The Lemming Simulation Pattern

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R. Reed

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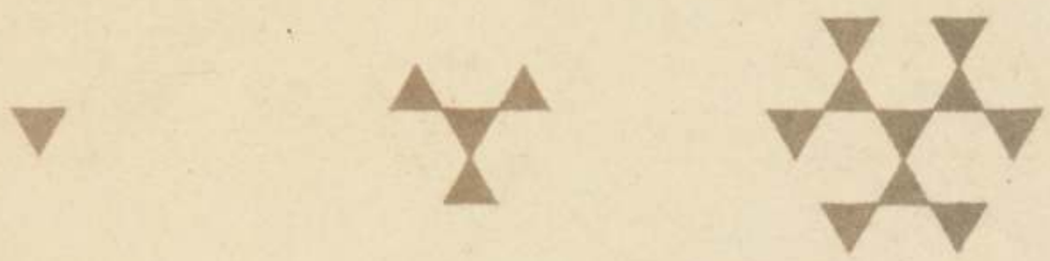
by R. Reed, Preston High School, North Tyneside
(Developed from an original idea by A. Ashbrook HMI)

How would you begin if you were asked to "start and develop a pattern you had never seen before"? Such a problem was put to one of the groups at a DES Course, in Birmingham last April, on the Development of the Mathematics Curriculum (which took the Association's 11-16 Report as its text).

Figure 1



(a) (b) (c)

The pattern began (using isometric graph paper) with a single triangle and the first "generation" was created by the addition of a triangle at each vertex. The second generation grew similarly. (See figures 1(a), (b), (c).) The third generation posed the first major problem: a choice had to be made;

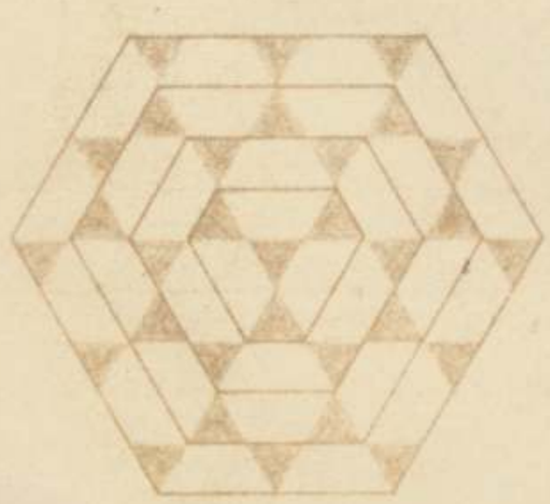
(a) should two "parents" provide only one "offspring" in a space?

(b) could two parents provide two offspring in the same space?

(c) should the space remain empty since any offspring would "die" through overpopulation?

Let us consider the first choice (a). The continuation of generations by the method already indicated produced a hexagonal pattern exhibiting three lines of

Figure 2



symmetry. (See figure 2.) The hexagonal border surrounding the nth generation offspring has sides of length n and $n+1$ units. The number of offspring in the nth generation is $3n$, and the total number of triangles is P where

$$P = 1+\sum_1^n 3r = 1+\frac{3n(n+1)}{2}$$ (a) A5448

Now consider the second choice (b). By giving parents and offspring numerical values it is possible to show which offspring in a particular generation have more ancestors. For example, one parent, value 1, gives an offspring of value 1, whilst two parents, each of value 1, give an offspring of value 2; and, further, one parent value 2 and one parent value 3 give an offspring of value 5. By drawing the pattern with number values instead of colours, a startling result emerges! Successive sides of the hexagon give rows of Pascal's Triangle. (See

