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FREE SPECTRA OF 3-ELEMENT ALGEBRAS

Joel Berman

If A is an algebraic system, then the free spectrum of A is the sequence s(n) of cardinalities of the free algebras on n free generators in the equational class generated by A. This paper is a catalog of such free spectra for several hundred different 3-element algebraic systems. The catalog is organized lexicographically by the sequences <s(0), s(1), s(2),...>.

1. USES OF FREE SPECTRA. Several points of view are possible in describing free spectra. In universal algebra the value s(n) is the number of distinct n-ary polynomials on an algebra A. If the algebra A is finite of size k, then a result of Birkhoff states that s(n) ≤ k**(k**n). (Here, and elsewhere in this paper, r**s means r rasied to the power s. Also, C(n,i) denotes the binomial coefficient n choose i.) Standard sources on universal algebra and free spectra include Birkhoff(1967) and Gratzer(1979).

Another place where the sequence s(n) occurs is in logic, especially many-valued logic. Given a system of propositional logic, s(n) counts the ~~number of~~ possible distinct truth tables that can be constructed in this system using the given connectives of the system. For classical logic this is of course 2**(2**n). For the nonclassical 3-valued systems of Post, Heyting, or Lukasiewicz the number of such truth tables is given by the entries #235#, #187#, or #201# respectively, in the catalog. In the logic literature the set of operation tables for the fundamental connectives is often called a matrix. Note that in computing the values of s(n) the so called designated elements of the matrix play no role. Another way of describing the free spectra in this setting is that the value s(n) is the cardinality of the Lindenbaum-Tarski algebra of n variable formulas in the given

logical system. Rescher(1969) contains a very extensive presentation of the various many-valued logics and also has a detailed bibliography. Wolf(1977) contains an updating of this bibliography.

The theory of switching functions provides another place where free spectra occur. Here the numbers s(n) can be interpreted as the number of inequivalent circuits that can be built using a specified family of components and n input signals. In the case where there are two possible choices for each input this corresponds to the usual Boolean valued switching theory. If instead the signals have three possible values, then the s(n) sequences that arise correspond in a natural way to the free spectra of 3-element algebras. The Proceedings of the International Symposia on Multiple-valued Logic for the last ten years contain numerous papers on the theory of such switching functions. The book Moisil(1969) is devoted to this many-valued switching theory and the books Carvallo(1968) and Thielliez(1973) are devoted solely to the three-valued case. The paper Rosenberg(1977) has an extensive bibliography.

In Berman(1980) I considered the free spectra of 2-element algebras. In the 2-element case all the possible distinct equational classes that can be generated have been described by Post(1941). They form a countably infinite well-behaved family. Indeed, in my paper a description of all these equational classes, a list of their free spectra, and a tabulation of some other properties they possess all fit nicely onto one page.

The 3-element case is much more complicated. Firstly, there are an uncountable number of inequivalent equational classes that can be generated by 3-element algebras. Simple proofs of this are given in Janov and Mucnik (1959) and Hulanicki and Swierczkowski(1960). In fact, the equational classes they give have pairwise different free spectra. Also, many algebraic properties that hold for equational classes generated by 2-element algebras fail for the 3-element case. Berman(1980) contains about a half-dozen of such properties. The cause of this is not known except of course that 3 is bigger than 2. My motivation for writing this paper is in part an attempt to understand this.

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Another motivation for this paper is that there appears to be a strong connection between the free spectrum, especially its rate of growth, and some important algebraic properties the equational class may possess. In Berman(1980) and Berman(1982) I investigated this and I felt that many more examples of free spectra would be needed in order to pursue this idea. So this catalog is a large collection of experimental results to be used, I hope, in suggesting theorems about algebras and their equational classes; theorems involving numerical conditions on the free spectrum.

Yet another motivation for compiling this list of free spectra is that the spectrum of an algebra is an important invariant for the algebra, and the first few terms of this invariant are easily computed (on a computer). This catalog is thus a bestiary of many known 3-element algebras, indexed by their free spectra. My experience with the extremely useful book, A Handbook of Integer Sequences by N. Sloane led me to compile the smaller, more specialized catalog given in this paper. As it turned out, the intersection of the sequences in Sloane's book and the free spectra listed below is a very small set.

2. THE CATALOG. The free spectra are arranged in lexicographic order. The values of s(n) are obtained by the computer program described in Berman and Wolk(1980). Given the operation tables of an algebra A and an integer n, this program explicitly constructs the free algebra on n free generators for the equational class generated by A. That is, it produces a list of the distinct n-ary polynomials that can be built by composition from the fundamental operations of the algebra A. A detailed discussion of this program and an exact FORTRAN listing of it may be found in that paper.

In the catalog the values s(0), s(1),...,s(4) are explicitly listed. Following these first few values of the s(n) is an explicit formula for s(n) if such a formula is known. In some cases this formula for s(n) only applies to n > 0. In a few cases such a formula is followed by a question mark; this indicates a conjectured closed form for evaluating s(n). If fewer than five values of the s(n) are given, and if no general formula for s(n) is presented, then the missing values are not known. The value for s(0) is defined to be the number of constant functions that are generated by the program when computing the free algebra on 1 free generator. Note that s(0) can be positive even if there are no constants in the given similarity type of A.

The line following the values of s(n) in the catalog is the set of functions of the algebra, unary operations written first. The algebra, A say, always has the same underlying set: {0,1,2}. Then a unary operation g(x) is written as the 3-tuple (g(0) g(1) g(2)). Binary operations are written as 9-tuples, read across rows. Constants are treated as unary operations and appear as (ccc) for some c = 0,1, or 2. This catalog includes only algebras that have unary or binary operations.

The next lines for a given spectrum are a description of the algebra, logical system, set of switching functions, or clone or whatever. Most of the algebras etc. considered are taken from the literature. References are given whenever possible. An asterisk following a bibliographic item indicates that the article explicitly deals with the free algebras or the free spectra of the algebra A. Many of the closed forms for s(n) presented here do not appear in the literature. Most, but not all, of these formulas are easy to derive. A paper describing general techniques for finding closed forms for free spectra is in preparation.

Two equational classes are called polynomially equivalent if there is a weak isomorphism between their countably generated free algebras that preserves free generators. See Goetz(1966) and Taylor(1973) for more details. In the catalog, algebras that have the same free spectra and generate polynomially inequivalent equational classes are distinguished by adding a suffix "1" or "2" etc. to the appropriate headings. The following procedure was used in deciding polynomial equivalence for algebras giving the same initial values for their free spectra. First the list of polynomials in the free algebra on two free generators was scanned to see if it contained the operations (or some isomorph) of the other system. This is relatively easy using the computer text editor. If this turns up nothing, then another computer program generates the principal congruences in the free algebra on one or two generators for each of the two algebras. Examination of this output usually provided conclusive proof of polynomial inequivalence, or else suggested the desired weak isomorphism.

In the description of the algebra the following terminology is used. Unary operations are frequently described by a phrase describing their diagram as a directed graph. Thus the unary function (100) is a 2-cycle with tail. The names for the other nonisomorphic unary functions are given in the index under "unary algebra". An algebra whose set of fundamental operations consists of a single binary operation is called a groupoid. A groupoid is a semigroup if the binary operation is associative. An algebra A is idempotent if $f(x,\dots,x)=x$ for all of its operations f and all x in A. Note that if A is idempotent, then $s(0)=0$ and $s(1)=1$. A zero of a groupoid is an element z for which $zx = xz = z$ for all x. A groupoid element z is a unit if $zx = xz = x$ for all x.

To adjoin a constant to an algebra A means to add a particular constant function to the similarity type of A. If an algebra A has free spectrum $s(n)$, and if an algebra A' is obtained from A by adjoining a constant to A, the the free spectrum $s'(n)$ of A' satisfies the inequality

$$s(n) \le s'(n) \le s(n+1)$$

The adjoined constant is called generic if the equality $s'(n) = s(n+1)$ holds for all n. For example, the middle element in a 3-element distributive lattice is generic, the two other elements are not.

Given an algebra A there several ways to adjoin a new element z to A in order to create a new algebra B of the same similarity type as A. The element z is called an absorbing element if for any operation f of B, f restricted to A behaves as f does on A, otherwise f evaluates to z. In this case, B is obtained from A by adjoining an absorbing element to A. Absorbing elements come up in the study of regular equations. See Lakser & Padmanabhan & Platt(1972), Jonsson & Nelson(1974), or John(1976) for some results on this. The element z is said to be analogous to the element a of A if for any operation f of B, f restricted to A behaves as f does on A, otherwise for arguments of f that involve z, replace z by a, and evaluate as in A. In such a case the algebra B is said to be analog of the algebra A. (See Smiley(1962) or some of the "weak variants" in

Rescher(1969).) For example, the algebra in sequence #038# in the catalog is the 3-element analog of the 2-element distributive lattice: here z is 1 and a is 0. Finally, the element z is called invisible if there is some element a in A such that for any operation f of B, f restricted to A behaves as f does on A, otherwise f evaluates to a. The algebra in #049# is obtained by adjoining an invisible element to the 2-element distributive lattice.

