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Subject: A Handbook of Integer Sequences

Dear Dr. Sloane,

In the cited referenced text SI2 on page 27, there is the following theorem numbered three; "If **n** is a natural number > 2, then between **n** and **n**! there is at least one prime number." This beings up several questions. Only three will be addressed here. One, how many primes are there between **n** and **n**! ? Or better yet, just how many primes are less than or equal to **n** factorial? Two, what is the last prime before **n** factorial? And three, what is the difference between **n**! and the Prime just preceeding it?

n	Last Prime before **n**!	the Diff.	Nbr of Primes < **n**!
1	0	0	0
2	0	0	0
3	5	1	3
4	23	1	9
5	113	7	30
6	719	1	128
7	5,039	1	675
8	40,289	31	4,231
9	362,867	13	30,969
10	3,628,789	11	258,689
11	39,916,787	13	2,428,956
12	479,001,599	1	25,306,287
13	6,227,020,777	23	289,620,751
14	87,178,291,199	1	3,610,490,805

n	Last Prime before **n**!	the Diff.
15	1,307,674,367,953	47
16	20,922,789,887,947	53
17	355,687,428,095,941	59
18	6,402,373,705,727,959	41
19	121,645,100,408,831,899	101
20	2,432,902,008,176,639,969	31
21	51,090,942,171,709,439,969	31
22	1,124,000,727,777,607,679,927	73
23	25,852,016,738,884,976,639,911	89

24	620,448,401,733,239,439,359,927	73
25	15,511,210,043,330,985,983,999,851	149

From the above table, it is easily discernable that the prime quoted in column two is the $n^{\underline{th}}$ prime as noted is column four. Therefore; for **n** ≥0 the following sequence for π (n!) [KN1] is : 0, 0, 1, 3, 9, 30, 128, 675, 4231, 30969, 258689, 2428956, 25306287, 289620751, 3610490805, ~48686917622, ~706003775292, ~10953618190634, ~181035031636349, ~3175094502815204, ~58893601707947620, ⋯ . The tilde "~" means about and is not necessarily exact! and the hyphen "-" means the number is seperated at that point and continues to the next line, but it is still one number.

Two of the above sequences grow so quickly that they will exceed the two lines given in your forthcoming tome. However, the sequence of the differences between **n**! and its immediate preceeding prime is restated for index **n** and from the beginning it is as follows: 0, 0, 1, 1, 7, 1, 1, 31, 13, 11, 13, 1, 23, 1, 47, 53, 59, 41, 101, 31, 31, 73, 89, 73, 149, 37, 43, 101, 31, 1, 61, 1, 1, 193, 113, 127, 97, 1, 73, 83, 131, 79, 109, 109, 53, 89, 79, 103, 59, 97, 179, 67, 59, 127, 61, 461, 277, 109, 137, 139, 71, 71, 101, 359, 127, 317, 191, 251, 103, 97, 751, 163, 373, 199, 167, 157, 491, 317, 257, 103, 83, 151, 353, 463, 383, 103, 911, 131, 197, 97, 1013, 379, 113, 1, 109, 163, 311, 173, 571, 271, 479, ⋯ .

In the preceeding series, there are instances where **n**! −1 is a prime. The following is that sequence for the index **n**: 3, 4, 6, 7, 12, 14, 30, 32, 33, 38, 94, 166, 324, 379, 469, 546, 974, and no others less than 1156.

Not only is the result of the first difference produce a prime, but so is the result of the difference between the **n**! and the second preceeding prime, beginning with **n** =3. They are as follows: 3, 5, 11, 11, 17, 37, 17, 17, 17, 13, 61, 17, 59, 71, 61, 43, 113, 71, 41, 101, 191, 103, 191, 179, 71, 127, 37, 79, 113, 163, 47, 373, 293, 157, 149, 79, 167, 211, 151, 89, 131, 113, 73, 107, 179, 227, 173, 113, 257, 239, 151, 227, 163, 509, 293, 347, 643, 373, 457, 109, 199, 661, 317, 659, 277, 547, 197, 139, 887, 211, 953, 499, 223, 353, 569, 347, 739, 107, 337, 409, 1051, 557, 787, 443, 953, 331, 281, 103, 1031, 467, 409, 449, 127, 613, 467, 607, 619, 839, 631, ··· .