Figure 3

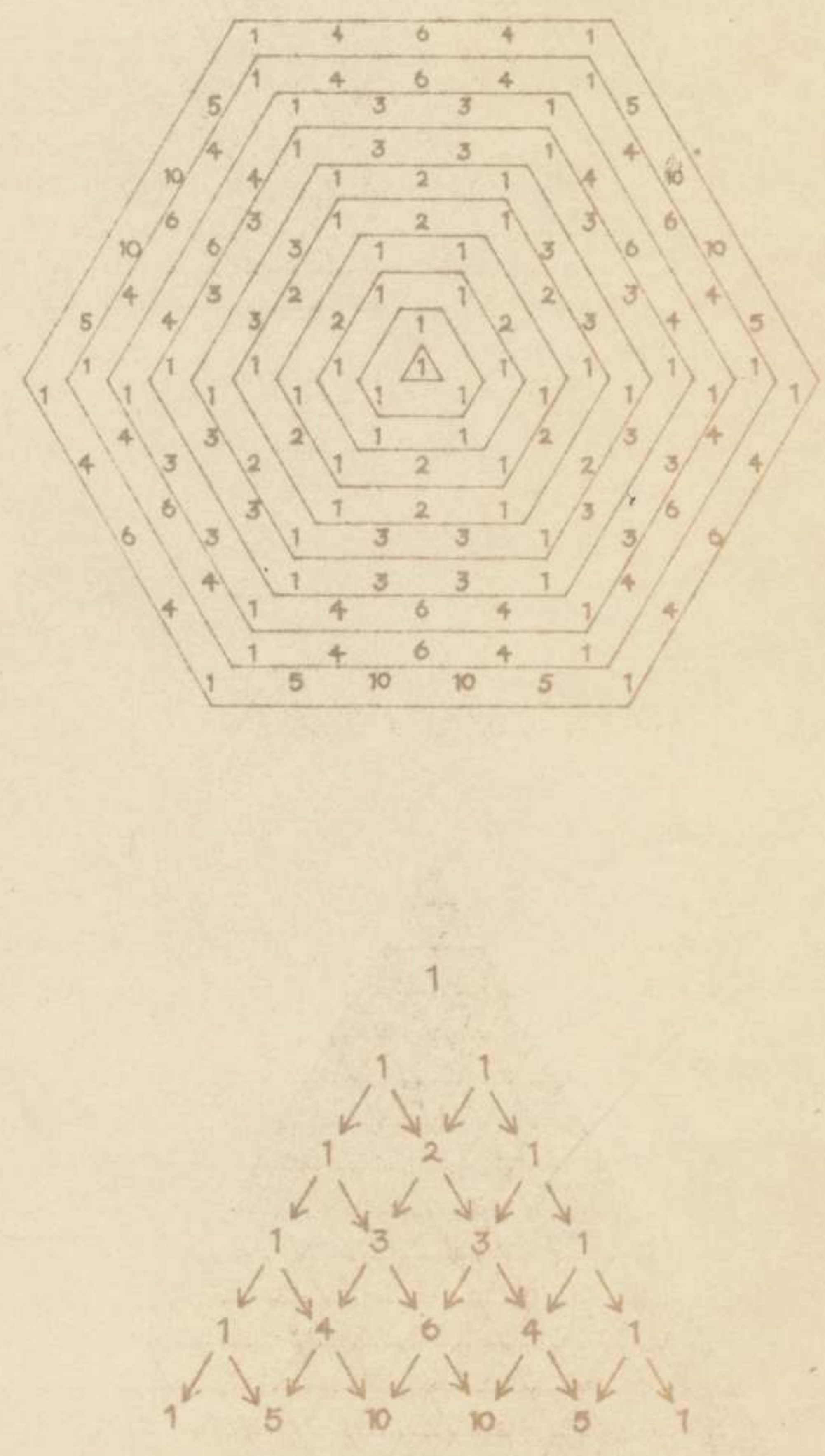


figure 3. The numbers are also the binomial coefficients.)

The number patterns obtained from this pattern are as follows:

Total Population	1	4	10	22	40	70	112	178	...
Each Generation		3	6	12	18	30	42	66	...
First Difference			3	6	6	12	12	24	...