Many of the entries in the catalog are derived from 3-valued propositional logics. Typically there are the binary operations of conjunction, disjunction, implication, and equivalence; the unary negation operation; and perhaps some logical constants. Of course frequently some of these operations can be defined in terms of the others. The reader is cautioned that there is no consistent pattern for which of the values 0,1, or 2 correspond to True or False in the logical system. A fragment of a logical system is a system in which only a subset of these connectives is allowed. For example if only implication is used, then the system is called an implicational fragment. If the point of view of universal algebra is used, then a fragment corresponds to a reduct of the algebra. In proving the independence of a set of axioms for a logical system, various ad hoc matrices are presented which satisfy some but not all of the given axioms. Some of the more interesting examples of such are also found in the catalog. For example, an algebra producing sequence #129# is a groupoid used in Sheffer(1913) to show the independence of his axioms for what is now called the Sheffer stroke. The groupoid has appeared sporadically in the literature since then.

3. THE INDICES. There are four indices following the catalog. The first is an index of all the binary operations that appear. If the 19683 possible binary operations on the set {0,1,2} are partitioned into classes of isomorphic or anti-isomorphic operations, then each class has at most 12 members. If an operation appears in the catalog, then the operation in its class having least value as a base 3 number when written as a 9-tuple is the representative chosen for the index. These representatives are then listed in increasing lexicographic (=numerical) order.

Next is an index of the number of essentially n-ary operations for each of the 235 free spectra listed. The number of essentially n-ary operations in an equational class is usually denoted by the sequence $p(n)$ for $n=0,1,2,\ldots$. For a given locally finite equational class the sequences $s(n)$ and $p(n)$ are related by

$$s(n) = \sum_{i=0}^{n} C(n,i)p(i) \qquad\qquad p(n) = \sum_{i=0}^{n} (-1)^{n-i} C(n,i)s(i)$$

Gratzer(1970) contains a survey of results on $p(n)$ sequences. The papers by Marczewski, Plonka, and Urbanik in the bibliography also contain work on these sequences.

The last two indices are more traditional. One is just an alphabetized list of words used in the description of the algebras and the equational classes of the catalog. The other is the bibliography, which is an index, since at the end of each item is a list of those sequences that cite it.

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CATALOG OF FREE ALGEBRAS IN EQUATIONAL CLASSES GENERATED BY 3-ELEMENT ALGEBRAS

line s: sequence of cardinalities of free algebras on 0,1,2,3,4 free generators, followed by general form if known. x*y means x times y; x**y means x raised to power y. C(n,i) is the binomial coefficient n choose i.
line f: the fundamental operations for the 3-element algebra. Parentheses enclose each operation of the algebra, binary functions are read across rows of Cayley table.
line d: description of the algebra, common name, properties etc.
line r: references, if any, to this variety or algebra. if reference contains information on free algebra, then this is indicated by a * .