So is the result of the difference between the **n**! and the third preceeding prime also produces a prime, beginning with **n** =4. They are as follows: 7, 13, 19, 19, 43, 29, 23, 43, 17, 67, 43, 71, 89, 239, 47, 197, 151, 43, 139, 197, 191, 239, 191, 173, 197, 47, 97, 223, 373, 71, 439, 307, 263, 157, 241, 199, 233, 337, 131, 179, 149, 113, 227, 269, 409, 197, 193, 379, 271, 181, 419, 367, 701, 751, 379, 811, 401, 463, 263, 839, 773, 683, 811, 983, 571, 359, 293, 1039, 281, 1399, 523, 271, 523, 877, 461, 743, 109, 491, 433, 1229, 991, 1987, 661, 1307, 887, 683, 139, 1213, 487, 557, 479, 179, 719, 829, 1103, 1901, 1427, 659, ··· .

What follows is a table of **n**! until the difference of the previous prime becomes composite.

	The **k**$^{\underline{th}}$ term is composite
1! :	0
2! :	0
3! : 1, 3, 4=2*2	3
4! : 1, 5, 7, 11, 13, 17, 19, 21=3*7	8
5! : 7, 11, 13, 17, 19, 23, 31, 37, 41, 47, 49=7*7	11
6! : 1, 11, 19, 29, 37, 43, 47, 59, 61, 67, 73, 77=7*7	12
7! : 1, 17, 19, 29, 31, 37, 41, 47, 53, 67, 71, 73, 83, 89, 97, 103, 107, 109, 121=11*11	19
8! : 31, 37, 43, 67, 79, 83, 89, 107, 127, 131, 143=11*13	11
9! : 13, 17, 29, 79, 121=11*11	5
10! : 11, 17, 23, 37, 41, 53, 89, 103, 121=11*11	9
11! : 13, 17, 43, 73, 83, 89, 97, 113, 149, 151, 163, 167, 199, 247=13*19	14
12! : 1, 13, 17, 37, 61, 73, 83, 107, 139, 169=13*13	10
13! : 23, 61, 67, 89, 101, 103, 109, 163, 191, 193, 223, 227, 233, 269, 281, 283, 289=17*17	17
14! : 1, 17, 43, 47, 61, 67, 89, 131, 197, 199, 211, 227, 233, 313, 347, 353, 359, 373, 419, 449, 461, 499, 601, 607, 659, 667=23*29	26
15! : 47, 59, 71, 79, 89, 97, 139, 157, 163, 167, 227, 251, 257, 293, 313, 317, 347, 367, 401, 419, 449, 461, 463, 467, 493=17*29	25
16! : 53, 71, 89, 107, 139, 163, 181, 227, 239, 263, 307, 317, 337, 353, 359, 367, 421, 431, 433, 449, 457, 467, 563, 569, 571, 587, 601, 641, 673, 677, 701, 739, 743, 757, 787, 911, 947, 967, 983, 1007=19*53	40
17! : 59, 61, 239, 281, 307, 367, 373, 389, 401, 487, 491, 547, 587, 619, 631, 653, 673, 719, 743, 751, 811, 821, 823, 827, 839, 859, 883, 941, 947, 1009, 1151, 1163, 1201, 1271=31*41	34
18! : 41, 43, 47, 61, 101, 109, 131, 139, 157, 193, 257, 283, 311, 317, 331, 349, 421, 431, 439, 457, 499, 547, 557, 557, 589=19*31	25
19! : 101, 113, 197, 199, 251, 263, 307, 331, 353, 359, 409, 541, 659, 887, 929, 961=31*31	16

20! : 31, 71, 151, 163, 179, 191, 251, 257, 269, 353, 367, 397, 433, 443, 521, 601, 613, 631, 641, 659, 661, 797, 823, 853, 857, 859, 863, 877, 1019, 1063, 1097, 1109, 1171, 1181, 1213, 1229, 1279, 1291, 1297, 1361, 1423, 1427, 1493, 1537=29*53

The above table suggests yet another sequence, that being the number of terms necessary to reach a difference that is composite. It is as follows: 0, 0, 3, 8, 11, 12, 19, 11, 5, 9, 14, 10, 17, 26, 25, 41, 34, 25, 16, 44, 28, 33, 25, 22, 13, 18, 29, 23, 22, 22, 21, 36, 24, 24, 22, 26, 19, 21, 35, 19, 32, 30, 38, 32, 27, 23, 30, 37, 55, 23, 35, 33, 34, 55, 39, 46, 24, 39, 24, 19, 39, 26, 35, 28, 37, 27, 20, 44, 27, 44, 46, 44, 25, 32, 50, 87, 39, 46, 46, 31, 23, 32, 25, 41, 41, 42, 35, 46, 29, 49, 49, 30, 67, 42, 44, 30, 38, 40, 22, 48, 40, ⋯.

