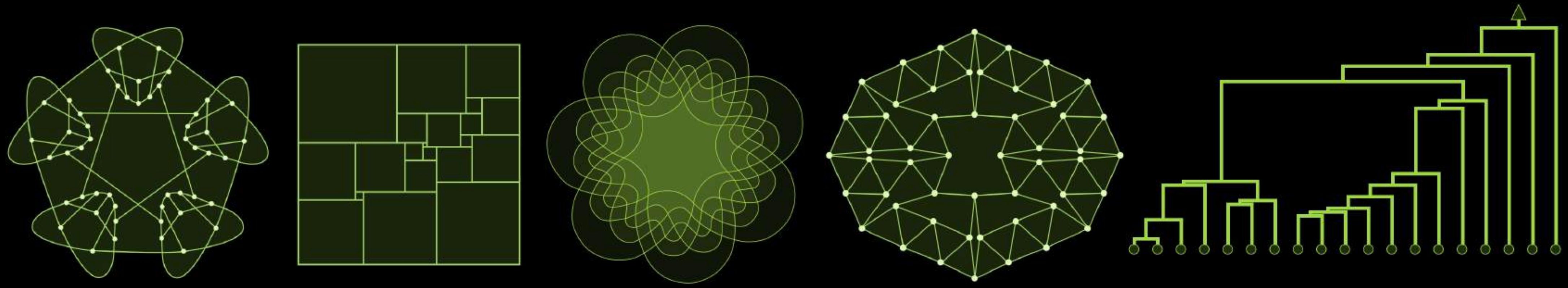
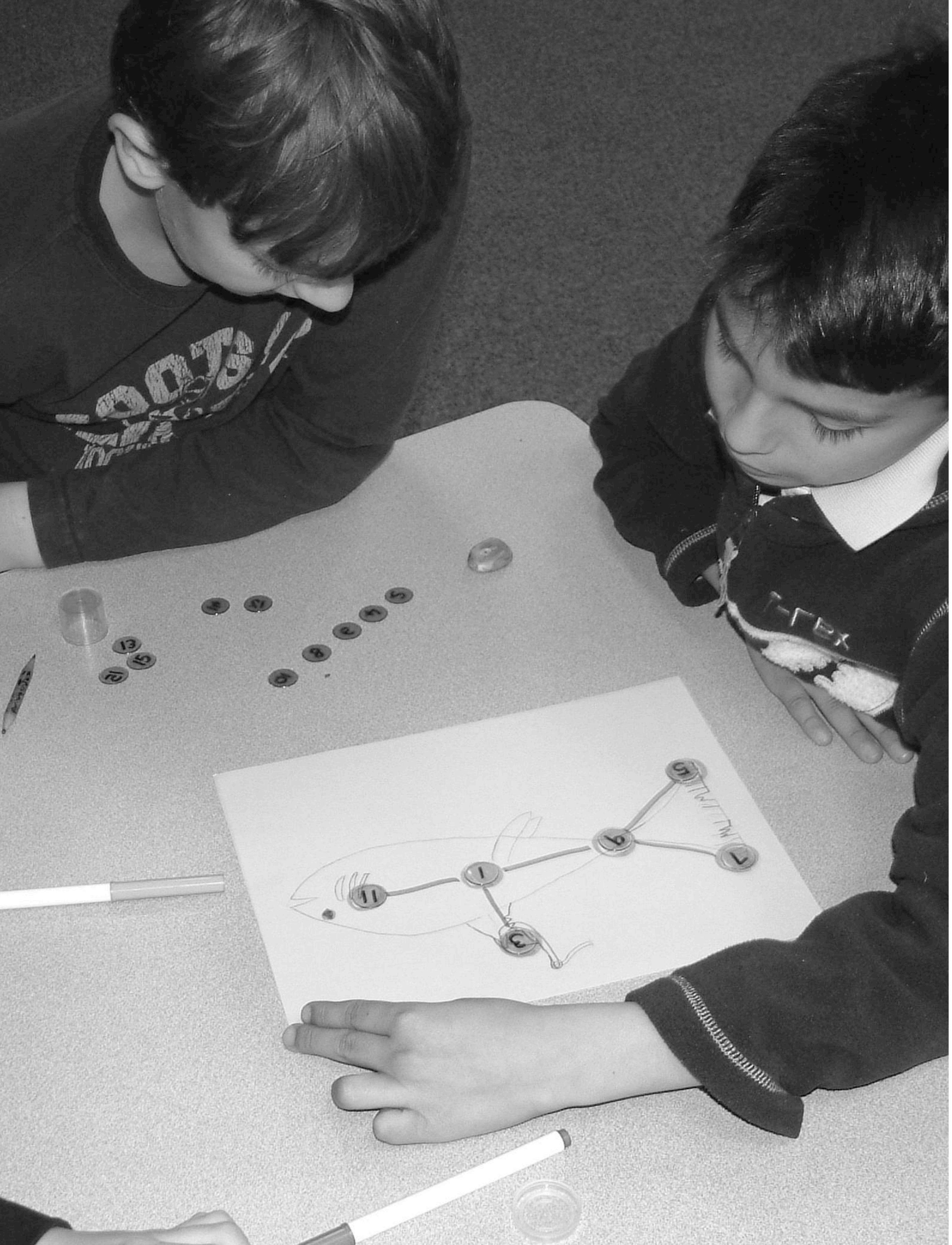


Integer Sequences K-12

A conference for mathematicians and educators

Feb. 27-Mar. 1 2015
Banff International Research Station



Integer Sequences K-12

The On-Line Encyclopedia of Integer Sequences (OEIS) has many pedagogic gems that remain undiscovered by K-12 educators. These sequences need to be lifted out of obscurity and become a part of every child's experience of mathematics.

The primary objective of this conference is to bring together educators and mathematicians to select 13 curricular integer sequences - one for each grade K-12. The secondary objective is to initiate a practical campaign to get the selected sequences the wide exposure they deserve.

It is not obvious that integer sequences - beyond those already in the curriculum - deserve pedagogic attention. It is easy to imagine their uninspired use in the classroom… for example: all students asked to independently reproduce a meaningless sequence term by term.

We won’t let that happen. Our strength is in the collaboration between mathematicians (who can identify sequences that will reward a young explorer,) and educators (who know the curriculum and can predict classroom challenges.)

The integer sequences we select do not all have to be worthy of a 45 minute exploration. In fact, the sequence of all 1s would be a very nice addition to the kindergarten or grade 1 classroom. It’s so ridiculously simple, yet can be used to draw attention to a pattern even more basic than ABABABAB. The all 1s sequence would also be good to remind students what skip counting by zero looks like. Usually students learn to skip count by 2s, 5s and 10s… but not by 0s.

If a sequence is worthy of a full period (or more) of exploration, it needs to engage students of diverse ability. Top students should be engaged because the sequence is intriguing and leads them to wonder, hypothesize and problem solve which are never a waste of time. Struggling students need to be engaged in acquiring curricular skills.

The target for the thirteen Integer Sequences that we select are not necessarily teachers from those states who adopted the Common Core State Standards, but if consensus can be reached without compromising quality let’s try to fit within that framework.

Gordon Hamilton
MathPickle

Je n'ai fait celle-ci plus longue que parce que je n'ai pas eu le loisir de la faire plus courte.

Blaise Pascal

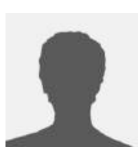
I would have precisely allocated all integer sequences to the Common Core Curricular Standards, but did not have the time.

Max Alekseyev is an Associate Professor at the Department of Mathematics and the Computational Biology Institute at the George Washington University in Washington, DC, USA. He holds M.S. degree in mathematics (1999) the Lobachevsky State University of Nizhni Novgorod, Russia, and Ph.D. degree (2007) in computer science from the University of California at San Diego, USA. Max's research interests range from combinatorics and number theory to computational molecular biology. The foundation for his scientific career and admiration of mathematics and computation was, in fact, laid during his school years when Max used to solve challenging mathematical problems and participate in various mathematical and computer programming contents. He now finds joy in maintaining the Online Encyclopedia of Integer Sequences (OEIS) as an Editor-in-Chief, extending some computationally hard sequences in the OEIS and proving interesting theoretical results about others. Max gladly shares with students his research expertise and passion for challenging mathematical and/or computational problems, which are often inspired by the OEIS.