lines f,d,r are repeated for polynomially equivalent algebras

lines f1,r1, etc. algebras with the same free spectra polynomially inequivalent to the previous ones

```
#001#s  0,1,2,3,4    n
#001#f  (012)
#001#d  1-unary: identity function
#001#f  (000 111 222)
#001#d  semigroup  xy=x

#002#s  0,1,3,7,15   2**n-1
#002#f  (000 010 002)
#002#d  semilattice
#002#f  (000 011 012)
#002#d  semilattice: chain

#003#s  0,1,3,9,27     3**(n-1)
#003#f  (021 210 102)
#003#d  groupoid: Steiner quasigroup, idempotent,  2x+2y (mod 3)
#003#r  Urbanik (1965), Gratzer & Padmanabhan(1971)*
#003#r  Baldwin & Lachlan(1973), Quackenbush(1976), Csakany(1980)

#004#s  0,1,3,15,531487  sum i=1 to n (C(n,i)*3**(3**(i-1)-2**i+1))
#004#f  (010 112 022)
#004#d  upper bound algebra; minimal binary clone; quasitrivial groupoid
#004#r  Kaiser(1975), Park(1976), Winker & Berman (1979)*
#004#r  Demetrovics & Hannak & Marchenkov(1980), Kepka(1981a), Csakany(1982)

#005#s  0,1,4,12,32      n*2**(n-1)
#005#f  (000 010 222)
#005#d  semigroup: idempotent, xyz=xzy, nearly quasitrivial
#005#r  Płonka(1971)*, Gerhard(1971)*, Jezek & Kepka(1978)
#005#f  (000 011 022)
#005#d  semigroup: idempotent, has zero, quasitrivial
#005#r  Plonka(1971)*, Gerhard(1971)*, Kepka(1981a)
#005#f1 (000 211 122)
#005#d1 groupoid: idempotent, (xy)y=x, quandle, kei
#005#r1 Takasaki(1943), Plonka(1971)*, Pierce(1978), Winker(1981)
#005#f2 (000 211 222)
#005#d2 groupoid: idempotent, (xy)y=xy, nearly quasitrivial
#005#r2 Plonka(1971)*, Day(1973)*, Jezek & Kepka(1978)

#006#s  0,1,4,15,64      sum i=0 to n (C(n,i)*(i factorial))
#006#f  (000 012 222)
#006#d  semigroup: idempotent, left distributive; minimal binary clone
#006#r  Gerhard(1971)*, Taylor(1976), Kepka(1981), Csakany(1982)
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%oR ~~MSG ... 103 ...~~ Berm 83.

#007#s 0,1,4,18,140
#007#f (000 011 222)
#007#d groupoid: quasitrivial, left distributive; minimal binary clone
#007#r Kepka(1981), Kepka(1981a), Csakany(1982)

#008#s 0,1,4,18,166,7579,7828352,2414682040996 ? new A7153
#008#f (000 011 012) (012 112 222)
#008#d distributive lattice
#008#r Birkhoff(1967),p.63*, Church(1965)*, Berman & Kohler(1976)*

#009#s 0,1,4,30
#009#f (000 110 202)
#009#d groupoid; minimal binary clone
#009#r Csakany(1982)

#010#s 0,1,4,36
#010#f (000 112 212)
#010#d groupoid: quasitrivial, left distributive
#010#r Kepka(1981), Kepka(1981a)

#011#s 0,1,4,54
#011#f (000 011 212)
#011#d groupoid: quasitrivial; minimal binary clone
#011#r Kepka(1981a), Csakany(1982)

X #012#s 0,1,4,162,88219206 #008#*3**(3**(n-1)-2**n+1) new
#012#f (002 011 212) (010 112 022)
#012#d tournament: triangle
#012#r Quackenbush(1972), Fried & Gratzer(1973)*

#013#s 0,1,5,28
#013#f (000 010 012)
#013#d groupoid: has zero

#014#s 0,1,5,96
#014#f (012 110 202)
#014#d groupoid: idempotent, has unit
#014#r Rose(1961), Marczewski(1964), Robinson(1971), Leigh(1972)

#015#s 0,1,6,33,266 sum i=1 to n (C(n,i)*#008#)
#015#f (000 011 012) (000 012 022)
#015#d distributive bisemilattice, distributive quasilattice
#015#d Bochvar fragment: disjunction, conjunction
#015#r Plonka(1967), Padmanabhan(1971)*, Plonka(1971a)*

#016#s 0,1,6,39,316
#016#f (000 111 102)
#016#d groupoid: left distributive
#016#r Kepka(1981)

#017#s 0,1,6,60,2367
#017#f (000 011 012) (012 111 212)
#017#d bisemilattice: bichain with one distributive law
#017#r Padmanabhan(1971)*, Romanowska(1980)*, see #018#

#018#s 0,1,6,60
#018#f (000 011 012) (000 010 002)
#018#d bisemilattice: satisfies no distributive laws
#018#r Dudek & Romanowska(1981)*, see #017#

#019#s 0,1,6,89
#019#f (001 011 112)

#019#d groupoid: idempotent and commutative
#019#r Keir(1964), Gutierrez & Moraga(1974)

#020#s 0,1,6,100
#020#f (000 011 012) (022 212 222)
#020#d bisemilattice
#020#r Bielecka-Holda(1980), Dudek & Romanowska(1981)*

#021#s 0,1,6,183
#021#f (000 012 212)
#021#d groupoid: quasitrivial
#021#r Kepka(1981a)

#022#s 0,1,6,213
#022#f (000 011 012) (010 111 012)
#022#d bisemilattice: bichain satisfies no distributive law
#022#r Dudek & Romanowska(1981)*

#023#s 0,1,7,505
#023#f (012 110 212)
#023#d groupoid: idempotent, has unit

#024#s 0,1,8,285
#024#f (010 011 012) (012 012 222)
#024#d system used for independence of lattice axioms
#024#r Croisot(1951)

#025#s 0,1,8,331
#025#f (001 012 122) (011 111 112)
#025#d fragment of Hanson ternary threshold logic
#025#r Hanson(1963)

#026#s 0,1,9,129
#026#f (011 111 112) (000 011 012) (012 112 222)
#026#d distributive lattice with third semilattice operation
#026#r Arnold(1951)

#027#s 0,1,9,489
#027#f (000 011 012) (012 112 222) (000 012 022)
#027#d lattice ordered semigroup
#027#r Gabovich(1976), Saito(1977)

#028#s 0,1,9,6561 3**((3**n-3)/3)
#028#f (010 112 022) (021 210 102)
#028#d clone of self dual functions preserving 0; quasiprimal
#028#r Demetrovics & Hannak & Marchenkov(1980), Csakany & Gavalcova(1982)

#029#s 0,1,10,411
#029#f (010 011 102)
#029#d groupoid: not entropic, preserves sums of subgroupoids
#029#r Evans(1962)

#030#s 0,1,14
#030#f (000 011 102)
#030#d groupoid: nonassociative, but satisfies the inclusion property
#030#r Salomaa(1959), p.138

#031#s 0,1,15
#031#f (001 010 102)
#031#d groupoid: idempotent, commutative
#031#r Keir(1964)

Spectrum of a certain 3-element algebra.

%N ~~Number of free algebras on n generators in a certain 3-element~~

#032#s 0,1,16
#032#f (000 010 002) (011 111 112)
#032#d bisemilattice

#033#s 0,1,27,531441 3**((3**n-3)/2)
#033#f (021 110 202)
#033#d groupoid; quasiprimal
#033#r Takasaki(1943), Csakany & Gavalcova(1982)

#034#s 0,1,729,282429536481 3**(3**n-3)
#034#f (012 112 222) (021 210 102)
#034#d all idempotent functions; quasiprimal
#034#r Quackenbush(1974)*

#035#s 0,2,4,6,8 2n
#035#f (002)
#035#d 1-unary: 2-chain with fixed point, f(f(x))=f(x)
#035#f (000 000 222)
#035#d semigroup: xy=xz
#035#f1 (021)
#035#d1 1-unary: 2-cycle with fixed point, involution, f(f(x))=x

#036#s 0,2,5,10,19 n+2**n-1
#036#f (000 000 002)
#036#d semigroup: has zero, analog of semilattice
#036#f (000 011 011)
#036#d semigroup: has zero, analog of semilattice
#036#r Bernstein(1921), Smiley(1962), Rescher(1969) p.336, Moraga(1975)

#037#s 0,2,6,18,68 n + #006#
#037#f (000 111 010)
#037#d groupoid: not entropic, preserves sums of subgroupoids
#037#r Evans(1962)

#038#s 0,2,6,21,170 n+#008#
#038#f (000 000 002) (002 002 222)
#038#d analog of distributive lattice
#038#d mutually distributive associative disjunction and conjunction
#038#r P. Dienes(1949), Rescher(1969) p.336

#039#s 0,2,7,19,47 (n+2)*2**(n-1)-1
#039#f (000 000 012)
#039#d semigroup: has zero; implication
#039#r Reichenbach(1944), Goddard & Routley(1973) p.351, Baker(1981)*

#040#s 0,2,7,22,69 n+3**n-2**n
#040#f (000 000 022)
#040#d groupoid: has zero

#041#s 0,2,7,25,181 #008# + 2**n -1 (?)
#041#f (000 000 002) (002 012 222)
#041#d mutually distributive conjunction and disjunction
#041#r P. Dienes(1949)

#042#s 0,2,8,24,64 n*2**n
#042#f (000 022 011)
#042#d groupoid: has zero, adjoin absorbing element to negation
#042#r Plonka(1971a)*, Baker(1981)*

#043#s 0,2,8,26,80 3**n-1
#043#f (000 001 012)
#043#d semigroup: has zero, xx=xxx; reduct of Chang MV algebra

#043#r Chang(1958)
#043#f1 (000 012 021)
#043#d1 semigroup: has zero, x=xxx
#043#d1 equivalential fragment of Bochvar and Kleene system
#043#r1 Rescher(1969), Chajda(1980)
#043#f1 (002 012 220)
#043#d1 semigroup: x=xxx

#044#s 0,2,8,35,212
#044#f (000 002 022)
#044#d groupoid: has zero

#045#s 0,2,8,59
#045#f (001 011 111)
#045#d groupoid
