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$$r_{n+1}(x) = (n+1)xr_n(x) + x(1-x)r_n'(x) + (1-x)^2 r_{n-1}(x) \qquad (n \geq 1) \tag{1.6}$$

with $r_0(x) = r_1(x) = 1$.

If we put

$$r_n(x) = \sum_{k=0}^{n} R_{n,k} x^k, \tag{1.7}$$

then, by (1.6), we get the recurrence

$$(n - k + 2)R_{n,k-1} + kR_{n,k} + R_{n-1,k} - 2R_{n-1,k-1} + R_{n-1,k-2}. \tag{1.8}$$

By means of (1.8) the following table is easily computed.

$n \backslash k$	0	1	2	3	4	5	6	7
0	1							
1	·	1						
2	1	-1	2					
3	·	3	·	3				
4	1	·	14	4	5			
5	·	8	22	60	22	8		
6	1	6	99	244	279	78	13	
7	·	21	240	1251	2016	1251	240	21

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It follows from (1.6) that

$$R_{n+1,n+1} = R_{n,n} + R_{n-1,n-1}.$$

Hence, since $R_{0,0} = R_{1,1} = 1$,

$$R_{n,n} = F_{n+1} \qquad (n = 0, 1, 2, \ldots). \tag{1.9}$$

Hoggatt and Bicknell [2] have conjectured that

$$R_{2n+1,k} = R_{2n+1,2n-k+2} \qquad (1 \leq k \leq 2n+1). \tag{1.10}$$

We shall prove that this is indeed true and that

$$R_{2n,2n-k+1} + (-1)^k \binom{2n+1}{k} = R_{2n,k} \qquad (1 \leq k \leq 2n). \tag{1.11}$$

The proof of (1.10) and (1.11) makes use of the relationship of $r_n(x)$ to the polynomial $A_n(x)$ defined by [1], [3, Ch. 2]

$$\frac{1-x}{1-xe^{(1-x)z}} = 1 + \sum_{n=1}^{\infty} A_n(x)\frac{x^n}{n!} \tag{1.12}$$

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etc

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Extract from a paper

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