Rosa Anajao is a graduate student in Mathematical Physics in University of Alberta. She is working in mathematical aspects of String Theory. Her research is about the interface between Topological String theory and Algebraic Geometry. She is also a teaching assistant in 100- and 200- level mathematics courses in the university. In her spare time, she enjoys solving puzzles and exploring recreational mathematics.

Mike Cavers, an instructor at the University of Calgary, has a passion for teaching and research. He particularly enjoys combinatorics, graph theory, and related applications. He enjoys helping out during the weekly Math Nites at the UofC, a weekly problem solving session for talented elementary and high school students. In his spare time, he publishes a math based **webcomic**.

Vincent Chan has always taken a keen interest in mathematics education and enrichment, ever since creating a Math Club in high school. He was involved with organizing two annual math competitions in the University of Waterloo: the "Integration Bee" (like a spelling bee, but with integrals) and "It's Over 9000!" (or is it? This is what participants must decide of the expressions provided). There, he also wrote a column on interesting math, aptly named "Interesting Math," for the newsletter "Math News." During his PhD at the University of British Columbia, Vincent began seriously focusing on mathematics education, facilitating training for new TAs, running a teaching mini-course, and directing the Math Learning Centre. Now in Calgary, he's excited to continue his efforts



Olive Chapman is currently Professor of mathematics education at the University of Calgary. Prior to this, she taught Physics and Mathematics in Colleges inToronto. Currently Editor-in-Chief of the International Journal of Mathematics Teacher Education and member of the International Committee of the International Group of the Psychology of Mathematics Education. Research interests include mathematics knowledge for teaching, mathematical problem solving, and inquiry-based mathematics pedagogy to facilitate mathematical thinking.

Gina Cherkowski is the education entrepreneur behind STEM Alberta.

Tom Edgar grew up in Colorado Springs, Colorado before OEIS was born but after the Handbook. I moved to Carlisle, PA for undergraduate studies at Dickinson College around the time of A047098. After four years, I decided to study mathematics in depth and moved back to Colorado (Fort Collins this time) around the time of A072361, where I studied finite projective geometries and linear codes. After two years, I switched programs and moved to South Bend, Indiana around the time of A096385 to study Coxeter groups under Matthew Dyer. After marrying my wife and finishing my program, I took a job as a professor at Pacific Lutheran University in Tacoma, Washington at the time of A164949 and have been here since. I am currently an associate professor, I volunteer as an associate editor for OEIS, and my first child will be here in a month. I enjoy thinking about triangular arrays in OEIS and my best submitted sequence is A235384.

Richard Guy has endeavored to teach mathematics at all levels from kindergarten to PhD students in Britain, Singapore, India and Canada.

He believes that math is fun

And available to everyone.

Mathematics keeps him alive (he's just celebrated his 97th birthday). When he discovered that he couldn't solve problems he specialized in Unsolved Problems: give him a problem and he can unsolve it!

Gordon Hamilton is a board game designer and the director of MathPickle. He experiments on K-12 students with unsolved problems, perplexing puzzles, great games and obscure integer sequences..In an argument between truth and beauty - Gordon is on the side of beauty. Too much of K-12 mathematics is trite & true.

Gael James is a graduate from the University of Calgary B. Ed. program and is teaching Grade 6 at River Valley School in Calgary. She was passionate about math puzzles as a child and is now equally passionate about sharing this enthusiasm with her students.

Veselin Jungic is a Senior Lecturer in the Department of Mathematics and a Deputy Director of the IRMACS Centre, Simon Fraser University. Most of his work is within Ramsey theory and the field of mathematics education. Veselin is a recipient of the Canadian Mathematical Society Excellence in Teaching Award.

Oluwaseun Okemakinde (MSc. Mathematics, University of Ibadan, Nigeria) taught mathematics in Yaba College of Technology, Nigeria and later moved to Canada where she is a Mathematics instructor in Calgary with a passion to help students in grade 10 - 12 understand Mathematics in an interesting way. Research interest is in Mathematical Physics.

Henri Picciotto is a math consultant and author. He received his BA and MA in Mathematics from the University of California, Berkeley. He has taught math for over 40 years, at every level from counting to calculus, including a 32-year stint at the Urban School of San Francisco, where he chaired the math department and directed the Center for Innovative Teaching. He has written and co-authored many books, including Algebra: Themes, Tools, Concepts and Geometry Labs. He has written articles, and edited activities for The Mathematics Teacher. He is the inventor of the Lab Gear, a hands-on environment for algebra, and is in fact a leading authority on the use of manipulatives and geometric puzzles in secondary school. He has been an enthusiastic (and skeptical) user of electronic learning environments since the early days of the personal computer. He has presented hundreds of workshops to teachers, at conferences, in summer institutes, and in school professional development programs. Henri shares his ideas about teaching, and much curriculum, through his Math Education Page <www.MathEducationPage.org>. His cryptic crosswords appear in The Nation every week.

Paulino Preciado, a PhD in Mathematics Education, has focused in the last six years on mathematics teacher, K to 12, professional development. His work is based on the believe that young kids are capable of approaching though problems and engage intellectually in mathematical inquiry, as long as they are immersed in a proper learning environment. Before embracing graduate studies in mathematics education, Paulino Preciado earned a PhD candidacy in pure mathematics and was a trainer and promoter of the Mathematics Olympiad at both National (Mexico) and international levels.

Lora Saarnio is the Math and Tech Integration Specialist at The Nueva School in Hillsborough, California. Her teaching interests: K-4th grade math and computer science; puzzle and strategic game design; mathematical modeling; digital art and storytelling. She also designs Math Night, Science Night, Engineering Night, and STEM Night for schools in the San Francisco Bay Area. She is also an avid strategic games player/collector and former competitive chess player (previously ranked #36 for US women players).

In his tenth year teaching high school math, Zaak Robichaud is excited to meet and discuss math education with other math enthusiasts. The joy of mathematics began at a young age and it stuck with him through university where he took math courses to complement his history and education degrees. His math classes are typically full of laughter. He is currently teaching math at Bearspaw Christian School in Calgary, AB.

Amanda Serenevy is the director of Riverbend Community Math Center, a non-profit organization located in South Bend, Indiana. She has been active with the Math Circle movement to connect mathematicians with young students interested in mathematics. In the South Bend area, Amanda leads many professional development sessions for K-12 teachers, as well as Math Circles and a variety of other math outreach programs.

Rakhee Vijairaghavan (B.Math, Waterloo) is one of the math specialists for the Calgary board of education. She has experience teaching grades 5-9 math and enjoys making her students struggle with fun math every day. Rakhee is always looking for ways to challenge her students and herself to gain a deeper understanding and appreciate the beauty of the mathematics we teach in schools today.

Robert Woodrow obtained his PhD in mathematical logic at Simon Fraser University and since then has continued to work in that area and related areas of cominbatorics. He has been involved in mathematics enrichment activities at the University of Calgary, organizing the Calgary Junior Mathematics Contest and other activities. He produced the Olympiad Corner for Crux Mathematicorum for a number of years.

Joshua Zucker is a freelance math teacher. After a decade of high school teaching, he co-founded the Math Teachers' Circle program and was the founding director of the Julia Robinson Mathematics Festivals. Now, he maintains his involvement with those programs, as well as teaching for Art of Problem Solving and at several different math circles around the San Francisco area. He has also been a member of the US Sudoku team at the World Sudoku Championship.

Kindergarten

A000012
The simplest sequence of positive numbers: the all 1's sequence.

1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1

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A002024
n appears n times; floor(sqrt(2n) + 1/2)
1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 6, 7, 7, 7, 7, 7, 7, 7, 8, 8, 8, 8, 8, 8, 8, 8, 9, 9, 9, 9, 9, 9, 9, 9, 9, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 13, 13, 13, 13, 13, 13

Grade 1

A034326
Hours struck by a clock.

