Formula for Central Terms in Triangle A008287

Center-justified Triangle of Coefficients in Expansion of $(1 + x + x^2 + x^3)^n$

Row n	Sum S_n	
0	4^0	1
1	4^1	1 1 1 1
2	4^2	1 2 3 4 3 2 1
3	4^3	1 3 6 10 12 12 10 6 3 1
4	4^4	1 4 10 20 31 40 44 40 31 20 10 4 1
5	4^5	1 5 15 35 65 101 135 155 155 135 101 65 35 15 5 1
6	4^6	1 6 21 56 120 216 336 456 546 580 546 456 336 216 120 56 21 6 1
7	4^7	1 7 28 84 203 413 728 1128 1554 1918 2128 2128 1918 1554 1128 728 413 203 84 28 7 1

The central terms in expansions of $(1 + x + x^2 + x^3)^n$ are $1,\ 4,\ 44,\ 580,\ \cdots$ (highlighted above).

The central terms are given by the following formula

$$a_n = \sum_{i=0}^{n} \sum_{j=n}^{2n} \frac{(2n)!}{(j-n)!\ (3n+i-2j)!\ (j-2i)!\ i!}$$

$$\frac{(2n)!}{(j-n)!\,(3n+i-2j)!\,(j-2i)!\,i!} = 0 \ \textit{for}\ (3n+i-2j) < 0\ \textit{or}\ (j-2i) < 0$$

REFERENCE:

Shara Lalo and Zagros Lalo, Polynomial Expansion Theorems and Number Triangles, Zana Publishing, 2018, ISBN: 978-1-9995914-0-3.