Scan A 3116
elr

H P Robinson

letter + notes

4 pages

19 Nov 1973

3116

→ 3105

HERMAN P. ROBINSON
31 DIABLO CIRCLE
LAFAYETTE, CA 94549
(415) 283-1861

A3319
A111537

19 November 1973

Dr. Neil J.A. Sloane
Bell Laboratories
600 Mountain Avenue
Murray Hill, New Jersey 07974

Dear Neil:

Here's another set of sequences. I needed the coefficients in the reciprocals of $1 \pm x + 2!x^2 \pm 3!x^3 + \ldots$ and $1 \pm 2!x + 3!x^2 + \ldots$, so here they are, for a few terms.

After calculating the above I realized that the program could be used to reciprocate Lehmer's series, so I ran that also, finding that one of the terms I gave you in my last letter should have been 72 instead of 74. My program is not nearly as versatile as ALTRAN. It is intended only for reciprocation with integer coefficients. It will handle 100, provided that the coefficients do not exceed 12 digits. So, no need for you to bother with the problem, if you haven't already done so.

Sincerely,

Herman

HERMAN P. ROBINSON
31 DIABLO CIRCLE
LAFAYETTE, CA 94549
(415) 283-1861

18 November 1973

These are the coefficients of the reciprocal of

A3319	A111537
$1+1!x+2!x^2+3!x^3+4!x^4+\cdots$	$1+2!x+3!x^2+4!x^3+\cdots$
1	1
-1	-2
-1	-2
-3	-8
-13	-44
-71	-296
-461	-2312
-3447	-20384
-29093	-199376
-273343	-2138336
-2829325	-24936416
-31998903	-314142848
-392743957	-.425277382400ex 10
-.520106145500ex 10	-.615948473600ex 11
-.739434244130ex 11	

A3319 again	A111537 again
$1-1!x+2!x^2-3!x^3+4!x^4-\cdots$	$1-2!x+3!x^2-4!x^3+\cdots$
1	1
1	2
-1	-2
3	8
-13	-44
71	296
-461	-2312
3447	20384
-29093	-199376
273343	2138336
-2829325	-24936416
31998903	314142848
-392743957	-.425277382400ex 10
.520106145500ex 10	.615948473600ex 11
-.739434244130ex 11	

3116
= N 403.5

Coefficients of the reciprocal of $P(\sqrt{x}) = \begin{vmatrix} 1 & \sqrt{x} & 0 & \cdots & \\ \sqrt{x} & 1 & x & 0 & \cdots \\ 0 & x & 1 & x^{3/2} & \cdots \\ 0 & 0 & x^{3/2} & 1 & x^2 \cdots \\ \vdots & \vdots & & & \end{vmatrix} = 1 - x - x^2 - x^3 + x^6 + x^7 + \ldots$

1	1	2	4	7	13	23	41	72	127
222	388	677	1179	2052	3569	6203	10778	18722	32513
56455	98017	170161	295389	512755	890043	1544907	2681554	4654417	8078679
14022089	24337897	42242732	73319574	127258596	220878683	383371749	665405119	.115492020200ex 10	.200455363800ex 10

.347923117800ex 10 .603877462600ex 10 .104812796080ex 11 .181919710500ex 11 .315751324060ex 11

.548037891140ex 11 .951209063330ex 11 .165097829503ex 12 .286554175616ex 12 .497361442811ex 12

H.P. Robinson 19 November 1973

Coefficients of $P(\sqrt{x})$

1	-1	-1	-1	0	0	1	1	2	1
2	1	1	0	0	-2	-1	-3	-3	-4
-3	-5	-3	-4	-2	-3	0	-1	3	2
5	5	9	7	11	9	13	10	13	9
12	7	9	3	5	-3	-1	-9	-7	-17
-15	-24	-21	-31	-27	-37	-31	-40	-33	-41
-31	-39	-27	-33	-18	-24	-6	-11	9	5
26	23	48	42	67	62	88	80	107	96

31/6
= N403.5

Lehmer's Determinant

$$f_n = \det \begin{vmatrix} 1 & q & & & & & & 0 \\ q & 1 & q^2 & & & & & \\ & q^2 & 1 & q^3 & & & & \\ & & q^3 & 1 & q^4 & & & \\ & & & q^4 & 1 & \ddots & & \\ & & & & \ddots & \ddots & & \\ & 0 & & & & & 1 & q^{n-1} \\ & & & & & & q^{n-1} & 1 \end{vmatrix}_{n\times n}$$

Also let

$$g_n = \det \left|\begin{array}{cccc|c} 1 & q & 0 & & \\ q & 1 & q^2 & & \\ & q^2 & 1 & q^{n-2} & \\ 0 & & q^{n-2} & 1 & q^{n-1} \\ \hline & & & \cdot & q^n \end{array}\right| = q^n f_{n-1}$$

Then $f_n = f_{n-1} - q^{n-1} g_{n-1}$

$$\boxed{f_n = f_{n-1} - q^{2n-2} f_{n-2}}$$

$f_1 = 1$

$f_2 = 1 - q^2$

$f_3 = (1-q^2) - q^4 = 1 - q^2 - q^4$

$f_4 = (1-q^2-q^4) - q^6(1-q^2) = 1 - q^2 - q^4 - q^6 + q^8$

$f_5 = (1-q^2-q^4-q^6+q^8) - q^8(1-q^2-q^4)$

$= 1 - q^2 - q^4 - q^6 + q^{10} + q^{12}$

$\boxed{f_\infty = f_n \bmod q^{2n}}$ ie discard q^{2n} & above