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D. B. Shapiro

1 page

Ent'd 2 new sequences

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~~August 9, 1974~~

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N. J. A. Sloane
Mathematics Research Center
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Dear Dr. Sloane:

I recently obtained a copy of your book, A Handbook of Integer Sequences, and thought it an excellent idea. However, I was surprised to find the Hurwitz-Radon function was not mentioned. You have probably received other letters remarking on this sequence, but I think it warrants another.

The function is defined: if $n = 2^r n_1$, where n_1 is odd, then express $r = 4a + b$, where $0 \leq b \leq 3$. Then, define $\rho(n) = 8a + 2^b$. Another way to express this definition:

(rho)

$$\rho(n) = \rho(2^r) = \begin{cases} 2r+1 & \text{if } r \equiv 0 \\ 2r & \text{if } r \equiv 1 \\ 2r & \text{if } r \equiv 2 \\ 2r+2 & \text{if } r \equiv 3 \end{cases} \pmod 4$$

3484 = N63.5

Then, the sequence $\rho(n)$ is:

new 1,2,1,4,1,2,1,8,1,2,1,4,1,2,1,9,1,2,1,4,1,2,1,8,1,2,1,4,....

N414.5 = 3485

The sequence $\rho(2^r)$ is:

new 1,2,4,8,9,10,12,16,17,18,20,24,25,26,28,32,33,34,36,40,....

This function turns up in the solution of the Hurwitz problem on the composition of quadratic forms. References to the original papers by Hurwitz, Radon, Albert, Eckmann, Jacobson, etc., are not hard to find, though I do not have them at hand. This algebraic theory is also described (via Clifford algebras) in T.-Y. Lam's book The Algebraic Theory of Quadratic Forms, Benjamin, 1973.

These "orthogonal" Clifford algebra representations lead to the existence of $\rho(n)$ independent (non-vanishing) vector fields on the n sphere S^n. J. F. Adams proved that, in fact, the maximum number of independent vector fields on S^n is exactly $\rho(n)$. There is some discussion of these results in Husemöller's book on fiber bundles.

I hope this letter is of some use to you. If any supplements to your book have been published, I would appreciate receiving one.

Sincerely,

Daniel B. Shapiro

Daniel B. Shapiro

ref [T. Y. LAM, The Algebraic Theory of Quadratic Forms, Benjamin, Reading, Mass., 1973, p. 131] N