Scan A3102

Silverman

1 sheet

hyperbola exactly once each. Whence B_3 lies in region I or III and has the same sign as A_1.

Also solved by H. Kaye, D. Smith and the proposer.

159.* Proposed by David L. Silverman, Beverly Hills, California.

If A_n denotes the largest integer divisible by all the integers less than its n^{th} root, show that $A_4 = 24$ and $A_5 = 420$. Find a general formula for A_n.

Editorial Note: A partial answer is given by Mathematics Review 1085, Aug., 1965:

"Ozeki, Nobuo

On the problem 1, 2, 3, ..., $[n^{1/k}] | n$.

J. College Arts Sci. Chiba Univ. 3 (1961/62), 427-431.

It is proved that 720720 is the largest integer which is divisible by all the positive integers that do not exceed the 5^{th} root of n. Similar results for k^{th} roots are proved in the cases $6 \leq k \leq 10$. The results for k = 2, 3, 4 are known."

160. Proposed by Sidney Kravitz, Dover, New Jersey.

"I have here," said the editor, "a cryptarithm which shows a two digit number being multiplied by itself. You will note that the subproducts are not shown, only the number being squared and the final product."

"Well," said the reader, "I've tried to solve this cryptarithm but the solution is not unique. It is possible that I might be able to give you the answer if you told me whether the number being squared is odd or even."

"The number being squared is odd," said the editor.

"Good," said the reader. "I was hoping you would say that. I now know the answer."

What is the solution to this unique cryptarithm?

Solution by Charles W. Trigg, San Diego, California.

Consider all the following possible patterns with their solutions:

1. AB^2 = CDEA; $42^2 = 1764$, $48^2 = 2304$, $93^2 = 8649$.
2. AB^2 = CDEF; AB = 53, 57, 59, 79, or 54, 72, 84.
3. AB^2 = BCDE; AB = 52 or 87.
4. AB^2 = ACDB; AB = 95 or 96.
5. AB^2 = CDDB; AB = 35, 65, 85, or 46.
6. AB^2 = CDCB; AB = 45, 81, 91, or 56.

[MR 30 (1965) p213 #1085]

7. AB^2 = CDEB; AB = 36, 86, or 51, 61, 71.
8. AB^2 = CCDE; AB = 34, 58, or 47, 67.
9. AB^2 = CAAB; AB = 76.
10. AB^2 = BCDB; AB = 41 or 75.
11. AB^2 = CDBE; AB = 32, 78, or 82.
12. AB^2 = CBDE; AB = 73 or 89.
13. AB^2 = BCDA; AB = 64.
14. AB^2 = CBAD; AB = 74.
15. AB^2 = BCAC; AB = 63.
16. AB^2 = ACDE; AB = 98.
17. AB^2 = CADE; AB = 37 or 49.
18. AB^2 = CDAE; AB = 43 or 69.
19. AB = CADC; AB = 68.
20. AB = CAAD; AB = 83.
21. AB = ACDA; AB = 97.
22. AB = CDEC; AB = 39.
23. AB = CDED; AB = 92.
24. AA = CDEF; AB = 33 or 44.
25. AA = CDEA; AB = 55 or 66.
26. AA = ACDE; AB = 99.
27. AA = CDED; AB = 77.

The three digit possibilities are given by

AB^2: CDB(16,31); ACD(13,14); CDE(17,29,18,24); CCB(15,21); CAB(25); BCB(26); BAC(27); CBD(28); CAD(23); CDA(19).

AA^2: ACA(11); CDC(22).

From the question asked and the answer given it follows that the particular pattern must lead to only one odd value of AB and more than one even value. This corresponds to (1) and the value

$$93^2 = 8649.$$

Also solved by H. Kaye, Paul Meyers, K. S. Murray, M. Wagner, F. Zetto and the proposer.

162. Proposed by M. S. Klamkin, Ford Scientific Laboratory.

If a surface is one of revolution about two axes, show that it must be spherical.