#045#r Moraga(1975)

#046#s 0,2,10,62,1138 sum i=1 to n (C(n,i)*#095#)
#046#f (000 011 021)
#046#d groupoid: has zero, adjoin absorbing element to implication
#046#d Bochvar fragment: implication
#046#r Plonka(1971a)*, Kalman(1980)*

#047#s 0,2,11,52,247
#047#f (011 122 222)
#047#d groupoid
#047#r Kabat & Wojick(1981)

#048#s 0,2,11,64,523
#048#f (000 001 022)
#048#d groupoid: has zero, not finitely based
#048#r Murskii(1965)

#049#s 0,2,11,492
#049#f (010 110 000) (000 010 000)
#049#d adjoin invisible element to distributive lattices

#050#s 0,2,12,114
#050#f (000 011 012) (012 111 211)
#050#d system used to show independence of lattice axioms
#050#r Sobocinski(1972)

#051#s 0,2,12,120
#051#f (112) (000 012 022) (012 112 222)
#051#d Conway's Kleene algebras
#051#r Conway(1971)

#052#s 0,2,12,158,33336 sum i=1 to n (C(n,i)*(2**(2**i-1)))
#052#f (000 012 021) (000 012 022)
#052#d adjoin zero to Boolean ring
#052#d Bochvar fragment: implication and equivalence

#053#s 0,2,12,174
#053#f (012 002 022)
#053#d regular implication
#053#r Cleave(1980)

#054#s 0,2,13,147
#054#f (000 011 012) (012 112 222) (111 111 112)
#054#d lattice ordered semigroup
#054#r Gabovich(1976), Saito(1977)

#055#s 0,2,13,174
#055#f (000 011 022) (012 122 222)
#055#d system used to show independence of lattice axioms
#055#r Sobocinski(1972)

#056#s 0,2,13,673
#056#f (012 101 212)
#056#d groupoid: has unit

#057#s 0,2,14,272
#057#f (000 001 012) (012 112 222)
#057#d system used to show independence of lattice axioms
#057#r Croisot(1951)

#058#s 0,2,16,659
#058#f (012 000 222)
#058#d implication; left distributive groupoid
#058#r McCarthy(1963), Bandler & Kohout(1979), Cleave(1980), Kepka(1981)

#059#s 0,2,18,1119
#059#f (000 110 210)
#059#d Kleene fragment: implication
#059#r Z.P. Dienes(1949), Church(1953)

#060#s 0,2,20,822
#060#f (012 121 221)
#060#d groupoid: has unit, adjoin unit to complementation

#061#s 0,2,22
#061#f (012 111 210) (012 112 222)
#061#d Kleene fragment: equivalence, conjunction

#062#s 0,2,25
#062#f (000 011 012) (012 111 210)
#062#d Kleene fragment: equivalence, disjunction

#063#s 0,2,28
#063#f (012 100 202)
#063#d groupoid: has unit, x(xx) and (xx)x need not be equal
#063#f1 (012 111 221)
#063#d1 groupoid: has unit, x(xx)=(xx)x=x, adjoin unit to implication

#064#s 0,2,60
#064#f (022 012 000)
#064#d implication
#064#r Sugihara(1955), Sobocinski(1952), Rose(1953), Dunn(1970)
#064#r Tokharz(1975)*, Biela(1975), Mortensen(1978)

#065#s 0,2,648,49589822592 2**(2**n-1)*3**(3**n-2**n-1)
#065#f (011 002 202) (011 002 222) (012 112 222) (000 011 012)
#065#d quasiprimal
#065#r Csakany & Gavalcova(1982)

#066#s 0,3,6,9,12 3n
#066#f (100)
#066#d 1-unary: 2-cycle with tail, pseudocomplementation
#066#f (002) (220)
#066#d 2-unary: 2-chain with fixed point, 2-cycle with tail
#066#f1 (120)
#066#d1 1-unary: 3-cycle, Post negation
#066#r1 Post(1921)
#066#f2 (002) (022)

#066#d2 2-unary: two 2-chains with fixed points
#066#r2 Belkin(1971), Gorbunov(1977)
#066#f3 (002) (112)
#066#d3 2-unary: two 2-chains with fixed points

#067#s 0,3,8,17,34 2**(n+1) + n - 2
#067#f (000 010 000) (222 212 222)
#067#d two analogs of semilattice

#068#s 0,3,9,27,81 3**n
#068#f (102 021 210)
#068#d groupoid: quasigroup, 2x+2y+1 (mod 3)
#068#r Clark & Krauss(1976)

#069#s 0,3,11,39,154 n + sum i=1 to n (C(n,i)*Bell number(i+1))
#069#f (000 010 001)
#069#d groupoid: has zero

#070#s 0,3,13,75
#070#f (000 010 011)
#070#d groupoid: has zero

#071#s 0,3,24
#071#f (000 010 021)
#071#d groupoid: has zero

#072#s 0,3,27,19683 3**(3**(n-1))
#072#f (122 020 110)
#072#d groupoid: satifies Martin's t-closing condition
#072#r Foxley(1962)
#072#f (120) (002 011 212)
#072#d upper bound algebra with 3-cycle adjoined
#072#d maximal clone of self dual functions; quasiprimal
#072#r, Jablonskii(1958)*, Kaiser(1975), McKenzie(1982)*,
#072#r, Demetrovics & Hannak & Ronyai(1982)*, Csakany & Gavalcova(1982)

#073#s 0,3,46
#073#f (012 122 222) (000 001 012)
#073#d reduct of Chang MV algebra S(2)
#073#d system used for independence of axioms for Kleene algebra
#073#r Chang(1958), Mukaidono(1981)

#074#s 0,3,68
#074#f (000 010 000) (012 012 102)
#074#d system used for independence of field axioms
#074#r Bernstein(1921)

#075#s 0,3,90
#075#f (000 110 120)
#075#d Rescher's version of Post implication
#075#r Rescher(1969) p.53

#076#s 0,3,138
#076#f (001 011 120)
#076#d groupoid: unknown if finitely based (due to Grzegorczyk)
#076#r Karnofsky(1968)

#077#s 0,3,168
#077#f (012 110 201)
#077#d groupoid: has unit

#078#s 0,3,432
#078#f (001 012 122) (012 111 210)
#078#d saturation arithmetic
#078#r Motil(1974)

#079#s 0,3,2187 3**(3**n-2)
#079#f (011) (012 112 222) (021 210 102)
#079#d quasiprimal
#079#r Csakany & Gavalcova(1982)

#080#s 0,4,8,12,16 4n
#080#f (002) (102)
#080#d 2-unary: 2-chain with fixed point, 2-cycle with fixed point

#081#s 0,4,16,52,160 2(3**n-1)
#081#f (021) (000 021 012)
#081#d adjoin zero to complemented Boolean group; Bochvar fragment: equivalence
#081#r Rescher(1969) p.29

#082#s 0,4,24,316,66672 sum i=1 to n (C(n,i)*2**(2**i)))
#082#f (000 021 011)
#082#d groupoid: has zero, adjoin absorbing element to Sheffer stroke
#082#r Plonka(1969), Tamthai & Chaiyakul(1980)
#082#f (210) (010 111 012)
#082#d Bochvar system of logic: negation and conjuction as primitive
#082#r Bochvar(1939), Church(1953), Rescher(1969) p.29, Goddard & Routley(1974) p.261

#083#s 0,4,56
#083#f (102) (000 012 222) (012 111 222)
#083#d logical system: modification of Kleene system
#083#r McCarthy(1963), Zaslavskii(1979)

#084#s 0,4,82,43916,160297985224 A7154
#084#f (210) (000 011 012) (012 112 222)
#084#d Kleene algebra, fuzzy switching functions, Lukasiewicz fragment
#084#r Kleene(1952) p.332, Balbes & Dwinger(1974)
#084#r Preparata & Yeh(1972)*, Mukaidono(1982)*, Berman & Mukaidono(1982)*
#084#f (222 211 210)
#084#d Sheffer function for Kleene algebra
#084#r Monteiro & Pico(1963), Sestakov(1964), Meredith(1969), Mouftah & Jordan(1974)

#085#s 0,4,264
#085#f (012 121 211)
#085#d groupoid: adjoin unit to Sheffer stroke
#085#f (210) (022 012 000)
#085#d Sobocinski system; Fragment Sugihara system; deontic logic
#085#r Sugihara(1955), Sobocinski(1952), Fisher(1961), Dunn(1970), Tokharz(1975)*

#086#s 0,4,1296,99179645184 2**(2**n)*3**(3**n-2**n-1)
#086#f (210) (000 011 012) (022 012 000)
#086#d Sugihara system; quasiprimal
#086#r Sugihara(1955), Dunn(1970), Tokharz(1975)*, Csakany & Gavalcova(1982)

#087#s 0,5,10,15,20 5n
#087#f (002) (200)
#087#d 2-unary: 2-chain with fixed point, 2-cycle with tail
#087#r Gratzer(1979) pp. 213-214.
#087#f1 (002) (221)
#087#d1 2-unary: 2-chain with fixed point, 2-cycle with tail
#087#f1 (100) (200)
#087#d1 2 unary: two 2-cycle with tail
#087#f2 (100) (101)
#087#d2 2-unary: two 2-cycle with tail

#088#s 0,6,12,18,24 6n
#088#f (002) (210)
#088#d 2-unary: 2-chain with fixed point, 2-cycle with tail
#088#f (021) (121)
#088#d 2-unary: 2-cycle with fixed point, 2-cycle with tail
#088#f1 (021) (100)
#088#d1 2-unary: 2-cycle with fixed point, 2-cycle with tail
#088#f2 (021) (102)
#088#d2 2-unary: two 2-cycle with fixed point
#088#f2 (021) (120)
#088#d2 2-unary: 2-cycle with fixed point, 3-cycle

#089#s 1,2,3,4,5 n+1
#089#f (000)
#089#d 1-unary: constant function
#089#f (000 000 000)
#089#d semigroup: has zero, xy=uv

#090#s 1,2,4,8,16 2**n
#090#f (000) (000 011 012)
#090#d semilattice with constant zero
#090#f1 (222) (000 011 012)
#090#d1 semilattice with constant unit

#091#s 1,2,5,13,33 1+n*2**(n-1)
