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April 3, 1973

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Professor R.K. Guy Mathematics Department University of Calgary Calgary, Alberta

Dear Professor Guy:

Suppose there are n objects each of which is either good or bad and you want to know which are which. You could test all n items separately but in certain applications it is more efficient to conduct a series of group tests. You subject a subset X of the items to a test that reveals either that all members of X are good or that at least one member of X is bad. In a nested group testing procedure we select the subset to be tested at each stage from the last subset we have isolated that we know contains at least one bad object -- provided there is such a subset and it contains at least two objects. If there isn't we choose the next subset for testing from the objects that have not yet been classified. In general we have some discretion on the size of the subset to be tested at each stage and various papers have dealt with the problem of determining the sizes that minimize the expected number of tests needed to classify all the items under certain assumptions.

Milton Sobel was here recently and said that it would be of some interest to have a formula for the total number f(n) of nested group testing procedures for classifying n objects. I was able to show that

$$f(n) = C_{n+1} f(n-1) f(n-2) ... f(1)$$

where

$$C_k = \frac{1}{k} \binom{2k-2}{k-1}$$

which implies that

$$f(n) = 4^{2^{n}-1} \prod_{i=1}^{n} \{1-3/2(\xi+1)\}^{2^{n-i}}$$

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and that

$$\lim_{n \to \infty} f(n)^{1/2^n} = 1.5267...$$

The proof of (*) is by induction and is easy enough once you know the answer (actually you seem to need to introduce an extra parameter).

Sobel suggested I write up the argument with the view of perhaps submitting it as a joint note. I sent him a draft for polishing but I haven't heard from him yet.

Sincerely yours,

J.W. Moon

JWM/mj

cc. N.J.A. Sloane



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FACULTY OF ARTS AND SCIENCE / DEPARTMENT OF MATHEMATICS, STATISTICS AND COMPUTING SCIENCE

March 14, 1973.

Dr. Neil J.A. Sloane, Bell Laboratories, Murray Hill, NEW JERSEY 07974. U.S.A.

Dear Neil,

Thank you very much for sending copies of your papers, two in Discrete Math. and one in J.C. θ . I do in fact subscribe to both journals, but it is convenient to have separata for filing under author, etc.

I enclose copies of two circulars I am sending out in connexion with the Research Problems section of the Monthly. Your own problem won't appear in time to get external feedback, but you may learn something more about it before it is published; if so I would be glad to hear. Also, you take an interest in the section more generally, and may have interesting comments on other problems.

John Moon wrote to ask me if 1,1,2,10,280,235200,... is in the Handbook. It isn't! He said "each number is a Catalan number times the product of all the earlier numbers", i.e. "double factorial Catalan", but presumably it has arisen in some other context. "Single factorial Catalan" would be 1,2,10,140,5880,... which is also not in.

Another item: do you know anything about "U-numbers"? They are in the Handbook: 1,2,3,4,6,8,11,13,16,18, ... a self-generating sequence. They are the subject of some problems submitted to the Monthly Research Problems section, which I am inclined to publish, but I am not sure if the referee has all the latest dope on them. Perhaps there isn't any!

Best wishes,

Yours sincerely,

Richard K. Guy.

Enclosures.

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RKG: iw