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A6532
– A6535
A186438
A6593

Meeus ~~letters~~
& notes
Correspondence
8 pages

I did not
make a note of
all the sequences
mentioned in
these letters

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A6593

Jean Meeus
Heuvestraat 31
B-3071 Erps-Kwerps
BELGIUM

1974 December 26

Dr. N.J.A. Sloane,
Mathematics Research Center,
Bell Telephone Laboratories,
Murray Hill, New Jersey 07974
U.S.A.

Dear Dr. Sloane,

I am in possession of your interesting HANDBOOK OF INTEGER SEQUENCES. If you have supplements to the Handbook, I should be very obliged if you would send them to me. Many thanks in advance.

May I make the following comments:

1. The enclosed sheets contain some sequences not included in your book.
2. By definition, a polyomino is formed by connecting congruent squares. Thus "hexagonal polyominoes" or "polyominoes made from cubes" (your sequences 562 and 731) must be considered as incorrect.
Bodies made by connecting congruent cubes are called polycubes (and thus we have tricubes, tetracubes, pentacubes, etc.).
For connected hexagones, the name of polyhexes has been proposed; it's a good name, and it has been used by Martin Gardner.
For triangles, the name of polyiamond is found in the literature. However, I prefer the term polyamond, without the "i". Indeed, the name is a generalisation of "diamond", which must be regarding as di-amond, not d-iamond, because the prefix for two is di-, not d-.
3. It may be regrettable that all your sequences are systematically beginning with 1. For example, in sequence 2352 (amicable numbers), the first term is not 1, but 220.
4. I do not know the reason of including some sequences. For example sequence 397: why Fibonacci Numbers - 1? Or why sequence 966, the sequence $n^2/3$. Why not the squares divided by 2 or by 5? And why the squares written in base 9 (sequence 1361), and not the same squares written in base 8 or 7?
5. I understand that the book is a Handbook; its aim is to identify sequences and to serve as an index to the literature. Nevertheless, a dozen more pages could have been added in order to give the definition of some sequences. Generally, the reader does not know what are logarithmic numbers, Lah numbers or quartan primes!

Although a meteorologist by profession, I am principally an (advanced) amateur astronomer. You may find my photograph and a short biographical note in the Journal of Recreational Math., Vol. 6, page 265.

Yours very sincerely,

J. Meeus
Jean Meeus

SOME INTEGER SEQUENCES

A. Numbers the sum of whose divisors is a square: 3, 22, ... A6532
See BEILER, Recreations in the Theory of Numbers, page 8.

B. Prime rpunit numbers: 1111...111 (y units): 1, 2, 19, 23
See BEILER, ibid., page 84.

C. Least number which may be the side of a specified number of pythagorean triangles: 3, 5, 16, 12, 15, 125, 24, 40, 75, 48, ... A6593
See BEILER, ibid., page 114.

~~D. Pythagorean triangles with consecutive legs.~~

D. Regular polygons constructable by straightedge and compass:
3, 4, 5, 6, 8, 10, 12, 15, 16, 17, 20, 24, 30, 32, 34, 40, 48, 51, 60, ...
See BEILER, ibid., page 183.

E. Places of zero in the decimals of Pi: 32, 50, 54, 65, 71, 77, 85, 97, 106, ...

F. Non-bissextile century years in the Gregorian Calendar:
1700, 1800, 1900, 2100, 2200, ...

G. Date of Easter in the Gregorian Calendar (beginning with 1583) (32 stands for 1 April, 33 for 2 April, etc): 41, 32, 52, 37, 29, 48, 33, 53, 45, 29, ...

H. Numbers whose ~~square ends~~ with the same digits: N1530.5
6, 76, 376, ~~9376, 09376~~, ... Sameas

I. Numbers whose square ends with two like digits: 1, 10, 12, 20, 30, 38, 40, 50, ... A186438

J. Numbers whose cube ends with two like digits: 1, 10, 14, 20, 30, 40, 42, ...

N 182 K. 1, 7, 19, 37, 61, 91, ... See Martin GARDNER, Sc.American, July 1974, page 117.