Just like the preceeding sequences are on the side less that the factorial of **n**, so do we have sequences on the greater side of **n**! as well.

The mirror image of the first such sequence is the difference between **n** factorial and the first prime that exceeds it. Keep in mind that 0! = 1 by definition [Graham p.111] so this sequence begins with the index **n** = 0. It is as follows: 1, 1, 1, 5, 7, 7, 11, 23, 17, 11, 1, 29, 67, 19, 43, 23, 31, 37, 89, 29, 31, 31, 97, 131, 41, 59, 1, 67, 223, 107, 127, 79, 37, 97, 61, 131, 1, 43, 97, 53, 1, 97, 71, 47, 239, 101, 233, 53, 83, 61, 271, 53, 71, 223, 71, 149, 107, 283, 293, 271, 769, 131, 271, 67, 193, 283, 73, 83, 131, 139, 857, 101, 1, 179, 229, 113, 1, 113, 271, 107, 701, 127, 157, 227, 131, 113, 367, 601, 109, 239, 149, 97, 137, 271, 547, 199, 229, 307, 103, 229, 139, ⋯ .

In the preceeding series, there are instances where **n**! +1 is a prime. The following is that sequence for the index **n**: 1, 2, 3, 11, 27, 37, 41, 73, 77, 116, 154, 320, 340, 399, 427, 872, 1477, and no others less than 2043.

Conjecture: If "we adopt the convention" of Knuth [KN1] that the Zeroth prime is the number One, just as "⋯ I. Chowla [I believe that his initilal is S.] has advanced the conjecture that if the number 1 is considered as a prime number (as some people did formerly), ⋯" [SI2] the above sequence (the amount by which the first prime number after **n**! exceeds **n**!) contains just primes!

These series are somewhat analogous to the "Fortunate" numbers, but I shall leave that sequence to a separate letter.

Here is the one which is analogous to the "Fortunate" numbers using the factorials versus the "primorials:" 2, 3, 5, 5, 7, 7, 11, 23, 17, 11, 17, 29, 67, 19, 43, 23, 31, 37, 89, 29, 31, 31, 97, 131, 41, 59, 47, 67, 223, 107, 127, 79, 37, 97, 61, 131, 311, 43, 97, 53, 61, 97, 71, 47, 239, 101, 233, 53, 83, 61, 271, 53, 71, 223, 71, 149, 107, 283, 293, 271, 769, 131, 271, 67, 193, 283, 73, 83, 131, 139, 857, 101, 79, 179, 229, 113, 149, 113, 271, 107, 701, 127, 157, 227, 131, 113, 367, 601, 109, 239, 149, 97, 137, 271, 547, 199, 229, 307, 103, 229, 139, ⋯ .

The preceeding sequence represents the difference between **n**! +1 and the first prime exceeding **n**! +1. This one is its mirror image; ie, the difference between **n**! −1 and the first prime preceeding **n**! −1 beginning with **n** =5: 7, 11, 17, 31, 13, 11, 13, 13, 23, 17, 47, 53, 59, 41, 101, 31, 31, 73, 89, 73, 149, 37, 43, 101, 31, 79, 61, 163, 47, 193, 113, 127, 97, 79, 73, 83, 131, 79, 109, 109, 53, 89, 79, 103, 59, 97, 179, 67, 59, 127, 61, 461, 277, 109, 137, 139, 71, 71, 101, 359, 127, 317, 191, 251, 103, 97, 751, 163, 373, 199, 167, 157, 491, 317, 257, 103, 83, 151, 353, 463, 383, 103, 911, 131, 197, 97, 1013, 379, 113, 449, 109, 163, 311, 173, 571, 271, 479, ⋯ .