#091#f (000 000 010)
#091#d groupoid: has zero
#091#r Wronski(1979)

#092#s 1,2,5,16,55 2+n+3**n-2**(n+1)
#092#f (000 001 020)
#092#d groupoid: has zero
#092#r Baker(1981)*

#093#s 1,2,5,19,167 1 + #008#
#093#f (000) (000 011 012) (012 112 222)
#093#d distributive lattice with constant minimal element
#093#f (222) (000 011 012) (012 112 222)
#093#d distributive lattice with constant maximal element

#094#s 1,2,6,19,57 1+n2**(n-1)+C(n,2)2**(n-2)
#094#f (000 001 010)
#094#d groupoid: has zero
#094#r Baker(1981)*

#095#s 1,2,6,38,942 sum i=0 to n ((-1)**(i-1)*C(n,i)*2**(2**(n-i))) A5530
#095#f (012 002 010)
#095#d implication algebra; BCK algebra; equivalent to 2-valued implication
#095#r Mitschke(1971), Iseki(1966)

#096#s 1,2,6,52
#096#f (012 100 200)
#096#d groupoid: has unit; pseudosum
#096#r Raca(1969)

#097#s 1,2,7,34,267 1+sum i=1 to n (C(n,i)*#008#)
#097#f (000) (000 011 012)
#097#d distributive quasilattices with 0 as constant

#098#s 1,2,7,46 n*(2**(2**(n-1)))-(n-1) (?)
#098#f (012 000 000)
#098#d implication
#098#r Brady(1971), Goddard & Routley(1973) p.324, Hardegree(1981), Cleave(1980)

#099#s 1,2,9,640
#099#f (012 102 220)
#099#d groupoid: has unit, Heyting fragment: equivalence
#099#r Kabzinski & Wronski(1975)*
#099#f (012 101 210)
#099#d groupoid: has unit, Lukasiewicz fragment: equivalence
#099#d distance function for Chang MV algebra
#099#r Chang(1958), Kabzinski(1979), Byrd(1979)

#100#s 1,2,12 2**n*3**((3**n-2*2**n+1)/2) (?)
#100#f (012 101 220)
#100#d groupoid: endomorphisms compose, but not "Abelian"
#100#r Lukasiewicz(1939), Klukovits(1973)

#101#s 1,2,14
#101#f (222 022 012)
#101#d Heyting fragment: implication; Hilbert algebra; BCK algebra
#101#r Jaskowski(1936), Skolem(1952)*, Henkin(1950), Horn(1962), Diego(1965)*,
#101#r Rielak(1974)*, Urquhart(1974)*, Iseki(1966)

#102#s 1,2,16
#102#f (012 112 222) (222 022 012)
#102#d Heyting fragment: disjunction, implication

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→ #103#s 1,2,18,39366 product i=0 to n-1 ((2**(2**i-1) + 1)**C(n,i))
#103#f (000 011 012) (222 022 012)
#103#d Heyting fragment: conjunction, implication, implicative semilattice
#103#r Nemitz & Whaley(1971), Balbes(1973)*, Landholt & Whaley(1974)*
#103#f (222) (000 011 012) (222 022 012)
#103#d Brouwerian semilattice
#103#r Kohler(1973)*, Kohler(1975)*, Davey(1976)*
#103#f (222) (000 011 012) (012 112 222) (222 022 012)
#103#d relative Stone algebra
#103#r Hecht & Katrinak(1972), Balbes & Dwinger(1974) pp. 166,176

#104#s 1,2,24,93312 2**(2**n-1)*3**((3**n-2*2**n+1)/2)
#104#f (022 212 222) (000 111 012) (210 021 012)
#104#d quasiprimal
#104#r Csakany & Gavalcova(1982)

#105#s 1,2,40
#105#f (012 001 000)
#105#d Lukasiewicz fragment: implication; BCK algebra
#105#f (012 001 010)
#105#d example of failure of p*(q*r)=(p*q)*(p*r)
#105#r Diego(1965) p.10, Iseki(1966), Byrd(1979)

#106#s 1,2,72,68024448 2**(2**n-1)*3**(3**n-2*2**n+1)
#106#f (012 001 000) (012 112 222)
#106#d complemented semigroup; quasiprimal algebra
#106#d Lukasiewicz fragment: implication, disjunction
#106#r Bosbach(1969), Csakany & Gavalcova(1982)
#106#f (012 001 000) (012 101 210)
#106#d Lukasiewicz fragment: implication, equivalence
#106#r Lukasiewicz(1920)
#106#f (012 101 210) (012 112 222)
#106#d Lukasiewicz fragment: equivalence, conjunction

#107#s 1,3,5,7,9 2n+1
#107#f (001)
#107#d 1-unary: 3-chain
#107#f (000) (001)

#107#d 2-unary: constant function, 3-chain
#107#f1 (000) (002)
#107#d1 2-unary: constant function, 2-chain with fixed point
#107#f1 (000) (011)
#107#d1 2-unary: constant function, 2-chain with fixed point
#107#f2 (000) (021)
#107#d2 2-unary: constant function, 2-cycle with fixed point

#108#s 1,3,6,10,15 C(n+2,2)
#108#f (000 000 001)
#108#d semigroup: has zero, xyz=rst
#108#d variety does not have definable principal congruence relations
#108#r Evans(1971) p. 31, Moraga(1975), Harrop(1976), Taylor(1977)

#109#s 1,3,6,11,20 n+2**n
#109#f (011 100 100)
#109#d semigroup: analog of Boolean group; equivalence operation
#109#r Salomaa(1959), p.136, Wesselkamper(1974)
#109#f (002 002 220)
#109#d semigroup: analog of Boolean group
#109#r Rescher(1969) p.336

#110#s 1,3,7,15,31 2**(n+1) - 1
#110#f (111) (000 011 012)
#110#d semilattice with generic constant

#111#s 1,3,8,20,48 1 + #039#
#111#f (000 000 111)
#111#d groupoid: has 0
#111#r Gutierrez & Moraga(1974), also see #039#r

#112#s 1,3,8,41,946 n + #095#
#112#f (000 000 220)
#112#d groupoid: analog of implication
#112#r Reichenbach(1944), Salomaa(1959), Rescher(1969) p.336, Bandler & Kohout(1979)

#113#s 1,3,9,26,72 (n**2+3n+8)2**(n-3)
#113#f (000 001 011)
#113#d groupoid: has zero
#113#r Klein-Barmen(1953), Moraga(1975)

#114#s 1,3,9,27,81 3**n
#114#f (000 020 001)
#114#d groupoid: has zero
#114#r Baker(1981)*
#114#f (021) (000 010 002)
#114#d involutory semigroup; fragment of Hanson threshold logic
#114#r Hanson(1963), Fajtlowicz(1972)
#114#f1 (012 120 201)
#114#d1 group: Z3
#114#f1 (021 102 210)
#114#d1 groupoid: quasigroup, x + 2y (mod 3)
#114#r1 Bernstein(1924), Clark & Krauss(1980)
#114#f1 (000 000 000) (012 120 201)
#114#d1 ring: trivial multiplication

#115#s 1,3,10,32,96 (n**2+n+4)*2**(n-2)
#115#f (000 001 111)
#115#d leader threshold function
#115#r Gutierrez & Moraga(1974)

#116#s 1,3,10,41,282 sum i=1 to n (C(n,i)*#093#)
#116#f (111) (000 011 012)
#116#d distributive quasilattices with non zero constant adjoined

#117#s 1,3,10,131,32772 n + 2**(2**n-1)
#117#f (000 000 002) (002 002 220)
#117#d analog of Boolean ring
#117#r Rescher(1969) p.336

#118#s 1,3,12,59,354
#118#f (000 000 100)
#118#d groupoid

#119#s 1,3,13,68,420
#119#f (001 000 100)
#119#d groupoid: commutative

#120#s 1,3,13,159 sum i=0 to n (C(n,i)*2**(2**i-1)) (?)
#120#f (000 012 021) (100 011 011)
#120#d two commutative semigroups
#120#r Wesselkamper(1974)

#121#s 1,3,15,273
#121#f (111) (000 012 020) (012 111 212)
#121#d Conway's Kleene algebras
#121#r Conway(1971)

#122#s 1,3,15,531487 (n+1)st term #004#
#122#f (000) (002 011 212)
#122#d adjoin generic constant to upper bound algebra

#123#s 1,3,16
#123#f (000 000 200)
#123#d groupoid
#123#r Gutierrez & Moraga(1974), Anderson & Belnap(1975) p. 85

#124#s 1,3,19,1120 1 + #059#
#124#f (000) (000 110 210)
#124#d Kleene fragment: implication with T as constant

#125#s 1,3,23 1 + #061#
#125#f (000) (012 111 210) (012 112 222)
#125#d Kleene fragment: equivalence, conjunction, T as constant

#126#s 1,3,26 1 + #062#
#126#f (000) (000 011 012) (012 111 210)
#126#d Kleene fragment: equivalence, disjunction, T as constant

#127#s 1,3,28,796
#127#f (001 000 000) (012 012 102)
#127#d system used to show independence of field axioms
#127#r Bernstein(1921)

#128#s 1,3,42
#128#f (012 102 221)
#128#d groupoid: has unit

#129#s 1,3,81,1594323 3**((3**n-1)/2)
#129#f (012 220 101)
#129#d groupoid: used in independence proof of axioms for Sheffer stroke
#129#r Sheffer(1912)
#129#f (210) (001 012 122)

#129#d fragment of Hanson threshold logic
#129#r Hanson(1963)
#129#f (120 201 012) (200 121 222)
#129#d complemented semigroup; quasiprimal algebra
#129#r Bosbach(1969), Csakany & Gavalcova(1982)

#130#s 1,3,81,2539107 one-half of #187#s
#130#f (000 011 012) (000 021 012) (012 100 200)
#130#d maximal subclone of #187#
#130#r Raca(1969)

#131#s 1,4,7,10,13 3n+1
#131#f (001) (002)
#131#d 2-unary: 3-chain, 2-chain with fixed point
#131#f1 (001) (010)
#131#d1 2-unary: 3-chain, 2-chain with fixed point
#131#f1 (001) (011)
#131#d1 2-unary: 3-chain, 2-chain with fixed point
#131#f2 (002) (010)
