1	1	1	1.
1	1	1	1.
1	1	1	1.
2	2	2	2.
3	3	3	3.
6	6	6	6.
10	11	11	11.
22	22	23	23.
45	46	46	47.
102	104	105	106.
226	231	232	235.
531	542	547	551.
1253	1281	1292	1301.
3044	3111	3139	3159.
7456	7626	7693	7741.
18604	19034	19205	19320.
46798	47911	48343	48629.
119133	122028	123148	123867.
305567	313200	316114	317955.
790375	810611	818302	823065.
2057523	2111630	2132030	2144505.
5390759	5536228	5590815	5623756.
14200122	14592522	14749361	14828074.
37598572	38667436	39064799	39299397.
100005401	102922632	104002872	104636890.
267131927	275124634	278074677	279792450.
716318650	738296510	746383687	751065460.
1927758155	1988386222	2010636670	2023443132.
5205240762	5372987480	5434402057	5469566585.
14098580633	14563962724	14723981734	14832871902.
38296720823	39591072272	40063022568	40330829030.
104308468102	107916705178	109230060068	109972410221.
284822276099	294002405596	298565659535	300628862420.
779575772518	807791947421	818031476010	823770631721.
2138509120146	2217637535736	2246315824848	2262366343746.
5878638470347	6100926139879	6191395134205	6226306037178.
16192131673139	16917590647147	17043769550634	17168677490714.
44683394099379	46445934196984	47282694445267	47426313524262.
123526445657157	128500236248793	130295715724948	131290543779126.

f  
91

2988

1			1
2			1
3			0
4	$b_1(x)$		1
5			1
6	trees		2
7			3
8	without		6
9			10
10	bunches		21
11			39
12			82
13			167
14			360
15			766
16		1	692
17		3	726
18		8	370
19		18	866
20		43	029
21		98	581
22		227	678
23		528	196
24		1 232	541
25		2 888	142
26		6 798	293

27				
28		16	061	348
29		38	086	682
30		90	607	902
31		216	230	205
32		517	482	053
33		1 241	778	985
34		2 987	268	628
35		7 203	242	490
36		17 407	585	653
37		42 155	516	330
38		102 287	480	657
39		248 655	424	863
40		605 531	960	590
41	1	477 070	392	668
42	3	608 722	924	344
43	8	830 000	291	196
44	21	636 715	579	281
45	53	090 458	066	417
46	130	439 469	811	033
47	320	880 862	850	687
48	790	309 595	013	536
49	1 948	710 391	176	557
50	4 810	312 306	273	531
51	11 886	548 808	279	065
	29 401	979 849	243	464

from KLM