The difference between the two sequences, the first one citing the differences between **n**! and its immediate preceeding prime and the second one citing the differences between **n**! and its immediate succeeding prime, is the "prime gap" surrounding the number **n**! The largest observed difference being about the number 71! and it is equal to 1608, which equals the spread of the pair (71! − 751, 71! + 857). Since the two sequences above contain only odd numbers, the differences are always even, therefore; the following series is the difference divided by two, beginning with **n** = 3: 1, 3, 7, 4, 6, 27, 15, 11, 7, 15, 45, 10, 45, 38, 45, 39, 95, 30, 31, 52, 93, 102, 95, 48, 22, 84, 127, 54, 94, 40, 19, 145, 87, 129, 49, 22, 85, 68, 66, 121, 90, 78, 146, 95, 156, 78, 71, 79, 225, 60, 65, 175, 66, 305, 192, 196, 215, 205, 420, 101, 186, 213, 160, 300, 132, 167, 117, 118, 804, 132, 187, 189, 198, 135, 246, 215, 264, 105, 392, 139, 255, 345, 257, 108, 639, 366, 153, 168, 581, 238, 125, 136, 328, 181, 270, 240, 337, 250, 309, ⋯ .

Not only is the result of the first difference above **n**! produce a prime, but so is the result of the difference between the **n**! and the second succeeding prime. They are as follows: 2, 3, 5, 7, 11, 13, 19, 31, 23, 19, 17, 43, 73, 41, 149, 41, 53, 61, 109, 37, 37, 71, 109, 193, 97, 173, 47, 101, 229, 163, 241, 83, 139, 103, 83, 577, 311, 47, 269, 61, 61, 107, 97, 89, 379, 149, 269, 83, 137, 167, 281, 89, 79, 443, 229, 157, 179, 563, 389, 277, 827, 281, 433, 151, 383, 1033, 79, 101, 137, 173, 971, 151, 79, 479, 449, 139, 149, 307, 839, 139, 757, 971, 227, 919, 229, 509, 593, 787, 1187, 1069, 251, 167, 193, 479, 557, 239, 257, 499, 109, 439, 233, ⋯ .

So is the result of the difference between the **n**! and the third succeeding prime after **n**! also produces a prime. They are as follows: 5, 7, 13, 17, 19, 37, 37, 31, 41, 19, 59, 109, 71, 179, 73, 59, 73, 113, 53, 47, 127, 149, 263, 107, 241, 59, 103, 317, 241, 317, 113, 197, 127, 109, 647, 397, 67, 281, 67, 211, 163, 109, 107, 439, 521, 709, 101, 383, 337, 397, 223, 337, 601, 281, 311, 389, 821, 421, 307, 911, 353, 787, 293, 431, 1051, 233, 461, 241, 307, 991, 251, 107, 487, 997, 151, 197, 647, 881, 157, 937, 1697, 487, 971, 311, 541, 1117, 1129, 1249, 1709, 727, 661, 257, 1153, 607, 461, 337, 523, 239, 547, 563, ⋯ .

What follows is a table of **n**! until the difference of **n**! and the subsequent prime becomes composite.

	The $\mathbf{k}^{\underline{th}}$ term is composite
1! : 1, 2, 4=2*2	3
2! : 1, 3, 5, 9=3*3	4
3! : 1, 5, 7, 11, 13, 17, 23, 25=5*5	8
4! : 5, 7, 13, 17, 19, 23, 29, 35=5*7	8
5! : 7, 11, 17, 19, 29, 31, 37, 43, 47, 53, 59, 61, 71, 73, 77=7*11	15
6! : 7, 13, 19, 23, 31, 37, 41, 49=7*7	8
7! : 11, 19, 37, 41, 47, 59, 61, 67, 73, 79, 107, 113, 127, 131, 139, 149, 157, 169=13*13	18
8! : 23, 31, 37, 41, 67, 103, 107, 109, 113, 139, 151, 163, 167, 173, 179, 187=11*17	16
9! : 17, 23, 31, 47, 61, 71, 73, 89, 97, 103, 107, 137, 139, 157, 163, 167, 179, 181, 187=11*17	19
10! : 11, 19, 41, 47, 53, 83, 97, 127, 131, 149, 167, 169=13*13	12
11! : 1, 17, 19, 29, 31, 37, 59, 109, 113, 131, 139, 149, 173, 179, 191, 211, 227, 239, 169=13*13	19
12! : 29, 43, 59, 83, 101, 137, 167, 191, 193, 197, 221=13*17	11

