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G J Simmons

Correspondence

1974–1975

3 pages

3407

Neil
you may want this for your collection of integer sequences
Gus

Sandia Laboratories

Albuquerque, New Mexico 87115

June 3, 1974

Problems Group
Mathematics Department
University of Maine
Orono, Maine 04473

Dear Sirs:

On December 5, 1973, in connection with an earlier submission of a solution to Problem E2440, I submitted a tabulation of the number of permutations P(n) on n symbols devoid of three term AP's for $1 \le n \le 10$. We have since, with a prodigious expenditure of computer time, extended these values for $11 \le n \le 20$. On the chance that you wish to include these results with my solution to E2440, I am including the complete table. We shall not compute any further values in this manner since the computing time is becoming excessive; almost forty minutes for n = 20!

n	P(n)
1	1
2	2
3	4
4	10
5	20
6	48
7	104
8	282
9	496
10	1066
11	2460
12	6128
13	12840
14	29380
15	74904 or ~~73904~~? 74904 is correct
16	212728
17	368016
18	659296
19	1371056
20	2937136

Sincerely,

Gustavus J. Simmons

Gustavus J. Simmons, Manager
Applied Mathematics Department - 5120

GJS:edw

3407

Sandia Laboratories

Albuquerque, New Mexico 87115

April 28, 1975

Dr. N. J. A. Sloan - 2C 363
Bell Telephone Laboratories, Inc.
600 Mountain Avenue
Murray Hill, New Jersey 07974

Dear Neal:

The value for P(15) published in the Math Monthly contained a typographical error. The correct value is 74,904. I verified that this was the case by going back to the original computer printout. I also checked my original letter to the Math Monthly to make sure that the correct value had been forwarded to them; which it was.

Sincerely,

Gus Simmons

Gustavus J. Simmons, Manager
Applied Mathematics
Department - 5120

GJS:edw

Amer. Math. Monthly 82#1 (Jan '75). N462.8 = 3407

E2460

E2440 [1973, 1058]

74 - 77

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thus generated are all distinct. The first time through Step 3 ($M = 2$), there is made one partition for a total of 2 subfiles. The second time through ($M = 4$), there are made two partitions for a total of $2^2 = 4$ subfiles. Continuing on, the kth time through ($M = 2^k$), there are made $\frac{1}{2}M = 2^{k-1}$ partitions for a total of $M = 2^k$ subfiles. Suppose that $2^k < n + 1 \leqq 2^{k+1}$ so that the construction terminates after $k + 1$ passes through Step 3. After the penultimate pass through Step 3, each of the $M = 2^k$ subfiles contains either one or two elements. Say p of them contain a single element and q of them contain two elements. Since $p + q = 2^k$ and $p + 2q = n + 1$, we see that $q = n + 1 - 2^k$ so that on the final pass through Step 3, only $n + 1 - 2^k$ new partitions are made. Thus the total number of partitions made is

$$1 + 2 + \cdots + 2^{k-1} + n + 1 - 2^k = n.$$

The same result can be obtained more simply by convincing oneself that rearrangement does not alter the number of partitions necessary to separate the file into singleton subfiles. Since there are $n + 1$ entries, it is obvious that n partitions are necessary to separate them.

With a little care, the same result can be derived for any set of $n + 1$ distinct integers. Note however that the construction does not yield in general all permutations without progressions. For example, if $n = 3$, there are actually 10 such permutations, whereas the algorithm generates only $2^3 = 8$: the permutations (1, 0, 3, 2) and (2, 3, 0, 1) are not constructed. We note that if $n + 1$ is a power of 2, then one of the constructions is essentially bit-reversal; for example, if $n = 7$, then bit-reversal on (0, 1, 2, 3, 4, 5, 6, 7) gives (0, 4, 2, 6, 1, 5, 3, 7). (This is the same as the second method of Odda. — Ed.)

IV. *Comment by G. J. Simmons, Sandia Laboratories, Albuquerque, New Mexico.* Let $P(n)$ denote the number of permutations on the n elements $\{0, 1, \cdots, n-1\}$ which contain no three-term arithmetic progressions. We have, with a prodigious expenditure of computer time, calculated $P(n)$ for $n \leqq 20$. The computing time for the case $n = 20$ alone was almost 40 minutes!

n	$P(n)$	n	$P(n)$
1	1	11	2460
2	2	12	6128
3	4	13	12840
4	10	14	29380
5	20	15	~~73904~~ 74904
6	48	16	212728
7	104	17	368016
8	282	18	659296
9	496	19	1371056
10	1066	20	2937136