Self Numbers

(last updated 1/3/2003)

Start with a number, known as the **generator**, and **repeatedly add the sum of the digits**. Numbers which **cannot be generated** by this process were originally called Columbian numbers and are now called **self numbers**. In **base b** we also call them **b-self numbers.**

The **first** few self numbers are (Sloane's A003052):

1, 3, 5, 7, 9, 20, 31, 42, 53, 64, 75, 86, 97, 108, 110, 121, 132, 143, 154, 165, 176, 187, 198, 209, 211, 222, 233, 244, ...

For **odd bases b the b-self numbers are exactly the odd numbers.** The proof is easy because for $x = a_k a_{k-1} \dots a_1 a_0$ with $0 \le a_i < b$:

$$x + SOD(x) = (2a_0) + a_1(b^1+1) + a_2(b^2+1) + \dots$$

and the right side is always even. Therefore all odd numbers are b-self. Conversely each even number can be written in the above form and is not b-self.

For **even bases** the b-self numbers up to b are again exactly the odd numbers but above b the situation is more complex and **no easy characterization is known.** However there are **infinitely many b-self numbers** since an infinite sequence can be generated from the following recurrence relation:

$$C_1 = b-1,$$
$$C_k = (b-2)b^{k-1} + C_{k-1} + b-2. \quad (k\ge 2)$$

(for b=2 use $C_k = 2^j + C_{k-1} + 1$, where j is the number of digits of C_{k-1})

For even bases b the following can be proven:
If $n \ge b$ is a self number then n has the form $n = (b-1) + a_1(b^1+1) + a_2(b^2+1) + \dots$ with $0 \le a_i < b$.

Note that the reverse direction is not true.

References

Cai, T.: On k-Self Numbers and Universal Generated Numbers,
Fibonacci Quarterly 34, 1996, pp. 144-146.
Joshi, V.S.: Ph.D. dissertation,
Gujarat University, Ahmadabad, 1973.
Patel, R.B.: Some Tests for k-Self Numbers,
Math. Student 56, 1991, pp. 206-210.
Weisstein, Eric W.: Self Number,
CRC Concise Encyclopedia of Mathematics, Chapman & Hall/CRC 1999.