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 1, 2,3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 1, 2, 3, 4,5, 6, 7, 8, 9, 10, 11, 12, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 1, 2, 3, 4, 5, 6, 7, 8, 9

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A002260
Triangle T(n,k) = k for k = 1..n.

1, 1, 2, 1, 2, 3, 1, 2, 3, 4, 1, 2, 3, 4, 5, 1, 2, 3, 4, 5, 6, 1, 2, 3, 4, 5, 6, 7, 1, 2, 3, 4, 5, 6, 7, 8, 1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 1, 2, 3

****

A031940 Length of longest legal domino snake using full set of dominoes up to [n:n].

1, 3, 6, 9, 15, 19, 28, 33, 45, 51, 66, 73, 91, 99, 120, 129, 153, 163, 190, 201, 231, 243, 276, 289, 325, 339, 378, 393, 435, 451, 496, 513, 561, 579, 630, 649, 703, 723, 780, 801
EXAMPLE
E.g. for n=4 [1:1][1:2][2:2][2:3][3:3][3:1][1:4][4:4][4:2].

The formula for this is known. What about domino loops as an extension exercise?

Grade 2

A072842
Largest m such that we can partition the set {1,2,...,m} into n subsets with the property that we never have a +b=c for any distinct elements a, b, c in one subset.

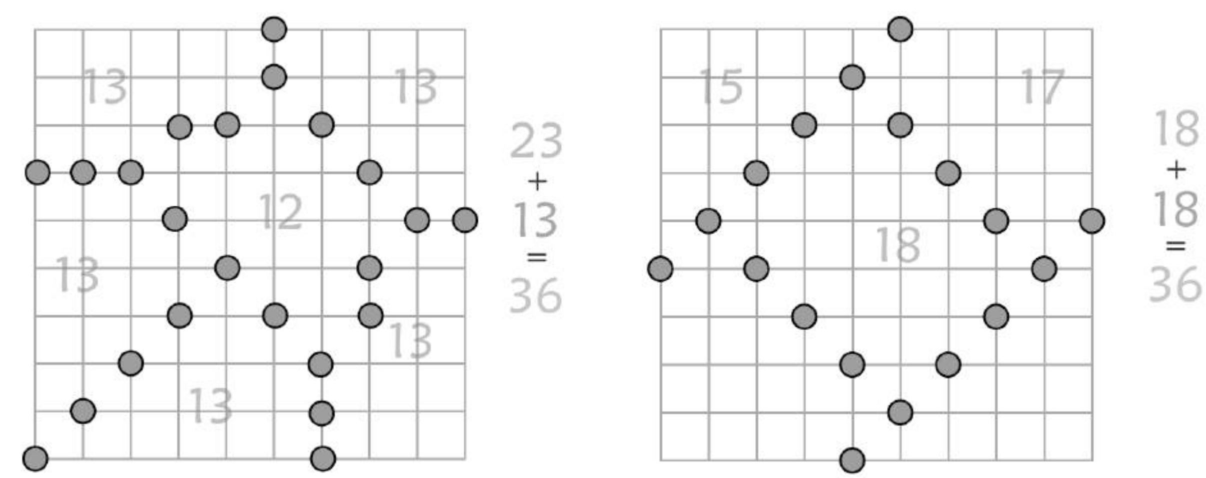
2, 8, 23, 66, 196

****

A243205
Consider the n X n Go board as a graph; remove i nodes and let j be the number of nodes in the largest connected subgraph remaining; then a(n) = minimum (i + j).

1, 3, 5, 9, 12, 16, 20, 25, 29

EXAMPLE
a(10) <= 36 because i + j = 18 + 18 = 36 in the left graph and i + j = 23 + 13 = 36 in the right graph. 36 may be the minimum.



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A002858

Ulam numbers: a(1) = 1; a(2) = 2; for n>2, a(n) = least number > a(n-1) which is a unique sum of exactly two distinct earlier terms.
1, 2, 3, 4, 6, 8, 11, 13, 16, 18, 26, 28, 36, 38, 47, 48, 53, 57, 62, 69, 72, 77, 82, 87, 97, 99, 102, 106, 114, 126, 131, 138, 145, 148, 155, 175, 177, 180, 182, 189, 197, 206, 209, 219, 221, 236, 238, 241, 243, 253, 258, 260, 273, 282, 309, 316, 319, 324, 339

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A249783

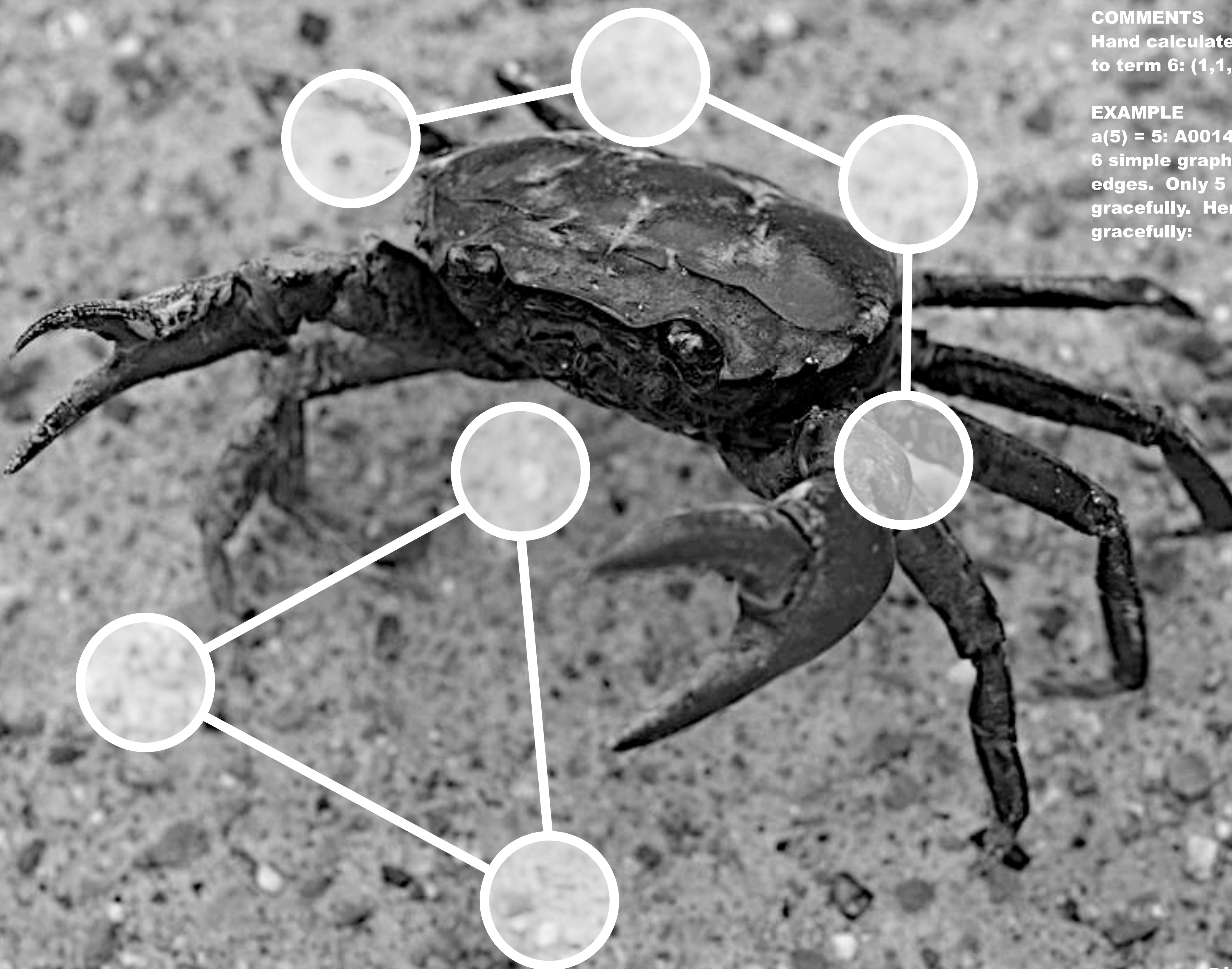
Smallest index of a Fibonacci-like sequence containing n, see comments.

