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THE NUMBER OF PARTIALLY ORDERED SETS

Marcel Erné and Kurt Stege, Hannover

Nov 89

√ 894

By a well-known one-to-one correspondence between finite topologies and finite quasiorders (= reflexive and transitive relations), the number of quasiorders on a set X with n points equals the number T(n) of topologies on this set, and the number of partial orders on X equals the number $T_0(n)$ of T_0-topologies on the same set. More precisely, the number $T_0(n,j)$ of T_0-topologies on X with exactly j open sets is just the same as the number of partial orders on X with exactly j antichains resp. lower sets (= order ideals).

In the past, there have been made so many fruitless attempts to establish a "reasonable" explicit or recursive formula for T(n) and $T_0(n)$ that there is little hope to discover such a formula in early future. However, there exist partial solutions to the problem. For example, it is easy to see that T(n) and $T_0(n)$ are related via the Stirling numbers, so that T(n) may be computed if $T_0(m)$ is known for all $m \le n$. Furthermore, in 1975, Kleitman and Rothschild have given an asymptotic formula for $T_0(n)$ which is based on the observation that almost all finite partially ordered sets are graded and have at most three "levels". The number of graded posets on n points has been determined explicitly by Klarner (1969).

Recently, Culberson and Rawlins have developed a fast algorithm for the computation of the numbers P(n,k) of posets with n points and exactly k unrelated pairs. On account of their numerical material, they have conjectured that for $k < n$, these numbers satisfy a recursion of the form

$$P(n,k) = \sum_{j=0}^{k} C_j \, P(n-j-1,\, k-j).$$

One can show that in fact such a recursion exists, but the computation of the coefficients C_j requires the knowledge of the numbers Q(n,k) of all ordinally indecomposable posets with n points and k unrelated pairs for $n \le j$.

Concerning the explicit computation of T(n), $T_0(n)$ and $T_0(n,j)$, some progress has been made until today. In 1967, $T_0(n)$ and T(n) have been computed by Evans, Harary and Lynn for $n \le 8$; the case $n = 9$ was solved in 1972 (Erné), the cases $n = 10$ and $n = 11$ have been supplemented in 1977 by Das. Recently, we have used a variant of the algorithm by Culberson and Rawlins in order to obtain the numbers $T_0(n,j)$ for $n \le 9$ and all j. A combination of these numerical results with a reduction formula that enables one to compute $T_0(n)$ if only the numbers $T_0(i,j)$ are known for $i \le n-3$, yields the new values $T_0(n)$ and T(n) for $n = 12$ and, with some more effort, for $n = 13$. The main ingredients for the reduction formula in question have been established already in 1972; the formula reads as follows:

$$T_0(n) = \frac{n(n+1)}{2} T_0(n-1) - \frac{n(n-1)(n-2)}{2} \sum_{i=0}^{n-3} \binom{n-3}{i} \frac{1}{n-i} \sum_{j=1}^{2^i} (-j)^{n-i} T_0(i,j).$$

$T_0(12) = 414\,864\,951\,055\,853\,499$
$T(12) = 319\,355\,571\,065\,774\,021$

n	partial orders on n points T_0-topologies on n points	quasiorders on n points topologies on n points	computed by
0	A1035 1	A 0798 1	
1	1	1	
2	3	4	
3	19	29	~~[illegible]~~
4	219	355	
5	4 231	6 942	Birkhoff 1948
6	130 023	209 527	
7	6 129 859	9 535 241	Comtet 1966
8	431 723 379	642 779 354	Evans, Harary, Lynn 1967
9	44 511 042 511	63 260 289 423	Erné 1972
10	6 611 065 248 783	8 977 053 873 043	
11	1 396 281 677 105 899	1 816 846 038 736 192	Das 1977
12	414 864 951 055 853 499	519 355 571 065 774 021	
13	171 850 728 381 587 059 351	207 881 393 656 668 953 041	Erné, Stege 1989

n	isomorphism classes homeomorphism classes	isomorphism classes homeomorphism classes	
0	A 0112 1	A 6870 1	
1	1	1	
2	2	3	
3	5	8	
4	16	26	
5	63	94	
6	318	435	Knopfmacher 1969
7	2 045	2 564	
8	16 999	19 983	
9	183 231	205 729	Wright 1979
10	2 567 284		Möhring 1984
11	46 749 427		Culberson, Rawlins 1989

Nov. 89