#131#d2 2-unary: two 2-chain with fixed point

#132#s 1,4,12,39,140
#132#f (010 121 010)
#132#d groupoid
#132#r Thelliez(1973), Wojciechowski & Wojcik(1979)

#133#s 1,4,17,72
#133#f (000 000 021)
#133#d groupoid: has zero

#134#s 1,4,18,166,7579 (n+1)st term of #008#
#134#f (111) (000 011 012) (012 112 222)
#134#d distributive lattice with one generic constant

#135#s 1,4,21,129, 991(?)
#135#f (000 001 022) (000 012 000) (000 000 012)
#135#d Murskii algebra with local discriminator operators
#135#r Pigozzi(1979)

#136#s 1,4,27,336
#136#f (000 001 021)
#136#d groupoid: has zero

#137#s 1,4,28
#137#f (000 002 021)
#137#d groupoid: has zero

#138#s 1,4,35
#138#f (002 002 100)
#138#d groupoid: leader for family of threshold functions
#138#r Gutierrez & Moraga(1974)

#139#s 1,4,49
#139#f (100 110 111)
#139#d groupoid: implication
#139#r Bernstein(1924), Turquette(1966), Biela(1975)

#140#s 1,4,56
#140#f (111) (000 011 012) (012 111 210)
#140#d Kleene fragment: equivalence, disjunction, U as constant
#140#r Kleene(1952) p.334

#141#s 1,4,64
#141#f (012 102 000)
#141#d implication
#141#r Goddard & Routley(1973) p. 365

#142#s 1,4,144,5038848 (2**(2**n-1))*#103#
#142#f (000 021 012) (200 121 222)
#142#d complemented semigroup; maximal subclone of #187#
#142#r Bosbach(1969), Raca(1969)

#143#s 1,4,162,68219206 (n+1)st term of #012#
#143#f (000) (002 011 212) (010 112 022)
#143#d tournament with one generic constant
#143#r Fried & Gratzer(1973)

#144#s 1,4,245
#144#f (222 201 212)
#144#d Rescher's version of Post equivalence operation
#144#r Rescher(1969) p.53

#145#s 1,4,576,8707129344 2**(2*2**n-2)*3**(3**n-2*2**n+1)
#145#f (022) (022 212 222) (000 111 012) (210 021 012)
#145#d quasiprimal
#145#r Csakany & Gavalcova(1982)

#146#s 1,5,9,13,17 4n+1
#146#f (002) (011)
#146#d 2-unary: two 2-chain with fixed point
#146#f (002) (212)
#146#d 2-unary: two 2-chain wih fixed point

#147#s 1,5,34,515
#147#f (200 002 022)
#147#d groupoid
#147#r Wojtylak(1979)

#148#s 1,5,43
#148#f (111) (000 110 210)
#148#d Kleene fragment: implication with u as constant

#149#s 1,5,63
#149#f (111) (012 111 210) (012 112 222)
#149#d Kleene fragment: equivalence, conjunction, u as constant
#149#r Kleene(1952) p.332

#150#s 1,5,114
#150#d (000 200 220)
#150#d implication
#150#r Saloman(1959), Thomas(1962), Goddard & Routley(1973) p.320
#150#r Meyer & Parks(1972), Epstein(1976), Zakrzewska(1976)

#151#s 1,5,130 2**(3**n-2**n+n)+n (?)
#151#f (022 202 220)
#151#d groupoid: equivalence
#151#r Reichenbach(1944), Muehldorf(1960), Goddard & Routely(1973) p.318

#152#s 1,5,136
#152#f (222 020 002)
#152#d groupoid: implication
#152#r Anderson & Belnap(1975) p. 40

#153#s 1,5,154
#153#f (110 010 000)
#153#d groupoid: adjoin invisible element to implication
#153#r Wajsberg(1937)

#154#s 1,6,11,16,21 5n+1
#154#f (001) (020)
#154#d 2-unary: two 3-chains
#154#f1 (001) (022)
#154#d1 2-unary: 3-chain, 2-chain with fixed point

#155#s 1,6,408
#155#f (000 100 201)
#155#d groupoid
#155#r Rose(1961)

#156#s 1,6,480
#156#f (022 001 000)
#156#d implication
#156#r Rescher(1969) p.135
#156#f (022 001 020)

#157#s 1,6,594
#157#f (022 001 010)
#157#d implication
#157#r Rescher(1969) p.135

#158#s 1,6,768
#158#f (001 100 201)
#158#d groupoid used to show independence of Sheffer axioms
#158#r Dines(1915), Taylor(1920)

#159#s 1,6,1944 2**(2**n-1)*3**(3**n-2**n)
#159#f (000 100 210) (012 122 222)
#159#d complemented semigroup; quasiprimal
#159#r Bosbach(1969), Werner(1978)

#160#s 1,7,13,19,25 6n+1
#160#f (001) (021)
#160#d 2-unary: 3-chain, 2-cycle with fixed point
#160#f (002) (021)
#160#d 2-unary: 2-chain with fixed point, 2-cycle with fixed point

#161#s 1,7,57
#161#f (000 020 011)
#161#d groupoid: has zero

#162#s 1,7,241
#162#f (021 000 021)
#162#d groupoid almost generating maximal clone
#162#r Jablonskii(1958) p. 111

#163#s 1,9,161
#163#f (102) (222) (012 111 222)
#163#d Zaslavskii system without all the constants specified
#163#r Zaslavskii(1979)

#164#s 1,9,6561,2541865828329 3**(3**n-1)
#164#f (000 012 021) (012 120 201)
#164#d ring: addition and multiplication (mod 3); maximal clone; quasiprimal
#164#f (012 202 121)
#164#d groupoid: used for complete independence proof

#164#r Dines(1915), Taylor(1920)
#164#f (201 201 202)
#164#r Carvallo(1968) p. 49

#165#s 2,3,4,5,6 n+2
#165#f (000) (111)
#165#d 2-unary: two constant functions

#166#s 2,3,5,9,17 1+2**n
#166#f (000) (222) (000 011 012)
#166#d semilattice with zero and unit as constants

#167#s 2,3,6,20,168 2+#008#
#167#f (000) (222) (000 011 012) (012 112 222)
#167#d bounded distributive lattice: zero and unit as constants
#167#r Balbes & Dwinger(1974)

#168#s 2,4,6,8,10 2n+2
#168#f (000) (102)
#168#d 2-unary: constant, 2-cycle with fixed point
#168#f1 (000) (110)
#168#d1 2-unary: constant, 3-chain
#168#f2 (000) (112)
#168#d2 2-unary: constant, 2-chain with fixed point

#169#s 2,4,8,16,32 2**(n+1)
#169#f (000) (111) (000 011 012)
#169#d semilattice: has zero as constant and generic constant
#169#f1 (111) (222) (000 011 012)
#169#d1 semilattice: has unit as constant and generic constant

#170#s 2,5,8,11,14 3n+2
#170#f (000) (100)
#170#d 2-unary: constant, 2-cycle with tail
#170#f (000) (101)
#170#d 2-unary: constant, 2-cycle with tail
#170#f1 (001) (110)
#170#d1 2-unary: two 3-chains
#170#f2 (001) (112)
#170#d2 2-unary: 3-chain, 2-chain with fixed point

#171#s 2,5,10,19,36 n+2**(n+1)
#171#f (220) (002 002 220)
#171#d analog of complemented Boolean group
#171#r Rescher(1969) p.336

#172#s 2,5,14,49,298 sum i=0 to n (C(n,i)*#167#)
#172#f (111) (222) (000 011 012) (000 012 022)
#172#d distributive quasilattices with two non-zero constants

#173#s 2,5,18,259,65540 n+2**(2**n)
#173#f (220) (002 002 222)
#173#d analog of negation and disjunction
#173#r Church(1953), Smiley(1962), Rescher(1969) p.336

#174#s 2,5,19,167,7580 1 + (n+1)st term of #008#
#174#f (000) (111) (000 011 012) (012 112 222)
#174#d distributive lattice: with zero constant and generic constant

#175#s 2,5,22,983(?)
#175#f (000) (222) (022 222 222) (000 000 002)
#175#d monotonic for 3-element chain with 2-element range

#176#s 2,5,23,311,66659 1+sum i=0 to n (C(n,i)*2**(2**i) - 1)
#176#f (200) (000 011 012)
#176#d pseudocomplemented semilattice
#176#r Jones(1972)*, Balbes(1973)*, Jones(1974), Taylor(1976)

#177#s 2,5,105
#177#f (000) (212) (012 112 222) (000 021 012)
#177#d maximal subclone of #187#
#177#r Raca(1969)

#178#s 2,6,10,14 18 4n+2
#178#f (002) (101)
#178#d 2-unary: 2-chain with fixed point, 2-cycle with tail
#178#f (001) (102)
#178#d 2-unary: 3-chain, 2-cycle with fixed point

#179#s 2,6,26,318,66674 sum i=0 to n (C(n,i)*2**(2**i))
#179#f (222) (000 011 021)
#179#d Bochvar system with two nonzero constants adjoined

#180#s 2,6,45
#180#f (111) (012 101 210)
#180#d Lukasiewicz fragment: equivalence with Slupecki constant
#180#r Slupecki(1936)
#180#f (220) (012 102 220)
#180#d Heyting fragment: equivalence, negation

#181#s 2,6,50
#181#f (200) (012 112 222)
#181#d Heyting fragment: disjunction, negation

#182#s 2,6,70
#182#f (200) (222 022 012)
#182#d Heyting fragment: implication, negation
#182#r McCall(1962)*, Horn(1962)

#183#s 2,6,84
#183#f (210) (012 101 210)
#183#d Lukasiewicz fragment: equivalence and negation
#183#f (111) (012 102 220)
#183#d Heyting fragment: equivalence, 1 as constant
#183#d compare #184#; not congruence distributive

#184#s 2,6,84,43918,160297985276 2+#084#
#184#f (000) (222) (210) (000 011 012)
#184#d Kleene algebra with two constants; compare #183#