0, 1, 1, 1, 2, 1, 2, 3, 1, 3, 2, 3, 4, 1, 4, 3, 2, 5, 3, 5, 4, 1, 6, 4, 3, 5, 2, 7, 5, 3, 6, 5, 4, 6, 1, 7, 6, 4, 7, 3, 5, 7, 2, 8, 7, 5, 8, 3, 6, 8, 5, 9, 4, 6, 9, 1, 7, 9, 6, 10, 4, 7, 10, 3, 8, 5, 7, 11, 2, 8, 11, 7, 9, 5, 8, 12, 3, 9, 6, 8, 10, 5, 9, 13, 4, 10, 6, 9, 11, 1, 10, 7, 9, 11, 6, 10, 12, 4, 11, 7, 10

COMMENTS
Any two nonnegative integers F0 and F1 generate a Fibonacci-like sequence where for n > 1, Fn = F[n-1] + F[n-2]. Call F0 + F1 the "index" of such a sequence. In this sequence, a(n) is the smallest occurring index of any Fibonacci-like sequence containing n.

Possible or Impossible?

Can you put odd consecutive integers into the circles so that the differences of connected circles are all unique?



A243013

Number of graphs with n vertices and n-1 edges that can be gracefully labeled.

1, 1, 1, 3, 5, 12, 36

COMMENTS

Hand calculated by grade 3 students up to term 6: (1,1,1,3,5,12...)

EXAMPLE

a(5) = 5: A001433 tells us that there are 6 simple graphs with 5 vertices and 4 edges. Only 5 of these can be labeled gracefully. Here is one of them labelled gracefully:

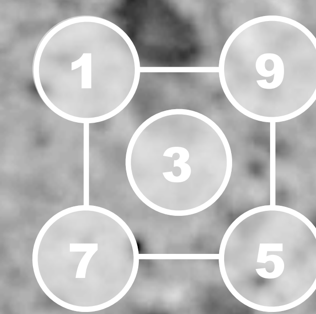


Photo by Peter Rowley


Photo by Alexandre Dulaunoy

Grade 3

Square Pairs numbers. The numbers from 1 to 18 can be organized in nine pairs, so that each pair sums to a perfect square. More generally, what are the positive integers 2n such that the numbers from 1 to 2n can be organized in n pairs whose sum is a perfect square? These are the Square Pairs numbers: 4, 7, 8, 9, and all numbers greater than 11.

n=4   1-8 2-7 3-6 4-5
n=7   1-8 2-14 3-13 4-12 5-11 6-10 7-9

****

A063865

Number of solutions to +- 1 +- 2 +- 3 +- ... +- n = 0.
1, 0, 0, 2, 2, 0, 0, 8, 14, 0, 0, 70, 124, 0, 0, 722, 1314, 0, 0, 8220, 15272, 0, 0, 99820, 187692, 0, 0, 1265204, 2399784, 0, 0, 16547220, 31592878, 0, 0, 221653776, 425363952, 0, 0, 3025553180, 5830034720, 0, 0, 41931984034, 81072032060

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A255253

Complete siteswap listing (indecomposable ground-state in concatenated decimal notation organized 1st by sum of digits and then magnitude)

0, 1, 2, 3, 4, 31, 40, 5, 6, 42, 51, 60, 312, 330, 411, 420, 501, 600, 7, 8, 53, 62, 71, 3122, 3302, 4013, 4112, 4130, 4202, 4400, 5111, 5120, 5201, 5300, 6011, 6020, 7001, 8000, 9, 423, 441, 450, 522, 531, 603, 612, 630

A065179

Number of site swap patterns with 3 balls and exact period n.

1, 3, 12, 42, 156, 554, 2028, 7350, 26936, 98874, 365196, 1353520, 5039580, 18831306, 70626140, 265741350, 1002984060, 3796211692, 14406086604, 54801192684, 208932673508, 798218225802, 3055417434732, 11716355452900

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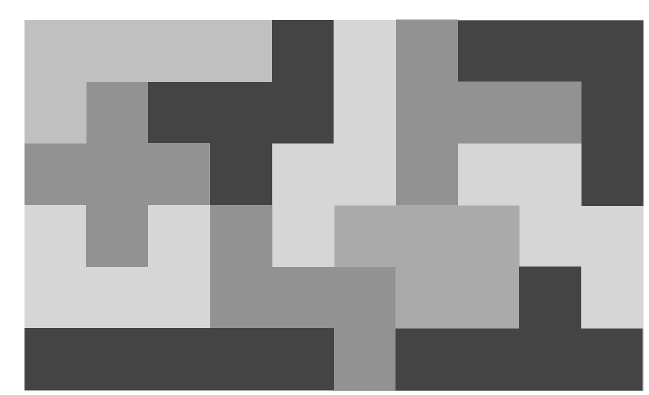
A000105 Number of free polyominoes (or square animals) with n cells.

1, 1, 1, 2, 5, 12, 35, 108, 369, 1285, 4655, 17073, 63600, 238591, 901971, 3426576, 13079255, 50107909, 192622052, 742624232, 2870671950, 11123060678



Grade 4

A036468

Number of ways to represent 2n+1 as a+b with 0 < a < b and a^2 + b^2 prime.

1, 2, 2, 2, 3, 3, 4, 4, 4, 3, 4, 8, 4, 6, 5, 4, 9, 8, 6, 9, 7, 7, 7, 5, 7, 9, 14, 8, 9, 11, 7, 17, 11, 10, 9, 11, 9, 8, 13, 9, 15, 20, 11, 14, 13, 8, 18, 14, 10, 18, 16, 10, 17, 16, 13, 20, 20, 13, 14, 17, 12, 23, 18, 14, 22, 15, 17, 18, 21, 12, 19, 29, 16, 23, 21, 14...

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A152242

Henri Picciotto's Slime numbers. Integers formed by concatenating primes.

2, 3, 5, 7, 11, 13, 17, 19, 22, 23, 25, 27, 29, 31, 32, 33, 35, 37, 41, 43, 47, 52, 53, 55, 57, 59, 61, 67, 71, 72, 73, 75, 77, 79, 83, 89, 97, 101, 103, 107, 109, 112, 113, 115, 117, 127, 131, 132, 133, 135, 137, 139, 149, 151, 157, 163, 167, 172, 173, 175, 177

A166504

Numbers which are the concatenation of primes, with "leading zeros" allowed.

2, 3, 5, 7, 11, 13, 17, 19, 22, 23, 25, 27, 29, 31, 32, 33, 35, 37, 41, 43, 47, 52, 53, 55, 57, 59, 61, 67, 71, 72, 73, 75, 77, 79, 83, 89, 97, 101, 103, 107, 109, 112, 113, 115, 117, 127, 131, 132, 133, 135, 137, 139, 149, 151, 157, 163, 167, 172, 173, 175, 177, 179

A number is slime if it is prime, or if (in base 10) it can be sliced into a sequence of primes. For example, 519 is slime (5|19), as is 777 (7|7|7)..

Exploring this is a good way for kids to become familiar with small primes. I have used this worksheet many times, with great success.

The culmination of Henri Picciotto's Slime numbers activity is the search for the super-slimes: numbers that are slime no matter how you slice them. 777 does not qualify because 77 and 777 are not prime. 53 is a super-slime since both 53 and 5|3 are sequences of primes. It turns out that there are only nine super-slimes, and finding them is a good exercise in checking for divisibility.

Also, what numbers, when sliced exactly once always produce two primes?

A255271

22, 23, 25, 27, 32, 33, 35, 37, 52, 53, 55, 57, 72, 73, 75, 77, 237, 297, 313, 317, 373, 537, 597, 713, 717, 737, 797, 2337, 2397, 2937, 3113, 3137, 3173, 3797, 5937, 5997, 7197, 7337, 7397, 29397, 31373, 37937, 59397, 73313, 739397

What is the maximum number of ways that a number can be sliced to produce two primes?

****

A214897 Conway's subprime Fibonacci sequence : cycle lengths.
1, 10, 11, 18, 19, 56, 136

Similar to the Fibonacci recursion starting with a pair of positive integers, but each new non-prime term is divided by its least prime factor. The recursion enters a loop of length a(n)after a finite number of steps. Conjecture: the list of loops is complete, loops of length a(n) are unique and no infinite chains exists.