13! : 67, 73, 109, 139, 157, 181, 199, 289=17*17 8
14! : 19, 41, 71, 79, 103, 127, 167, 193, 229, 353, 361=19*19 11
15! : 43, 149, 179, 223, 227, 361=19*19 6
16! : 23, 41, 73, 149, 163, 173, 197, 199, 233, 277, 283, 323=17*19 12
17! : 31, 53, 59, 83, 109, 167, 181, 263, 281, 283, 293, 307, 349,
367, 397, 421, 487, 599, 631, 739, 761, 769, 779=19*41 23
18! : 37, 61, 73, 101, 103, 107, 137, 173, 179, 239, 241, 257, 271,
293, 331, 337, 383, 397, 463, 541, 613, 617, 683, 713=23*31 24
19! : 89, 109, 113, 137, 139, 241, 281, 311, 389, 463, 467, 641,
659, 701, 713=23*31 15
20! : 29, 37, 53, 79, 89, 137, 139, 157, 167, 211, 271, 277, 283,
353, 367, 379, 439, 491, 571, 617, 619, 661, 673, 743, 839,
919, 941, 953, 997, 1091, 1189=29*41 31

The above table suggests yet another sequence, that being the number of terms necessary to reach a difference that is composite. It is as follows: 3, 4, 8, 8, 15, 8, 18, 16, 19, 12, 19, 11, 8, 11, 6, 12, 23, 24, 15, 31, 21, 27, 15, 16, 26, 25, 18, 17, 29, 20, 27, 27, 30, 23, 16, 28, 24, 25, 29, 15, 25, 19, 36, 36, 39, 15, 36, 24, 44, 35, 29, 27, 25, 36, 22, 37, 31, 32, 41, 29, 55, 27, 45, 29, 59, 34, 37, 24, 49, 25, 40, 29, 55, 39, 38, 34, 46, 31, 37, 41, 24, 25, 35, 45, 33, 41, 42, 63, 31, 49, 46, 40, 30, 28, 36, 50, 36, 26, 32, 31, 37, ⋯ . Please notice that in the above table for **n** > 0 and in the other table for **n** > 4, the first composite difference has as its prime factors, numbers which exceed **n**.

Given the sequence immediately above and its counterpart cited earlier, the sum of the two less the two end points states the number of prime differences surrounding **n** factorial. It is as follows: 2, 3, 9, 14, 24, 18, 35, 25, 22, 19, 31, 19, 23, 35, 29, 51, 55, 47, 29, 73, 47, 58, 38, 36, 37, 41, 45, 38, 49, 40, 46, 61, 52, 45, 36, 52, 41, 44, 62, 32, 55, 47, 72, 66, 64, 36, 64, 59, 97, 56, 62, 58, 57, 89, 59, 81, 53, 69, 63, 46, 92, 51, 78, 55, 94, 59, 55, 66, 74, 67, 84, 71, 78, 69, 86, 119, 83, 75, 81, 70, 45, 55, 58, 84, 72, 81, 75, 107, 58, 96, 93, 68, 95, 68, 78, 78, 72, 64, 52, 77, 75, ⋯ .

Conjecture: If "we adopt the convention" of Knuth [KN1] that the Zeroth prime is the number One; for **n** >2, there exists at least six primes - three below **n**! and three above **n**!; for **n** >9, there exits at least ten primes - five below **n**! and five above **n**!; and for **n** >15, there exits at least twenty primes - ten below **n**! and ten above **n**!; which represent the absolute difference of the consecutive primes and **n**! Additionally, as **n** grows so does the lower limit. Take for example, **n** = 10, there are 3 consecutive primes immediately below 10!, they being 3628777, 3628783 and

1 2 6 24

1 2 4 10 34

3628789, and 3 consecutive primes immediately above 10!, they being 3628811, 3628819 and 3628841. The differeces are −23, −17, −11, +11, +19 and +41, and these are all prime. This is rather astonishing since a cursory look at Table 1 on page 390 of Knuth [KN1] demonstrates the rarity of this condition.