#185#s 2,6,90,60750 product i=0 to n ((1+2**i)**C(n,i))
#185#f (200) (212) (000 021 012)
#185#d maximal subclone of #187#
#185#r Raca(1969)

#186#s 2,6,108,233280 product i=0 to n (i-th term #167#**C(n,i))
#186#f (200) (000 011 012) (012 112 222)
#186#d pseudocomplemented lattice; Stone lattice
#186#d Heyting fragment: conjunction, disjunction, negation
#186#r Balbes & Horn(1970)*, Gratzer(1971)*, Balbes & Dwinger(1974)*

#187#s 2,6,162,5078214 #103#*(2**(2**n-1)+1)
#187#f (000) (000 011 012) (222 022 012)
#187#d Heyting system: implication, conjunction, F as constant
#187#f (000) (000 011 012) (012 112 222) (222 022 012)

#187#d Heyting algebra; L-algebra; pseudo Boolean algebra
#187#r Heyting(1930), Godel(1932), Birkhoff(1940), Rasiowa & Sikorski(1963)
#187#r Raca(1966), Raca(1969)*, Horn(1969)*, Kohler(1973)*, Monteiro(1972)*,
#187#r Balbes & Dwinger(1974), Kohler(1975)*, Davey(1976)*

#188#s 2,7,12,17,22 5n+2
#188#f (001) (100)
#188#d 2-unary: 3-chain, 2-cycle with tail
#188#f (001) (101)
#188#d 2-unary: 3-chain, 2-cycle with tail

#189#s 2,8,14,20,26 6n+2
#189#f (002) (100)
#189#d 2-unary: 2-chain with fixed point, 2-cycle with tail

#190#s 2,9,16,23,30 7n+2
#190#f (002) (020) (200)
#190#d 3-unary: three peak functions
#190#r Rosser & Turquette(1945), Rosenberg(1976) p.11

#191#s 2,9,514,134217731 n + 2**(3**n)
#191#f (100) (110 010 000)
#191#d implicational logic; adjoin invisible element to implication and negation
#191#r Wajsberg(1937)
#191#f (101) (110 010 110)
#191#d variant of implicational logic
#191#r Wajsberg(1937), Zakrzewska(1976)

#192#s 2,9, >1000
#192#f (000) (111) (002 011 212) (010 112 022)
#192#d tournament: two elements as constants
#192#r Fried & Gratzer(1973)

#193#s 2,10,18,26,34 2+8n
#193#f (022) (202) (220) (210)
#193#d 3 peaks and negation function
#193#r Rosser & Turquette(1945)

#194#s 2,10,516
#194#f (102) (000 011 011)
#194#d negation and implication used in independence proof
#194#r Bernays(1926), Meredith(1953)

#195#s 2,10,562
#195#f (101) (000 011 012) (012 112 222)
#195#d distributive lattice with additional unary operation
#195#r Kollar(1980)

#196#s 2,10,622
#196#f (101) (000 012 012)
#196#d negation and implication used in independence proof
#196#r Bernays(1926)
#196#f (022) (202) (220) (012 002 000)
#196#d example of implication with Rosser & Turquette J functions
#196#r Shoesmith & Smiley(1978) p. 358
#196#f (002) (022) (200) (012 000 000)
#196#d matrix used for independence proof in modal logic
#196#r Lukasiewicz(1953), attributed to C. A. Meredith

#197#s 2,10,626
#197#f (200) (220) (000 011 012)
#197#d semilattice with pseudocomplement and dual pseudocomplement
#197#f (002) (020) (200) (000 011 012)
#197#r fragment Muehldorf system as in Rosenberg(1976) p.12

#198#s 2,10,642
#198#f (100) (000 012 222)
#198#d negation and implication used in independence proof
#198#r Bernays(1926)

#199#s 2,10,808
#199#f (111) (012 001 000)
#199#d Lukasiewicz implication with Slupecki constant
#199#r Slupecki(1936)

#200#s 2,12,656
#200#f (102) (011 000 000) (012 112 222)
#200#d logical system similar to Bochvar system
#200#r Pirog-Rzepcka(1973)
#200#f (002) (020) (200) (210) (012 111 212)
#200#d logical system similar to Bochvar system
#200#r Finn & Grigolija(1979)

#201#s 2,12,3888,297538935552 (2**(2**n))*(3**(3**n-2**n))
#201#f (000) (222) (210 121 012) (222 122 012)
#201#d Lukasiewicz system: implication, equivalence, two constants
#201#d maximal clone, quasiprimal
#201#r Lukasiewicz(1920)
#201#f (210) (012 001 000)
#201#d Lukasiewicz system: negation, implication as primitive
#201#d implicaton = max(0,x-y), negation = 2-x
#201#r Slupecki(1936)
#201#f (210 100 000)
#201#d Sheffer stroke for Lukasiewicz algebra
#201#r McKinsey(1936), Evans & Hardy(1957)
#201#f (111) (222 022 012) (000 011 012)
#201#d Heyting fragment: implication, conjunction, 1 as constant
#201#f (000) (210) (002) (022) (000 011 012) (012 112 222)
#201#d Lukasiewicz algebra, Moisil algebra
#201#r Moisil(1940), Cignoli(1970)*, Balbes & Dwinger(1974)*
#201#f (102) (000 012 020)
#201#d negation and implication used in indpendence proof
#201#r Bernays(1926)
#201#f (210) (022 002 000) (000 011 012)
#201#d Chang MV algebra
#201#r Chang(1958)
#201#f (001) (102) (012 111 222)
#201#d EFC system for recursively defined predicates
#201#r McCarthy(1963)
#201#f (011) (102) (000 012 022)
#201#d Aqvist logic of nonsense
#201#r Aqvist(1962)
#201#f (002) (020) (200) (000 011 012) (012 112 222)
#201#d max, min, three peak functions
#201#r Rosser & Turquette(1945), Rosenberg(1976) p.11
#201#f (200) (220) (000 011 012) (012 112 222)
#201#d regular double Stone algebra
#201#r Varlet(1972), Katrinak(1974), Hecht & Katrinak(1974)*
#201#f (200) (220) (210) (000 011 012)
#201#d pseudocomplemented Kleene algebra
#201#r Romanowska(1981)
#201#f (000) (222) (000 011 012) (012 112 222) (222 022 002)
#201#d P-algebra
#201#r Epstein & Horn(1974)

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#202#s 3,5,7,9,11 2n+3
#202#f (000) (122)
#202#d 2-unary: constant, 3-chain

#203#s 3,5,9,17,33 1+2**(n+1)
#203#f (000) (111) (222) (000 011 012)
#203#d semilattice: all three constants adjoined

#204#s 3,6,9,12,15 3n+3
#204#f (000) (120)
#204#d 2-unary: constant, 3-cycle
#204#f1 (000) (121)
#204#d1 2-unary: constant, 2-cycle with tail

#205#s 3,6,12,24,48 3*2**n
#205#f (111) (222) (000 010 002)
#205#d semilattice: has two constants

#206#s 3,6,20,168,7581 2+(n+1)st term of #008#
#206#f (000) (111) (222) (000 011 012) (012 112 222)
#206#d distributive lattice: all three constants named

#207#s 3,8,13,18,23 5n+3
#207#f (100) (221)
#207#d 2-unary: two 2-cycle with tail
#207#f1 (001) (221)
#207#d1 2-unary: 3-chain, 2-cycle with tail
#207#f1 (001) (220)
#207#d1 2-unary: 3-chain, 2-cycle with tail

#208#s 3,8,29,141
#208#f (000) (111) (222) (000 001 022)
#208#d Murski groupoid with all constants adjoined
#208#r Pigozzi(1979)

#209#s 3,8,38,566 #176# + 2**(2**n)-1 (?)
#209#f (111) (200) (000 011 012)
#209#d Heyting fragment: conjunction, negation, all constants
#209#d pseudocomplemented semilattice with all constants

#210#s 3,8,52
#210#f (000) (111) (222) (100) (221) (000 011 012) (012 112 222)
#210#d N-Boolean algebra
#210#r Schmidt(1972)

#211#s 3,9,15,21,27 6n+3
#211#f (002) (121)
#211#d 2-unary: 2-chain with fixed point, 2-cycle with tail
#211#f1 (000) (021) (120)
#211#d1 3-unary: all permutations and constants

#212#s 3,9,27,81,243 3**(n+1)
#212#f (111) (012 120 201)
#212#d group Z3 with all constants named; maximal clone
#212#r Jablonskii(1958), Clark & Krauss(1980)

#213#s 3,9,40,569
#213#f (000 010 000) (222 222 221)
#213#d analog of semilattice and analog of Sheffer stroke

#214#s 3,10,17,24,31 7n+3
#214#f (100) (121)

#214#d 2-unary: two 2-cycle with tail
#214#f (100) (220)
#214#d 2-unary: two 2-cycle with tail
#214#f1 (002) (020) (200) (111)
#214#d1 4-unary: three peaks and constant

#215#s 3,10,45,248
#215#f (000) (111) (222) (000 001 022) (000 012 000) (000 000 012)
#215#d Murski algebra with constants and local discriminators
#215#r Pigozzi(1979)

#216#s 3,10,46,244
#216#f (001) (002) (022) (111) (012 112 222)
#216#d maximal subclone of monotonic functions #220#
#216#r Machida(1979)

#217#s 3,10,59
#217#f (001) (111) (112) (022 222 222) (000 000 002)
#217#d maximal subclone of monotonic functions #220#
#217#d intersection of clone #220# and #234#
#217#r Kolpakov(1974), Machida(1979)

#218#s 3,10,83 n+3**(2**n) (?)
#218#f (211 100 100)
#218#d groupoid with large s(1)
#218#r Foxley(1962), Muzio(1971)