[136] A214892

4, 1, 5, 3, 4, 7, 11, 9, 10, 19, 29, 24, 53, 11, 32, 43, 25, 34, 59, 31, 45, 38, 83, 11, 47, 29, 38, 67, 35, 51, 43, 47, 45, 46, 13, 59, 36, 19, 11, 15, 13, 14, 9, 23, 16, 13, 29, 21, 25, 23, 24, 47, 71, 59, 65, 62, 127, 63, 95, 79, 87, 83, 85, 84, 13, 97, 55, 76, 131, 69, 100, 13, 113

COMMENTS
Similar to the Fibonacci recursion starting with (4, 1), but each new non-prime term is divided by its least prime factor. Sequence enters a loop of length 136 after 8 terms when hitting (11, 9).

[56] A214893
18, 5, 23, 14, 37, 17, 27, 22, 7, 29, 18, 47, 13, 30, 43, 73, 58, 131, 63, 97, 80, 59, 139, 99, 119, 109, 114, 223, 337, 280, 617, 299, 458, 757, 405, 581, 493, 537, 515, 526, 347, 291, 319, 305, 312, 617, 929
[19] A214894
10, 18, 14, 16, 15, 31, 23, 27, 25, 26, 17, 43, 30, 73, 103, 88, 191, 93, 142, 47, 63, 55, 59, 57, 58, 23, 27
[18] A214674
1, 1, 2, 3, 5, 4, 3, 7, 5, 6, 11, 17, 14, 31, 15, 23, 19, 21, 20, 41, 61, 51, 56, 107, 163, 135, 149, 142, 97, 239, 168, 37, 41, 39, 40, 79, 17, 48, 13, 61, 37, 49, 43, 46, 89, 45, 67, 56, 41, 97, 69, 83, 76, 53, 43, 48, 13
[11] A214895
23, 162, 37, 199, 118, 317, 145, 231, 188, 419, 607, 513, 560, 37, 199
[10] A214896
382, 127, 509, 318, 827, 229, 528, 757, 257, 507, 382, 127

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A020342

Vampire numbers (definition 1): n has a nontrivial factorization using n's digits.

126, 153, 688, 1206, 1255, 1260, 1395, 1435, 1503, 1530, 1827, 2187, 3159, 3784, 6880, 10251, 10255, 10426, 10521, 10525, 10575, 11259, 11439, 11844, 11848, 12006, 12060, 12384, 12505, 12546, 12550, 12595, 12600, 12762, 12768, 12798, 12843, 12955, 12964

EXAMPLE
E.g. 1395 = 31*9*5.

CROSSREFS
The following sequences are all closely related: A020342, A014575, A080718,A048936, A144563.

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A156209

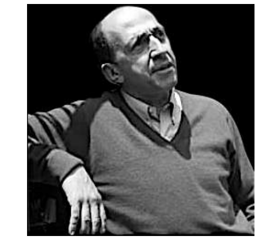
Number of possible values of C(v) = the number of valid mountain-valley assignments for a flat-foldable origami vertex v of degree 2n.

1, 3, 7, 13, 24, 39, 62, 97, 147, 215, 312, 440, 617, 851, 1161, 1569, 2098, 2778, 3649, 4764, 6163, 7939, 10160, 12924, 16361, 20613, 25833, 32259, 40097, 49667, 61272, 75337, 92306, 112755, 137272, 166654, 201734, 243582, 293288

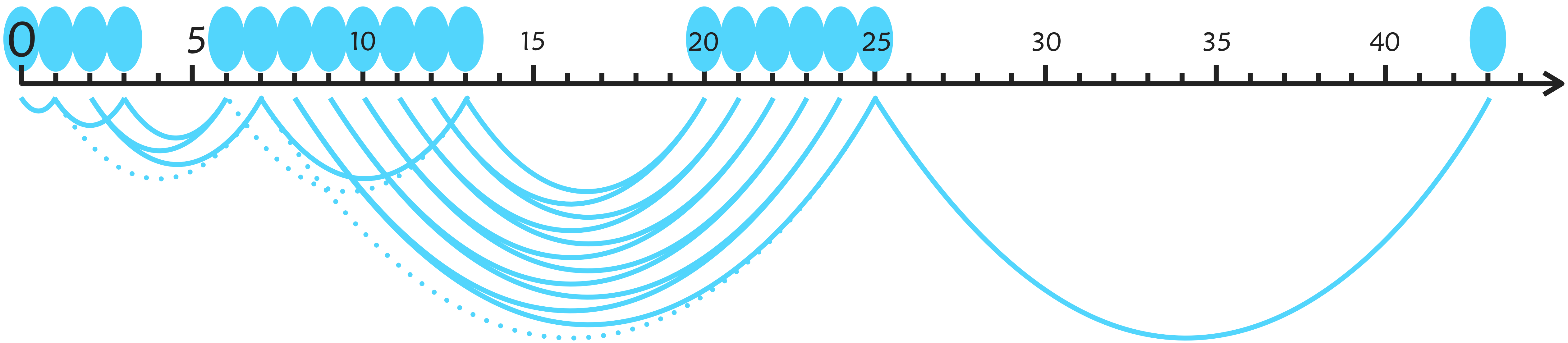
EXAMPLE
The n=1 case is degenerate; C(v) = 2 in this case, so a(1) =1. When n=2 we have a degree 4 vertex, and C(v) can take on the values 4, 6, or 8 depending on the angles between the creases, so a(2) =3. In the n=3 (degree 6) case, C(v) can be any of {8, 12, 16, 18, 20, 24, 30}. Thus a (3)=7. The possible values of C(v) can be determined by recursive equations found in (Hull, 2002, 2003).

Recamán Sequence

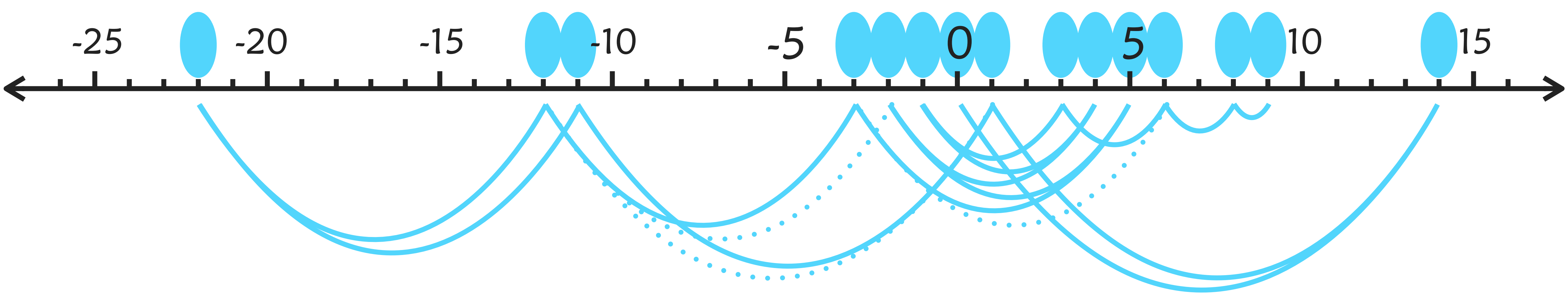
Bernardo Recamán, 1991



Start at zero. On step 1, move 1, on step 2 move 2, on step 3 move 3… The direction of the step must be to the left on the number line if you visit a non-negative integer that you have not visited before. Otherwise you must step to the right. Do you visit every positive integer? (unsolved)
Do you visit any integer twice? (solvable by grade 5s)



Wrecker ball sequences are a variant of the Recamán sequence that gives students practice moving on the number line. Start with any integer. On step 1, move 1, on step 2 move 2, on step 3 move 3… The direction of the step must be towards zero if you visit an integer that you have not visited before. Otherwise you must step away from zero. The sequence ends when you hit zero. All sequences that start with an absolute value of less than 12 are solvable in 26 or less steps with the exception of 7 which takes 1725 steps. Do all sequences end? Play videos of wrecker balls destroying buildings in the class when a number is hit twice.