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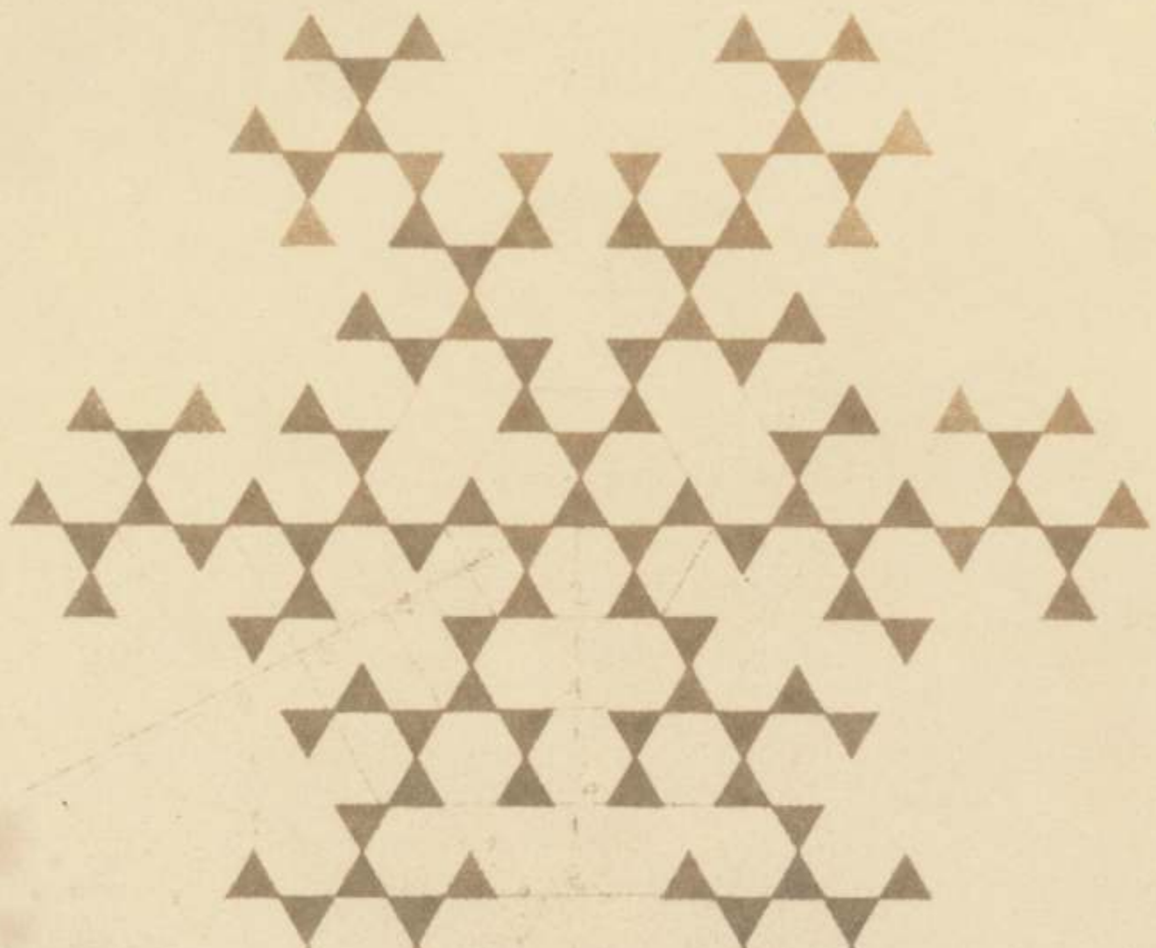
To construct an algebraic model for the number of triangles in each generation, consider the following:

Generation	*Number of Triangles*
1	3
2	6
3	3(1+2+1)
4	6(1+2+1)−6 = 6(2²−1)

Figure 4



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(NOT A161644 and A161645)

v. letter. $O(n^2)$ as opposed to $O(n^{1.5\cdots})$, I think.

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Generation $2n-1$ has a population $3(2^{n-1}-1)+3(2^n-1)$,
Generation $2n$ has a population $6(2^n-1)$,
where $n = 1, 2, 3, \ldots$.

Hence the total populations P_{2n} and P_{2n+1} (for patterns with an even or an odd number of generations respectively) may be found as follows:

$$P_{2n} = 6\sum_1^n(2^r-1)+3\sum_1^n(2^{r-1}-1)+3\sum_1^n(2^r-1)+1$$

$$= 6\left[\frac{2(2^n-1)}{2-1}-n\right]+3\left[\frac{2(2^{n-1}-1)}{2-1}-n\right]+3\left[\frac{2(2^n-1)}{2-1}-n\right]+1$$

$$= 21.2^n-12n-20$$

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$$P_{2n+1} = P_{2n}+(2n+1)\text{th generation}$$

$$= 30.2^n-12n-26$$

Now consider the third choice (c), where parents are not allowed to "breed" into the same space. In the tenth generation (see figure 4) there is another choice to make from the following:

(d) are parents only allowed to breed "outwards"?
or (e) are parents allowed to "breed-back" into the previous generation, or even further back into the "past"?

The front cover shows a section of the first forty generations using choice (d). What proves most fascinating about this particular pattern is that in the 8th, 16th and 32nd generations, only six "live" cells remain and yet the 30th generation had been teeming with 90 live cells. Hence the allusion to lemmings in the title of this pattern; after a few generations the pattern destroys itself leaving only a few to carry on. The number of triangles in the first few generations is