#219#s 3,10,88
#219#f (111) (200) (012 112 222)
#219#d Heyting fragment: disjunction, negation, all constants

#220#s 3,10,175
#220#f (111) (002) (022) (000 011 012) (012 112 222)
#220#d maximal clone: monotonic functions
#220#r Jablonskii(1958), Alexseev(1974)*, Schweigert(1979), Epstein & Liu(1982)*

#221#s 3,11,163 2 + #163#
#221#f (102) (111) (222) (012 111 222)
#221#r Zaslavskii(1979)

#222#s 3,11,197,129615,430904428717

#222#f (000) (111) (222) (210) (000 011 012)
#222#d Fragment of Slupecki variant of Lukasiewicz system; Regular functions
#222#r Kleene(1952) p.332, Berman & Mukaidono(1982)*

#223#s 3,12,21,30,39 9n+3
#223#f (001) (200)
#223#d 2-unary: 3-chain, 2-cycle with tail
#223#f (001) (121)
#223#d 2-unary: 3-chain, 2-cycle with tail
#223#f (001) (212)
#223#d 2-unary: 3-chain, 2-chain with fixed point
#223#f1 (001) (211)
#223#d1 2-unary: two 3-chains
#223#f1 (001) (202)
#223#d1 2-unary: two 3-chains

#224#s 3,12,207
#224#f (112 200 200)
#224#d groupoid with large s(1)
#224#r Muzio(1971)

#225#s 3,13,23,33,43 10n+3
#225#f (001) (210)
#225#d 2-unary: 3-chain, 2-cycle with fixed point
#225#f (021) (101)
#225#d 2-unary: 2-cycle with fixed point, 2-cycle with tail

#226#s 3,15,333
#226#f (121 100 100)
#226#d groupoid
#226#r Foxley(1962), Muzio(1971), Rose(1979)

#227#s 3,15,369
#227#f (111) (210) (012 101 210)
#227#d equivalence and negation in Lukasiewicz system with Slupecki constant
#227#f (111) (220) (012 102 220)
#227#d Heyting fragment: equivalence, negation, all constants

#228#s 3,15,525
#228#f (121 200 100)
#228#d groupoid with large s(1)
#228#r Muzio(1971)

#229#s 3,15,2275,473490375 product i=0 to n ((1+2**(2**i))**C(n,i))
#229#f (000) (111) (000 011 012) (222 022 012)
#229#d Heyting fragment: implication, conjunction, all truth values
#229#d Heyting algebra with all constants; maximal clone
#229#r Jablonskii(1958), McKenzie(1982)*, Demetrovics & Hannak & Ronyai(1982)*
#229#f (000) (112) (000 011 012) (222 022 012)
#229#d Heyting algebra with modal operator
#229#r Mcnab(1981)
#229#f (000) (122) (000 011 012) (222 022 012)
#229#d Heyting algebra with additional unary operation
#229#r Ursini(1979)

#230#s 3,17,1361
#230#f (222) (021 000 021) (001 020 100)
#230#d maximal clone: preserves central relation
#230#r Jablonskii(1958), Lau(1980), Demetrovics & Hannak & Ronyai(1982)*

#231#s 3,24,45,66,87 21n+3
#231#f (001) (120)
#231#d 2-unary: 3-chain, 3-cycle
#231#f (002) (120)
#231#d 2-unary: 2-chain with fixed point, 3-cycle
#231#f (100) (120)
#231#d 2-unary: 2-cycle with tail, 3-cycle

#232#s 3,27,51,75,99 24n+3
#232#f (001) (021) (120)
#232#d all unary operations
#232#r Picard(1935)

#233#s 3,27,105,399,1557 3*2**(2*n+1) + 6*n - 3
#233#f (120) (021) (001 001 110)
#233#d clone of quasilinear functions, maximal subclone of #234#
#233#r Burle(1967), Malcev(1972), Malcev(1973)
#233#r Rosenberg & Szendrei(1981), Berman & McKenzie(1982)*

#234#s 3,27,1545 3*2**(3**n) + 6*n - 3
#234#f (001) (021) (120) (011 111 111)
#234#d all unary and all non-onto n-ary operations; maximal clone
#234#r Jablonskii(1958), Burle(1967)

#235#s 3,27,19683,7625597484987 3**(3**n)
#235#f (201) (012 112 222)
#235#d Post system of 3-valued logic: negation, disjunction as primitive
#235#r Post(1921)
#235#f (121 222 120)
#235#d groupoid: commutative, min(x,y)+1 (mod 3)
#235#r Webb(1936)
#235#f (111) (210) (012 001 000)
#235#d Slupecki variant of Lukasiewicz system
#235#r Slupecki(1936)
#235#f (111) (000 012 021) (012 120 201)
#235#d ring of integers mod 3 with all constants; field GF(3)
#235#f (221 201 111)
#235#d Rescher's version of Post conjunction
#235#r Rescher(1969) p.53
#235#f (002) (200) (220) (022) (000 011 012) (012 112 222)
#235#r Yoeli & Rosenfeld(1965)
#235#f (000) (111) (222) (210) (022) (002) (000 011 012) (012 112 222)
#235#d Post algebra
#235#r Cignoli(1970)*, Dwinger(1972)*, Balbes & Dwinger(1974)*

INDEX OF BINARY OPERATIONS

The minimal member of isomorphism/anti-isomorphism class is given.

(000 011 012)	#002# #008# #015# #017# #018# #020# #022# #026# #027# #034# #041# #050# #051# #052# #054# #057# #061# #062# #065# #079# #082# #084# #086# #090# #093# #097# #102# #103# #106# #110# #116# #121# #125# #126# #130# #134# #140# #149# #166# #167# #169# #172# #174# #176# #177# #181# #184# #186# #187# #195# #197# #200# #201# #203# #206# #209# #210# #216# #219# #220# #222# #229# #235#
(000 011 021)	#046# #179#
(000 011 022)	#005# #024# #055# #196#
(000 011 102)	#030#
(000 011 212)	#011#
(000 011 222)	#007#
(000 012 021)	#043# #052# #061# #062# #078# #081# #120# #125# #126# #130# #140# #142# #149# #164# #177# #185# #235#
(000 012 111)	#058#
(000 012 122)	#023#
(000 012 212)	#021#
(000 012 220)	#063#
(000 012 222)	#006# #024# #083# #104# #145# #163# #198# #201# #221#
(000 020 001)	#114#
(000 020 011)	#161#
(000 021 011)	#082#
(000 021 021)	#042#
(000 021 222)	#053#
(000 022 112)	#065#
(000 100 100)	#200#
(000 100 111)	#191#
(000 100 200)	#098# #196#
(000 100 201)	#155#
(000 100 210)	#105# #106# #159# #199# #201# #235#
(000 100 220)	#101# #102# #103# #182# #187# #196# #201# #229#
(000 101 100)	#201#
(000 101 102)	#064# #085# #086#
(000 101 210)	#105#
(000 101 220)	#095# #129# #142#
(000 102 100)	#156#
(000 102 110)	#156#
(000 102 120)	#157#
(000 102 202)	#059# #124# #148#
(000 110 120)	#075#
(000 110 202)	#009#
(000 110 222)	#005#
(000 111 102)	#016#
(000 111 222)	#001#
(000 112 212)	#010#
(000 202 101)	#162# #230#
(000 211 122)	#005# #074# #127#
(001 000 100)	#119#
(001 000 220)	#138#
(001 001 110)	#233#
(001 010 102)	#031#
(001 011 111)	#045# #049#
(001 011 112)	#019#
(001 011 120)	#076#
(001 012 120)	#077#
(001 012 121)	#063#
(001 012 122)	#014# #025# #078# #129#
(001 020 100)	#230#
(001 021 111)	#084#
(001 100 201)	#158#
(001 101 000)	#139#
(001 101 100)	#152#
(001 101 111)	#153# #191#
(001 110 012)	#029#

(001 201 201)	#156#
(001 210 122)	#033#
(001 212 011)	#065#
(002 000 201)	#144#
(002 002 220)	#109# #117# #120# #171#
(002 002 221)	#132#
(002 011 212)	#004# #012# #028# #072# #122# #143# #192#
(002 012 220)	#043#
(002 012 221)	#056#
(002 022 220)	#147#
(002 101 200)	#141#
(011 100 100)	#109#
(011 101 110)	#151#
(012 100 200)	#096# #130#
(012 101 210)	#099# #106# #180# #183# #201# #227#
(012 101 220)	#100# #104# #145#
(012 102 221)	#128#
(012 120 201)	#114# #129# #164# #212# #235#
(012 121 211)	#085#
(012 121 221)	#060#
(012 201 120)	#114#
(012 202 121)	#156#
(012 220 101)	#129#
(021 210 102)	#003# #028# #034# #079#
(100 000 000)	#213#
(101 020 101)	#218#
(101 020 121)	#224#
(101 021 111)	#201#
(101 200 101)	#226#
(101 221 200)	#072#
(102 021 210)	#068#
(111 120 100)	#235#
(121 200 100)	#228#

TABLE OF p(n) SEQUENCES

Table of number of essentially n-ary operations for n=0,1,2,3,4. If fewer than five values are given it is because the values are too large to print or are unknown.

#001#p 0 1 0 0 0
#002#p 0 1 1 1 1
#003#p 0 1 1 3 5
#004#p 0 1 1 9 531441
#005#p 0 1 2 3 4
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%N Essentially #n#-operations in a certain 3-element algebra.

WORD INDEX

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The following abbreviations are used:

BSL - Polish Academy of Sciences. Institute of Philosphy and Sociology. Bulletin of the Section of Logic.
ISMVL - Proceedings of the International Symposium on Multiple-valued Logic.
MR - Mathematical Reviews.
NDJFL - Notre Dame Journal of Formal Logic
ZMLG - Zeitschrift fur Mathematische Logik und Grundlagen der Mathematik.

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