This variant has not been rigorously investigated.

Grade 5

A228474

This is a Recamán-like sequence (cf. A005132). Starting at n, a(n) is the number of steps required to reach zero. On the k-th step move a distance of k in the direction of zero. If the number landed on has been landed on before, move a distance of k away from zero instead.

0, 1, 4, 2, 24, 26, 3, 1725, 12, 14, 4, 26, 123, 125, 15, 5, 119, 781802, 20, 22, 132896, 6, 51, 29, 31, 1220793, 23, 25, 7, 429, 8869123, 532009, 532007, 532009, 532011, 26, 8, 94, 213355, 213353, 248, 33, 31, 33, 1000, 9, 144, 110, 112, 82, 84, 210, 60,

****

Nearest integer to 4n/3 unless that is an integer, when 2n/3. A006369
0, 1, 3, 2, 5, 7, 4, 9, 11, 6, 13, 15, 8, 17, 19, 10, 21, 23, 12, 25, 27, 14, 29, 31, 16, 33, 35, 18, 37, 39, 20, 41, 43, 22, 45, 47, 24, 49, 51, 26, 53, 55, 28, 57, 59, 30, 61, 63, 32, 65, 67, 34, 69, 71, 36, 73, 75, 38, 77, 79, 40, 81, 83, 42, 85, 87, 44, 89, 91, 46, 93, 95
Another function by Lothar Collatz (1932).

****

The "amusical permutation" of the nonnegative numbers
A006368: a(2n)=3n, a(4n+1)=3n+1, a(4n-1)=3n-1.
0, 1, 3, 2, 6, 4, 9, 5, 12, 7, 15, 8, 18, 10, 21, 11, 24, 13, 27, 14, 30, 16, 33, 17, 36, 19, 39, 20, 42, 22, 45, 23, 48, 25, 51, 26, 54, 28, 57, 29, 60, 31, 63, 32, 66, 34, 69, 35, 72, 37, 75, 38, 78, 40, 81, 41, 84, 43, 87, 44, 90, 46, 93, 47, 96, 49, 99, 50, 102, 52, 105, 53
0,3

Numbers n such that n and n+1 have same sum of divisors. A002961
14, 206, 957, 1334, 1364, 1634, 2685, 2974, 4364, 14841, 18873, 19358, 20145, 24957, 33998, 36566, 42818, 56564, 64665, 74918, 79826, 79833, 84134, 92685, 109214, 111506, 116937, 122073, 138237, 147454, 161001, 162602, 166934

****

A005349

Harshad numbers are numbers that are divisible by their sum: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 18, 20, 21, 24, 27, 30, 36, 40, 42, 45, 48, 50, 54, 60, 63, 70, 72, 80, 81, 84, 90, 100, 102, 108, 110, 111, 112,114, 117, 120, 126, 132, 133, 135, 140, 144, 150, 152, 153, 156, 162, 171, 180, 190, 192, 195, 198, 200, ...

****

A255198

EKG Ancestral Links: Take an EKG sequence starting with n (EKG-n). Observe where this sequence first starts to coincide with other EKG sequences.a(n) = the number of these EKG sequences.

DATA
1, 1, 1, 4, 1, 6, 2, 2, 5

EXAMPLE
a(5) = 4 because the EKG sequence starting with 5 (EKG-5) starts duplicating sequences EKG-3, EKG-6, EKG-9 and EKG-12 simultaneously (when all sequences hit 18).
EKG-3: 3, 6, 2, 4, 8, 10, 5, 15, 9, 12, 14, 7, 21, 18, 16, 20, 22, 11...
EKG-6: 6, 2, 4, 8, 10, 5, 15, 3, 9, 12, 14, 7, 21, 18, 16, 20, 22, 11...
EKG-9: 9, 3, 6, 2, 4, 8, 10, 5, 15, 12, 14, 7, 21, 18, 16, 20, 22, 11...
EKG-12: 12, 2, 4, 6, 3, 9, 15, 5, 10, 8, 14, 7, 21, 18, 16, 20, 22, 11...
EKG-5: 5, 10, 2, 4, 6, 3, 9, 12, 8, 14, 7, 21, 15, 18, 16, 20, 22, 11...
a(12) = 3 because the EKG sequence starting with 12 (EKG-12) starts duplicating sequences EKG-3, EKG-6, and EKG-9 simultaneously (when all sequences hit 14).

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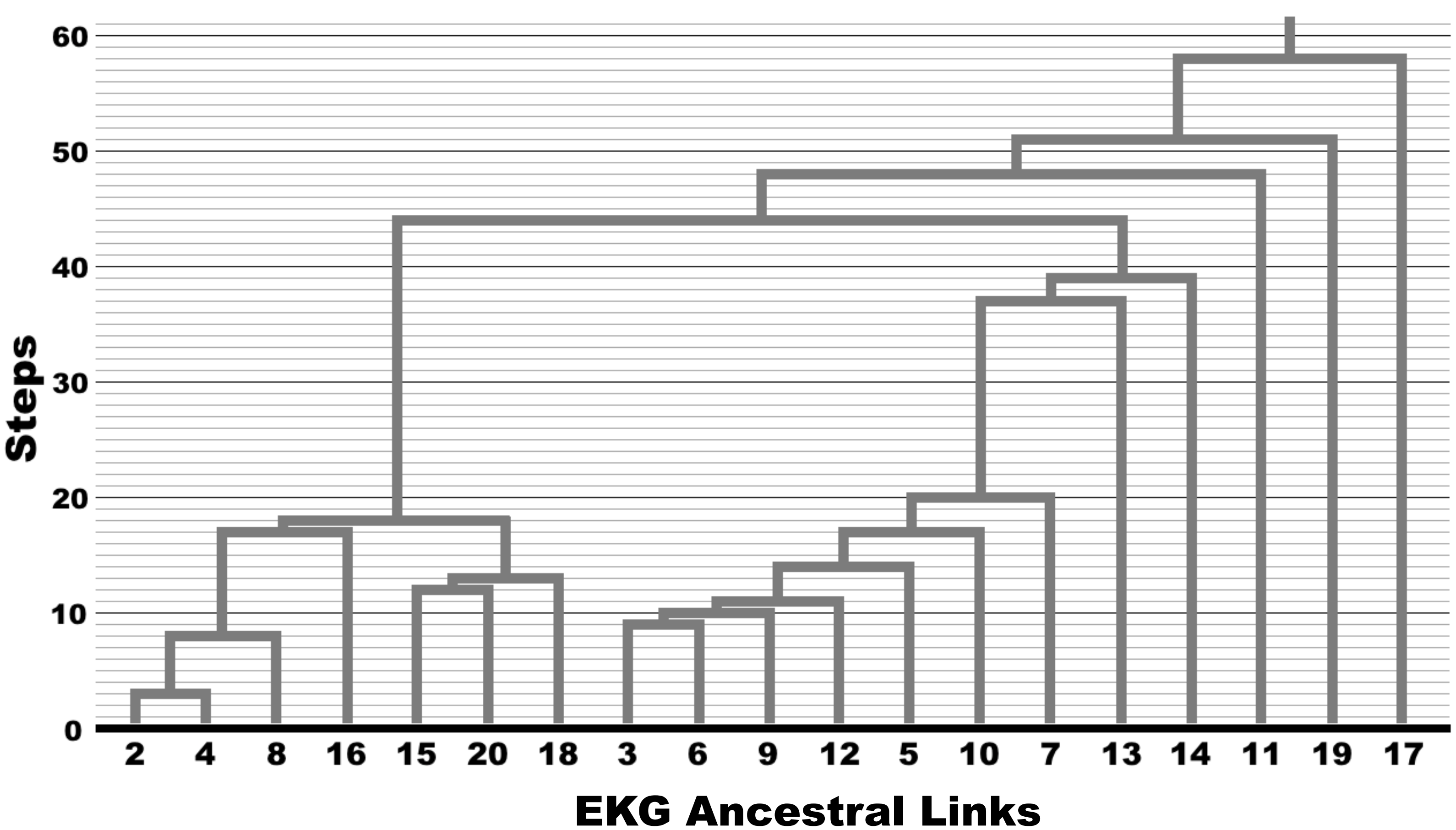

Choose any real r >= 0.
Starting with n = 1, on the first step add r, on subsequent steps either add r or take the reciprocal as you choose.
For example, if r = 1/4, we can generate the sequence
1, 5/4, 3/2, 7/4, 2, 1/2, 3/4, 1.
For which r is it possible to return to 1 as does this sequence?