L. 1, 13, 37, ... Ibid., page 118.

M. 1, 253, 49141, 9533161, ... Ibid., page 118.

N. 1, 121, 11881, ... Ibid., page 118.

O. Number of topologically different polyhedra: 1, 2, 7, ... have

P. Number of polyaboloes: 1, 3, 4, 14, 30, 107, ...
T.H. O'BEIRNE, New Scientist, 266, page 752 (21 December 1961).

Q. Number of edge-colored triangles: 1, 4, 11, 24, 45, 76, ...
The formula is $n(n^2+2)/3$

R. Number of edge-colored squares: 1, 6, 24, 70, 165, 336, ...
The formula is $n(n+1)(n^2-n+2)/4$

S. Edge-colored hexagons: 1, 14, 130, 700, 2635, 7826, ...
The formula is $n(n+1)(n^4-n^3+n^2+2)/6$

T. Number of vertex-colored squares: 1, 6, 24, 70, 165, ...
The formula is the same as for series R.

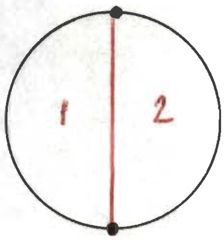
old 939 U. Symmetric polygons: 1, 2, 4, 11, 24, 67, ...
This is the same as your sequence 482, but now only the symmetric polygons are taken into account.

V. Dissection of a circle from **n** symmetrically placed points:
1, 2, 4, 8, 16, 30, 57, 88, 163, 230, 375, 456, 794, ... 6533
See more information on the enclosed page.

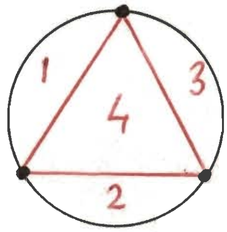
J. Meeus,
December 1974

A6533

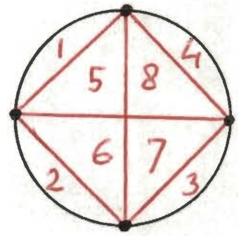
Illustration of Sequence V



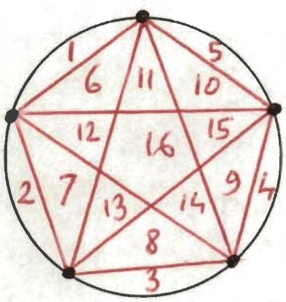
n=2



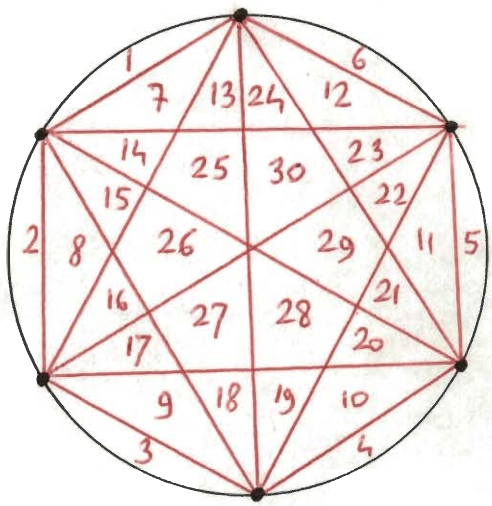
n=3



n=4



n=5



n=6

On a circumference, n equidistant points are taken, then they are connected with straight lines in all possible ways, forming a number of regions inside the circumference.

I have found the number of the regions up to n = 13, but could not found a general formula. I have published the values up to n = 13 in the Belgian journal "Wiskunde Post", Vol. 10, pages 62-63 (1972).

The series is particularly interesting, because the first terms are 1, 2, 4, 8 and 16, giving the impression that the general formula is 2^{n-1}.

Jean Meeus

December 29, 1974

Dear Dr. Sloane,

In addition to the sequences mentioned in my recent letter, the following ones may be of interest to you:

Number of one-sided polyamonds ("triangular polyominoes"): 6534
1, 1, 1, 4, 6, 19, 43, 121, ...

Number of one-sided polyhexes ("hexagonal polyominoes"): 6535
1, 1, 3, 10, 33, 147, ...

Further, I should be very obliged if you could give me briefly the following informations:

a. Who calculated the last terms of the sequences 561, 693, 941, 1072?

b. What is a colored polyomino (seq. 66)?

c. What is a restricted polyomino, and what is the difference between your sequences 295, 562, 694, 1145 and 1603 (they all are "restricted hexagonal polyominoes"!)?

d. What is a rectangular polyomino (seq. 334)?

e. What are board-pile and board-pair-pile polyominoes (sequences 639 and 640)?

Hoping that this will take not too much of your time, and thanking you in advance for your reply, I beg to remain, Dear Mr. Sloane,

Yours very sincerely,

J. Meeus

Jean Meeus,

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B-3071 Erps-Kwerps
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JAN 9 1975

Dr. Jean Meeus
Heuvestraat 31
B-3071 Erps-Kwerps
BELGIUM

Dear Dr. Meeus:

Thank you very much for your kind letters of December 26 and 29 and the many interesting sequences. A copy of Supplement I is enclosed herewith and two of the sequences you propose are in it (numbers K and L) - most of the others will go into the next supplement. By the way, I think your sequence O is already in the book as sequence 709.

