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M. Gardner &
NSAS,
correspondence,
1973 - 1974

6 pages

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3154
3052
3215
5188
6051

MARTIN GARDNER
10 EUCLID AVENUE
HASTINGS-ON-HUDSON
NEW YORK 10706

Dear Mr. Sloane: 18 Oct 73

Your Handbook is a thing of beauty! My first reaction is to keep its existence a secret, and use it as a secret tool in my work, but of course that wouldn't be fair to others. I shall certainly want to plug it in the column early next year, as soon as I know for sure when Academic expects to publish. I'll either do a column on puzzle sequences in general, or a column on Catalan numbers and their endless applications, or a column on some other particular sequence that is rich in recreational angles. Incidentally, a surprising new application of Catalan numbers was discovered recently by J. H. Conway. (See "Triangulated polygons and Frieze Patterns" by Conway and Coxeter, in Math. Gazette, Vol. 57, 1973, pp. 87-94.)

star numbers

I have a column coming up next year on a new type of figurate number which, so far as I know, I'm the first to worry about. I call them star numbers because they correspond to the six-pointed star configurations such as the one used in the game of Chinese checkers. The simple formula is $6x(x+1)+1$, and the series goes;

new 3154

1, 13, 37, 73, 121, 181,.....

This may have been studied before, but I couldn't locate any references. There are some interesting problems, such as proving that there is an infinity of square stars starting with 1, 121,...)*

I suppose you know of H. P. Robinson's table of Mathematical Constants (1971) which lists irrational decimal fractions in such a way that you can find on the list Any such number and find out what formula generates it. (If not, let me know and I'll tell you how to obtain it). But your table of integer sequences will be even more valuable for my purposes, and for thousands of others. Thank you for letting me see it in advance. I'll be giving it a good sendoff ~~[illegible]~~ early in 1974. You did a marvelous job of assembling the sequences, organizing them, and explaining them so simply and clearly.

Best,

Martin

*Next is 11881
where X = 44

√ 591

Bell Laboratories

600 Mountain Avenue
Murray Hill, New Jersey 07974
Phone (201) 582-3000

February 4, 1974

Mr. Martin Gardner
10 Euclid Avenue
Hastings-on-Hudson
New York, New York 10706

Dear Martin:

I have just added your star numbers $6n(n+1) + 1$ (from your letter of 18 October) to the supplement to my book. As far as I know you are the first to study these numbers.

Your columns and books have supplied quite a few very interesting sequences, and it occurs to me that perhaps you have obtained more terms in these sequences since the columns appeared. In particular, are any more terms known in the following sequences?

(a)							
n	1	2	3	4	5	6	7
a_n	1	1	3	5	10	19	39

a_n = number of topologically distinct patterns that can be made with n matches (from The Unexpected Hanging, p. 80)

(b)									
n	2	3	4	5	6	7	8	9	10
b_n	1	1	6	15	255	1897	92,263	1,972,653	213,207,210

b_n = number of ways of halving an n×n board (same ref., p. 189)

(c)									
n	3	4	5	6	7	8	9	10	11
c_n	3,	6,	11,	17,	25,	34,	44,	55,	72

Minimum-length Golomb rulers. (From your column of June 1972, p. 116.) Have the values $c_{12} = 85$, $c_{13} = 111$ been rigorously proved yet?

(d)

n	1	2	3	4	5
d_n	1	2	12	2688	1,813,091,520
d_n^1	1	2	18	5712	5,859,364,320

Hamilton paths on n-cubes (from your column of April 1973, p. 111).

Any new values you can send me in these sequences would be greatly appreciated. And of course, if you can suggest other sequences I should include they would be most welcome.

With best wishes,

MH-1216-NJAS-mv

Neil Sloane

√ 591

Bell Laboratories

600 Mountain Avenue
Murray Hill, New Jersey 07974
Phone (201) 582-3000

February 15, 1974

Mr. Martin Gardner
10 Euclid Avenue
Hastings-on-Hudson
New York, New York 10706

Dear Martin:

Thank you so much for the new sequences and suggestions - they are all very much appreciated. I shall certainly check the Numerology of Dr. Matrix.

If you could possibly lend me one of D. R. Kaprekar's pamphlets on self- or Colombian numbers, I promise to return it instantly. Is the first of the pamphlets the most typical?

By the way, sequence 1828 shows primes of the form x^3-y^3, which are what A. J. C. Cunningham calls cuban primes. So this is not in fact a subsequence of your sequence $x^3 - (x-1)^3$, which gives the differences of consecutive cubes.

Thank you again for all your help.

Best regards,

MH-1216-NJAS-mv

N. J. A. Sloane

Ent'd

[MG]

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MARTIN GARDNER

10 EUCLID AVENUE
HASTINGS-ON-HUDSON
NEW YORK 10706

Dear Neil: 5 Feb 74

I just checked my files and find that I have no higher values for any of those sequences you list. So far as I know, the Golomb rulers are still confirmed only through n = 11.

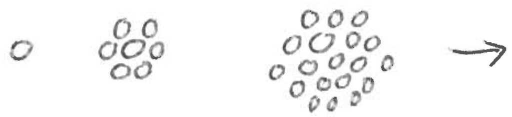
It's hard to find nontrivial sequences not in your handbook or the supplement! I can add one better reference. The so-called Colombian numbers in that problem proposed [redacted] in AMM, April/73:

1,3,5,7,9,20,31,42,53,64,75.... 3052

were studied in considerable depth by the Indian number theorist, D.R. Kaprekar. He published [redacted] several pamphlets on these numbers, which he called "self-numbers." The first one, Puzzles of the Self-Numbers, was published in English in 1959. You're welcome to borrow it if you like. I also have his later pamphlets on these numbers. The current year, 1974, is a self-number! (The next is 1985).

I wish I could tell you which month this year will be my column on star numbers, but I haven't yet scheduled it. I have worked out the formulas and recursive procedures for both square stars and triangular stars. Are you interested in these sequences? (They will be in the column)

You might also want to include what I call the "centered hexagonal" numbers:



1 7 19 . . .

3215

good lib

I was unable to find a reference on these either, but a friend writes that they were studied by Matila Ghylea, of Rumania, and that papers on them were published. A surprising formula for these numbers is the formula for partial sums. The sum of the first n hexagonal numbers is simply n cubed!

3215 The sequence is: 1, 7, 19, 37, 61, 91, 127, 169, 217, 271, 331....

[formula: $3x(x+1)+1$.] (very rich in primes!)

ya Your handbook lists something called Cuban primes that apparently are a subset of the above series.

The square hexagons (of course these "hexagons" are not to be confused with what the Greeks called hexagonal numbers.) are:

6051 → 1, 169, 32761, 6355441....

I cracked this by way of the Pell: $3x^2 + 1 = y^2$. This gives three sequences which are in the handbook:

> x = 1, 7, 104, 1455...
> y (square root of the square hexes) = 1, 13, 181, 2521...
> x+1 (side of the square hexes) = 1, 8, 105, 1456...

I haven't looked for triangular hexes.

It seems curious to me that the Greeks didn't investigate centered hexagonal numbers, but apparently they didn't.

Did you check my little book, The Numerology of Dr. Matrix? It has a good listing of known numbers of n digits that equal the nth powers of their digits. (Hardy mentions such numbers in his Apology as interesting to puzzlists, but not to mathematicians!)
5188 They are: 1, 153, 370, 371, 407, 1634, 8208, 9474, 54748.... and so on through n = 9. One order 10 number is known, 4679307774, but may not be unique. (See p.97f of the book on Dr. Matrix.)

If I think of any more sequences I'll let you know.

Best,

Martin