David Wilson

For example, 3/11 gives us:

1 8/11 5/11 2/11 11/2 115/22 109/22 103/22 97/22 91/22 85/22 79/22 73/22 67/22 61/22 5/2 49/22 43/22 37/22 31/22 25/22 19/22 13/22 7/22 1/22 22 239/11 236/11 233/11 230/11 227/11 224/11 221/11 218/11 215/11 212/11 19 206/11 203/11 200/11 197/11 194/11 191/11 188/11 185/11 182/11 179/11 16 173/11 170/11 167/11 164/11 161/11 158/11 155/11 152/11 149/11 146/11 13 140/11 137/11 134/11 131/11 128/11 125/11 122/11 119/11 116/11 113/11 10 107/11 104/11 101/11 98/11 95/11 92/11 89/11 86/11 83/11 80/11 7 74/11 71/11 68/11 65/11 62/11 59/11 56/11 53/11 50/11 47/11 4 41/11 38/11 35/11 32/11 29/11 26/11 23/11 20/11 17/11 14/11 1

Reverse the order to get the solution.

For example, r = sqrt(2)/2 seems to work:
+ / + + + + / +
should yield 1 (where / is reciprocal).

CCSS.MATH.CONTENT.5.NF.A.1
Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)

Grade 6

A006255

I have had great success getting some deep thinking about prime factorization by working with Graham's sequence (which is also a bijection between positive integers and composites!)
(Joshua Zucker)
1, 6, 8, 4, 10, 12, 14, 15, 9, 18, 22, 20, 26, 21, 24, 16, 34, 27, 38, 30, 28, 33, 46, 32, 25, 39, 35, 40, 58, 42, 62, 45, 44, 51, 48, 36, 74, 57, 52, 50, 82, 56, 86, 55, 60, 69, 94, 54, 49, 63, 68, 65, 106, 70, 66, 72, 76, 87, 118, 75, 122, 93, 77, 64, 78, 80, 134, 85, 92, 84
a(2) = 6 because the best such sequence is 2,3,6.
a(59) = 118 because one best such sequence is 59, 72, 118. In this case the answer is not unique. 59, 98, 118 works. 59, 60, 72, 75, 80, 88, 98, 99, 100, 118 also works.

For n = 3 through 6 the {smallest m then smallest t then smallest product} solutions are 3,6,8; 4; 5,8,10; 6,8,12.

****

A132199

Rowland's prime-generating sequence: first differences of A106108 (see below).
1, 1, 1, 5, 3, 1, 1, 1, 1, 11, 3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 23, 3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 47, 3, 1, 5, 3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 101, 3, 1, 1
COMMENTS
Rowland shows that the terms are all 1's or primes.

A106108

Rowland's prime-generating sequence: a(1) = 7; for n >1, a(n) = a(n-1) + gcd(n, a(n-1)).
7, 8, 9, 10, 15, 18, 19, 20, 21, 22, 33, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 69, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 141, 144, 145, 150, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168

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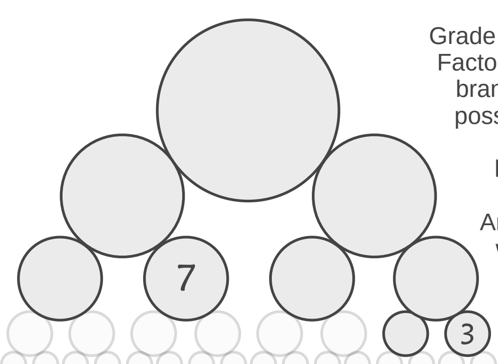
A214777

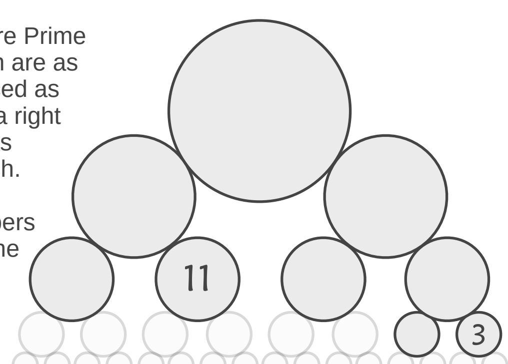
McNuggets numbers. You can buy 6, 9, or 20 McNuggets, and therefore 12 (=6+6), 15 (=6+9), and so on.
I strongly recommend using this sequence, probably as a middle school or upper elementary school topic. It is very accessible, requiring only addition. It is quite motivational, and can be introduced with a funny story. It generalizes nicely to pairs of numbers, and in fact the generalization leads naturally to the concept of greatest common factor (grade 6 in the Common Core): Given two numbers, you can get all the multiples of their GCF after a certain point. So it is extremely curricular.

CCSS.MATH.CONTENT.6.NS.B.4
Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2)…



Grade 7 Integer Fission are Prime Factorization Trees which are as branches are as balanced as possible and for which a right branch is at least as large as a left branch.

Are there more numbers with the shape on the left or right?

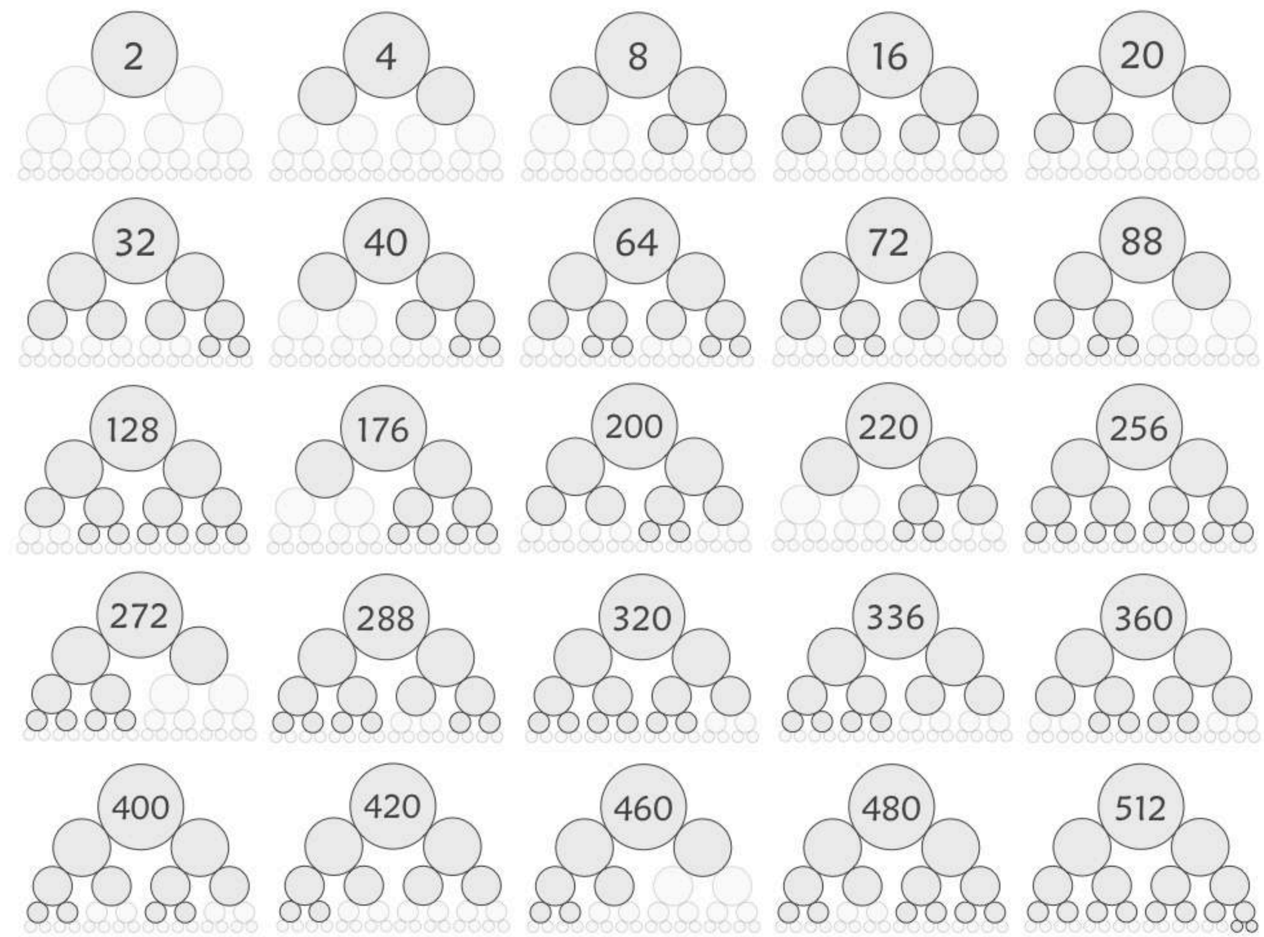


Grade 7

A125508

Integral Fission - Lowest number having a particular shape of factor decomposition binary tree.

