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THE COMBINATORICS OF RANDOM WALK WITH ABSORBING BARRIERS*

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0. Introduction

Consider a particle executing random walk on the line. The particle starts at the point 1 and arrives eventually at the point N in a total of $N+2l-1$ equally probable unit steps, l of which are in the negative direction. Both the origin and the point N are taken to be absorbing barriers, so that the particle may never visit 0, and may reach N only at the end of the walk. We seek the number $P_{N,l}$ of distinct walks satisfying these restrictions.

In the third edition of his celebrated book, Feller [4, p. 96] gives an explicit solution to this problem:

$$P_{N,l} = \sum_{k=-\infty}^{\infty} \left\{ \binom{N+2l-2}{kN+l-2} - \binom{N+2l-2}{kN+l-1} \right\}, \qquad \text{(I)}$$

with the usual convention that a binomial coefficient $\binom{a}{b}$ is zero if either $b>a$ or $b<0$; thus the right-hand side of (I) is really a finite sum. Apparently, this result has been widely ignored in the combinatorial literature. One possible reason for this is that (I) is a rather unwieldy formula from which it is hard to draw useful conclusions. Using the well-known identity [6, p. 20]:

$$\sum_{i=0} \binom{n}{iq+r} = \frac{1}{q} \sum_{k=0}^{q-1} \left(2\cos\frac{k\pi}{q}\right)^n \cos\frac{k(n-2r)\pi}{q}, \qquad 0 \le r < q,$$

we may verify that $P_{3,l} = 1$ (obvious), and $P_{4,l} = 2^l$. It is laborious, however, to work out even the next two cases:

$$P_{5,l} = \frac{1}{\sqrt{5}}\left(\frac{3+\sqrt{5}}{2}\right)^{l+1} - \frac{1}{\sqrt{5}}\left(\frac{3-\sqrt{5}}{2}\right)^{l+1} \quad \text{(alternate Fibonacci numbers)},$$

$$P_{6,l} = \tfrac{1}{2}(3^{l+1} - 1).$$

In the present paper we choose an approach different from Feller's, and derive a recurrence formula:

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Table 3. The numbers $P_{N,l}$

l \ N	7	8	9	10	11
1	5	6	7	8	9
2	19	26	34	43	53
3	66	100	143	196	260
4	221	364	560	820	1156
5	728	1288	2108	3264	4845
6	2380	4488	7752	12597	19551
7	7753	15504	28101	47652	76912
8	25213	53296	100947	177859	297275
9	81927	182688	360526	657800	1134705
10	266110	625184	1282735	2417416	4292145
11	864201	2137408	4552624	8844448	16128061
12	2806272	7303360	16131656	32256553	60304951
13	9112264	24946816	57099056	117378336	224660626
14	29587889	85196928	201962057	426440955	834641671
15	96072133	290926848	714012495	1547491404	3094322026
16	311945595	993379072	2523515514	5610955132	11453607152
17	1012883066	3391793664	8916942687	20332248992	42344301686
18	3288813893	11580678656	31504028992	73645557469	156404021389
19	10678716664	39539651584	111295205284	266668876540	577291806894
20	34673583028	134998297600	393151913464	965384509651	2129654436910

Table 3 (cont'd)

l \ N	12	13	14	15
1	10	11	12	13
2	64	76	89	103
3	336	425	528	646
4	1581	2109	2755	3535
5	6954	9709	13244	17710
6	29260	42504	60214	83490
7	119416	179630	263120	376740
8	476905	740025	1116765	1645605
9	1874730	2991495	4637100	7012200
10	7283640	11920740	18932940	29310996
11	28048800	46981740	76292736	120674568
12	107286661	183579396	304253964	490828828
13	408239530	712493461	1203322288	1976886616
14	1547129284	2750450981	4727337561	7898654920
15	5844716616	10572046555	18470700776	31351584145
16	22025185281	40495806764	71847381189	123762906805
17	82836630954	154683305139	278446103452	486339962755
18	311063682160	589504177384	1075843106843	1903826698410
19	1166646177136	2242448706435	4146266684560	7428725013375
20	4371207361885	8517201473375	15945859603310	28907860655466