Am I correct in assuming that the two sequences in your second letter (one-sided polyamonds and polyhexes) should be referred to as a "private communication" from you (as yet unpublished)?

To answer your comments:

(1) I completely agree with you that the terminology "hexagonal polyomino" etc. as used by Read, Klarner, Golomb, Lunnon and others is inconsistent. However, it does seem to be the most widely used, and it has the merit of being self-explanatory. "Polyhex" is not nearly so clear, I think. But in time I may come around to your way of thinking! Could you please tell me where polycube, and polyiamond (or polyamond) have been used in the literature?

(2) As explained on page 5, the initial 1 in each sequence acts as a marker and is not necessarily part of the sequence itself. There were several reasons for this. In many combinatorial problems, for example, the initial term (A_0) is usually defined to be 1 by convention. Also without this marker you would have to look in two places for a sequence - if you were trying to identify a sequence which appeared to begin 2,3,4,12,... you would have to look both under 1,2,3,5,12,... and 2,3,5,12,... . With the convention that the initial 1 is always present, you have only to look in one place.

(3) The rules by which I selected sequences are I think adequately described on page 3 of the book - see especially Rule 4! Suggestions for new sequences are always welcome, but I think it is harder to say that a certain sequence should not have been included!

(4) Yes, I agree that another chapter of definitions of sequences would be a very good idea. I shall suggest this to the publisher.

(5) The answers to all the questions in your second letters are the same, i.e.,

Dr. Fred Lunnon
Department of Computing Mathematics
University College
39 Park Place
Cardiff CF/3BB
WALES

He has carried out the most extensive calculations of polythings, and is responsible for the definitions of board-pile polynominoes etc. The easiest thing is for you to write to him for a set of his publications.

It was a pleasure to hear from another enthusiast of sequences.

With best wishes for 1975.

Yours sincerely,

MH-1216-NJAS-mv

N. J. A. Sloane

Enc.
As above

VERENIGING VOOR STERRENKUNDE

Jean MEEUS

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Belgium

1975 January 19

Dr. N.J.A. Sloane,
Mathematics Research Center,
Bell Telephone Laboratories,
Murray Hill, New Jersey 07974
U.S.A.

Dear Dr. Sloane,

Thank you very much for your reply of January 9, and for the copy of Supplement I to your Handbook. Here are some additional remarks, and reply to your questions.

1. My sequence H is incomplete and should be dropped. The correct series appears in your Supplement I as Sequence 1530.5.
2. The numbers in my sequence V (dissection of a circle) have been found by me. Perhaps these numbers have been published in the mathematical literature, but I don't know. Neither do I know any general formula for calculating these numbers.
3. The formulae for the calculation of sequences Q to T are given in JRM, Vol. 6, page 11 (1973). I have considered only triangles, squares and hexagons, because other polygons cannot be put together edgewise to cover the plane completely.
4. One-sided polyamonds and one-sided polyhexes: yes, you may refer to me ("personal communications) for these sequences. I don't know the next numbers in these series, unfortunately.

Poly(i)amond: This name has been proposed by T.H. O'Beirne (New Scientist, No. 259, page 316, 1961) and subsequently has been used by most puzzlists. See, for instance:
Martin Gardner, Sixth Book of Mathematical Games, Chapter 18 (pp. 173-182);
JRM, Vol. 2, pp. 216-227 (1969);
JRM, Vol. 6, pp. 215-220 (1973);
JRM, Vol. 7, pp. 1-7 (1974).

Polyhex. This name was proposed by David Klarner. It has been used, for instance, by Martin Gardner in Scientific American, Vol. 216, pp. 124-132 (June 1967).

Polycube. This term, or the corresponding terms tetracube, pentacube, etc., are used in many places in the mathematical literature, for instance:
JRM, Vol. 3, pp. 10-26 (1970);
JRM, Vol. 5, 20-21 (1972);
JRM, Vol. 5, 266-268 (1972);
JRM, Vol. 6, 211-214 (1973);
JRM, Vol. 6, 257-265 (1973);

and also at the end of Chapter 6 of Martin Gardner's "More Mathematical Puzzles and Diversions".

Thanks also for the address of Dr. Lunnon. I will write to him.

With very best wishes,

Yours sincerely,

J. Meeus

Jean Meeus,

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