This sequence is $\sum_{k=1}^{n} \mathbf{n}!$ is as follows: 1, 3, 9, 33, 153, 873, 5913, 46233, 409113, 4037913, 43954713, 522956313, 6749977113, 93928268313, 1401602636313, 22324392524313, 378011820620313, 6780385526348313, 128425485935180313, 2561327494111820313, 53652269665821260313, 1177652997443428940313, 27029669736328405580313, 647478071469567844940313, 16158688114800553828940313, 419450149241406189412940313, 11308319599659758350180940313, 316196664211373618851684940313, 9157958657951075573395300940313, 274410818470142134209703780940313, 8497249472648064951935266660940313, 271628086406341595119153278820940313, 8954945705218228090637347680100940313, 304187744744822368938255957323620940313, 10637335711130967298604907294846820940313, 382630662501032184766604355445682020940313, 14146383753727377231082583937026584420940313, 537169001220328488991089808037100875620940313, 20935051082417771847631371547939998232420940313, 836850334330315506193242641144055892504420940313, 34289376947494122614363304694584807557656420940313, 1439295494700374021157505910939096377494040420940313, 61854558558074209658512637979453093884758552420940313, 2720126133346522977702138448994068984204397080420940313, 122342346998826717539665299944651784048588130840420940313, 5624964506810915667389970728744906677010239883800420940313, 264248206017979096310354325882356886646207872272920420940313, 12678163798554051767172643373255731925167694226950680420940313, 620960027832821612639424806694551108812720525606160920420940313, 31035053229546199656252032972759319953190362094566672920420940313, ⋯ . This sequence for all **n** >4 have a digital root of 9 and for all **n** >10 are divisible by 99.

This sequence is when the above series less two is prime for the index **n**: 2, 3, 4, 5, 12, 13, 19, 65, 90, 123, ⋯ .

Notice that the lesser significant digits of $\sum_{k=1}^{n}$ **n**! become identical for increasing modulus powers of ten as **n** increases. Therefore; the sequence of the digits in reverse order (least to greatest significant digit) is as follows: 3, 1, 3, 0, 4, 9, 0, 2, 4, 0, 2, 9, 8, 2, 5, 6, 3, 3, 2, 4, 4, 6, 5, 5, 2, 5, 0, 9, 3, 0, 5, 0, 1, 3, 9, 5, 3, 2, 3, 4, 0, 8, 4, 9, 9, 7, 0, 1, 1, 2, 6, 8, 3, 7, 4, 8, 6, 8, 7, 4, 9, 7, 4, 7, 4, 2, 2, 9, 0, 0, 4, 3, 3, 0, 5, 6, 5, 8, 6, 5, 0, 0, 2, 6, 6, 5, 1, 5, 9, 7, 8, 8, 1, 6, 2, 0, 2, 8, 1, 2, 1, ⋯ .

This sequence is the first prime factor of $\sum_{k=1}^{n}$ **n**! which exceeds **n**, or if prime, then the entry will be a zero: 1, 1, 1, 11, 17, 0, 0, 11, 131, 11, 0, 23, 821, 2789, 107, 0, 163, 19727, 29, 53, 877, 0, 139, 37, 454823, 107, 431363, 191, 37, 0, 1231547, 41, 109, 0, 7523968684626643, 542410073, 379, 127, 956042657, 0, 1652359939, 0, 134593, 467, 2663, 18701, 0, 12890567, 0, 1361, 659, 331, 1301, 2927, 206766374172237107631553, 199, 79, 67, 1181, 149382661, 18492619, 312269-44590932009, 627898235566991, 733, 1427427733176073, 887, 211, 0, 17569, 139, 151, 9619, 229, 0, 239, 263, 443, 3023, 0, 8293, ???81???, 627078017207, 97, 4057, 131, 2252681, 163, ???88???, 1327, 30178433, 5107, 11013997953139, 5088213983, 519632149363, 149, 297509, 32869, 296340509, 1607, 168463, 4099, 148303, ⋯ . The two unknown entries (???**n**???) at indexes 81 and 88 are primes which exceed 3e9.

And this last sequence is when $\frac{1}{99} * \sum_{k=1}^{n}$ **n**! is a prime for the index **n**: 2, 4, 5, 6, 7, 11, 16, 22, 30, 34, 40, 42, 47, 49, 68, 74, 79, 168, 202, 245, and no others less than or equal to 262. Note that this series needs its own recitation, but notice that for all the above primes, the lesser significant digits become identical with increasing nodulus power of ten as **n** increases, just as the previous series does. Therefore; all that needs to be done is to take the preceeding sequence and divide it by 99.

What follows is not a series but are just observations in the gap between the primes (the size is followed by the factorial and which primes that surround it): 732=88!±1, 750=66!+1+2, 784=81!±1, 830=90!+1+2, 840=61!±1, 844=82!+1+2, 1078=89!+1+2, 1162=91!±1, 1200=85!−2−3, 1278=87!±1, 1282=99!−2−3, and lastly 1608=71!±1.

Reference: W.W. Rouse Ball and H.S.M. Coxeter, Mathematical Recreations and Essays, 13th edition, Dover Publ., N.Y., p66

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Sequentially yours,

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