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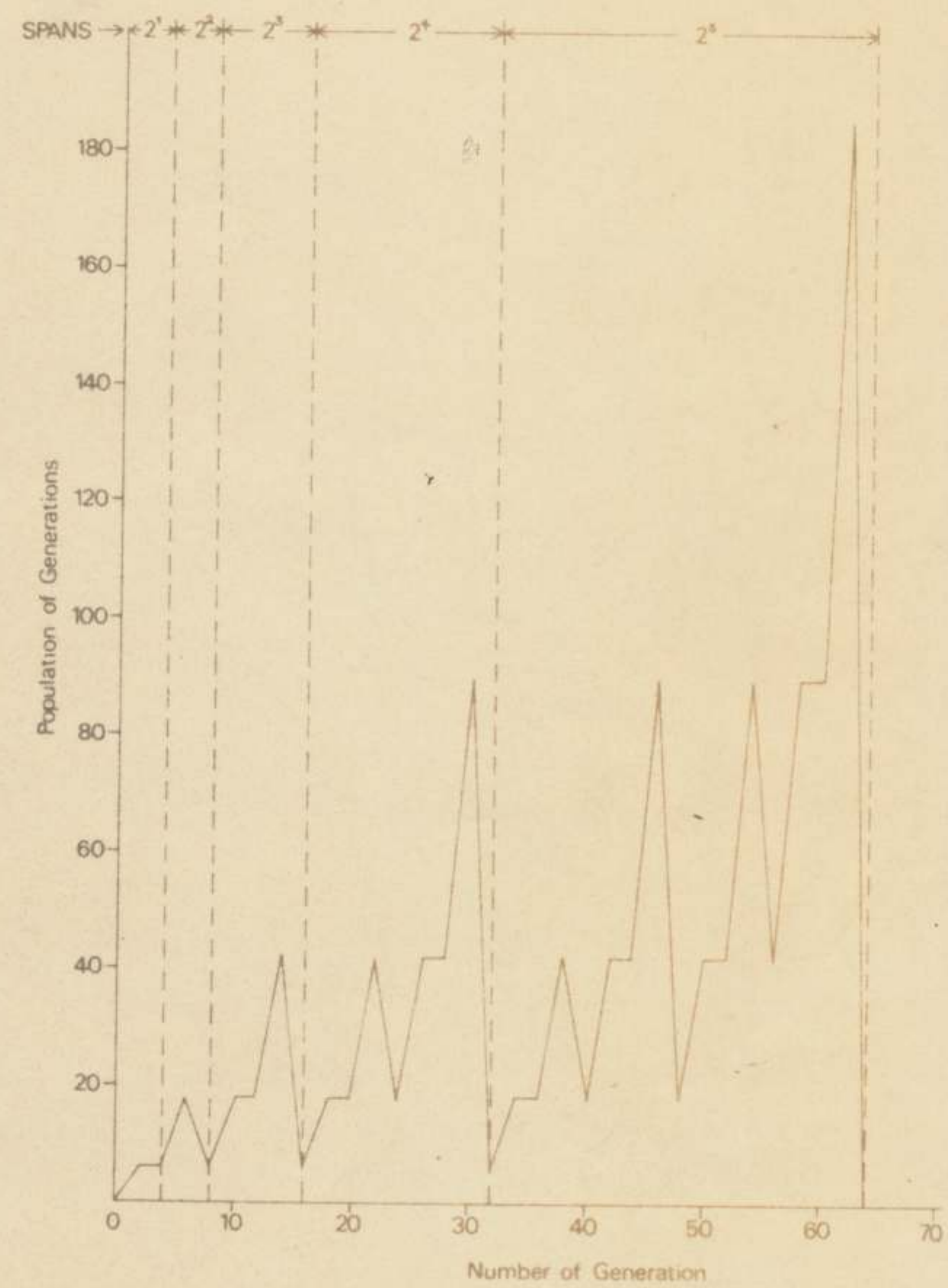
3, 6, 6, 6, 12, 18, 6, 12, 6, 18, 18, 18, 30, 42, 24, ...

The population of the first 64 generations is shown in figure 5. This readily shows how the pattern develops. As indicated in figure 5, the pattern can be split into "spans", the nth span having 2^n generations Of these, the first 2^{n-1} are repeated as the first half of the next span; the second half having a direct relationship with the first half. It seems likely that an algebraic model can be set up to give the total population in any given span and further work is being done on this problem. In order to predict the population in any given generation, only one-sixth of the pattern need be considered, namely that portion lying between two adjacent lines of symmetry. This portion can be considered in two separate parts, the left and right-hand sides and from this a rather cumbersome but apparently accurate method of predicting populations is possible. It still remains to explain why this empirical method works, and, also, to formulate a model which gives the total population after any specified number of generations.

The pattern obtained with choice ~~(d)~~ (e) which allows back-breeding is far denser than the previous pattern and gives the following sequence for total populations: 1, 4, 10, 16, 22, 34, 52, 64, ... (so far no difference!) and, as yet, no number pattern is forthcoming. Further work is necessary on the question of whether this last pattern is a "true pattern" in that it would be possible to predict the population in any specified generation.

If any reader takes up any of the problems outlined in this article, or tackles it from a different viewpoint, I would be pleased to hear from them at Preston High School, Preston North Road, North Shields, Tyne Wear. ■

Figure 5



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