1, 2, 4, 8, 16, 20, 32, 40, 64, 72, 88, 128, 160, 176, 200, 220, 256, 272, 288, 320, 336, 360, 400, 420, 460, 480, 512, 540, 544, 640, 704, 864, 880, 920, 1024, 1056, 1152, 1184, 1200, 1280, 1344, 1440, 1600, 1640, 1680, 1800, 1840, 1920, 2048, 2176, 2368



A002487

Stern's diatomic series.
a(0) = 0, a(1) = 1; for n > 0: a(2*n) = a(n), a(2*n+1) = a(n) + a(n+1).

0, 1, 1, 2, 1, 3, 2, 3, 1, 4, 3, 5, 2, 5, 3, 4, 1, 5, 4, 7, 3, 8, 5, 7, 2, 7, 5, 8, 3, 7, 4, 5, 1, 6, 5, 9, 4, 11, 7, 10, 3, 11, 8, 13, 5, 12, 7, 9, 2, 9, 7, 12, 5, 13, 8, 11, 3, 10, 7, 11, 4, 9, 5, 6, 1, 7, 6, 11, 5, 14, 9, 13, 4, 15, 11, 18, 7, 17, 10, 13, 3, 14, 11, 19, 8, 21, 13, 18, 5, 17, 12, 19

COMMENTS
Also called fusc(n) [Dijkstra].
a(n)/a(n+1) runs through all the reduced nonnegative rationals exactly once

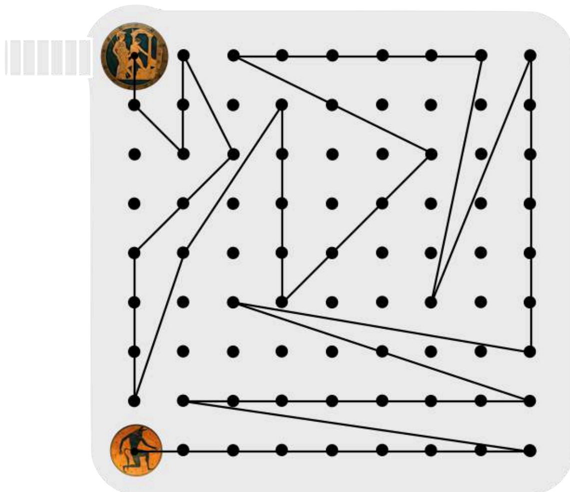
Grade 8

A226595

Lengths of maximal non touching increasing paths in n X n grids.

0, 2, 4, 7, 9, 12, 15, 17, 20...

EXAMPLE
An example for a(9)=20



Charles R Greathouse IV and Giovanni Resta, Jun 13 2013

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A234300

Number of unit squares, aligned with a Cartesian grid, partially encircled along the edge of the first quadrant of a circle centered at the origin ordered by increasing radius. Start with radius zero. Increase the radius gradually. Each time the circle expands to include a new intersection, write down two numbers: 1) the number of squares the circle intersects and 2) the number of squares the circle intersects an arbitrarily short time later.

0, 1, 1, 3, 2, 3, 3, 5, 3, 5, 4, 5, 5, 7, 5, 7, 5, 7, 7, 9, 7, 9, 8, 9, 7, 9, 7, 11, 9, 11, 9, 11, 10, 11, 9, 11, 11, 13, 11, 13, 11, 13, 11, 13, 11, 13, 13, 15, 12, 15, 13, 15, 13, 15, 13, 15, 13, 15, 15, 17, 13, 17, 15, 17, 16, 17, 15, 17, 15, 17, 15, 17, 17, 19, 17, 19, 15, 19, 17, 19, 17, 19, 17, 19, 18, 19, 17, 21, 19, 21, 19, 21, 19, 21, 19, 21, 19, 21, 19, 21, 19, 21

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A006720

Somos-4 sequence: a(0)=a(1)=a(2)=a(3)=1; for n >= 4, a(n)=(a(n-1)a(n-3)+a(n-2)^2)/a(n-4).

1, 1, 1, 1, 2, 3, 7, 23, 59, 314, 1529, 8209, 83313, 620297, 7869898, 126742987, 1687054711, 47301104551, 1123424582771, 32606721084786, 1662315215971057, 61958046554226593, 4257998884448335457, 334806306946199122193

COMMENTS
From the 5th term on, all terms have a primitive divisor; in other words, a prime divisor that divides no earlier term in the sequence. A proof appears in the Everest-McLaren-Ward paper. - Graham Everest

Grade 9

A103840

Number of ways to represent n as a sum of b^e with b >= 2, e >= 2, e distinct.

1, 0, 0, 0, 1, 0, 0, 0, 1, 1, 0, 0, 1, 0, 0, 0, 2, 1, 0, 0, 1, 0, 0, 0, 2, 2, 0, 1, 1, 0, 0, 1, 2, 2, 0, 0, 3, 0, 0, 0, 2, 2, 0, 2, 2, 0, 0, 1, 2, 3, 0, 0, 4, 0, 0, 0, 2, 3, 0, 2, 2, 0, 0, 2, 4, 3, 0, 0, 5, 0, 0, 0, 3, 4, 0, 2, 3, 0, 0, 2, 5, 5, 0, 0, 5, 1, 0, 0, 3, 7, 1, 3, 3, 1, 0, 2, 5, 5, 1, 0, 7, 0, 0, 0, 3 (list; graph; refs; listen; history; edit; text; internal format)

EXAMPLE
68 = 2^2+4^3 = 2^2+2^6 = 3^2+3^3+2^5 = 5^2+3^3+2^4 = 6^2+2^5 so a(68) = 5. Note that although 4^3 = 2^6, the exponents are different and so 2^2+4^3 and 2^2+2^6 are counted as distinct.

****

A248808

Dale Gerdmann lists the powers of phi used in golden ratio base
to represent the Fibonacci numbers. So, for example, 5 = phi^3 + phi^-1 +
phi^-4 = f(3)+f(-1)+f(-4)

[0] 1
[0] 1
[1, -2] 2
[2, -2] 3
[3, -1, -4] 5
[4, 0, -4] 8
[5, 1, -3, -6] 13
[6, 2, -2, -6] 21
[7, 3, -1, -5, -8] 34
[8, 4, 0, -4, -8] 55
[9, 5, 1, -3, -7, -10] 89
[10, 6, 2, -2, -6, -10] 144
[11, 7, 3, -1, -5, -9, -12] 233
[12, 8, 4, 0, -4, -8, -12] 377

****

A014556

Euler's "Lucky" numbers: n such that m^2-m+n is prime for m=0..n-1.

2, 3, 5, 11, 17, 41

****

A007623

Integers written in factorial base.

0, 1, 10, 11, 20, 21, 100, 101, 110, 111, 120, 121, 200, 201, 210, 211, 220, 221, 300, 301, 310, 311, 320, 321, 1000, 1001, 1010, 1011, 1020, 1021, 1100, 1101, 1110, 1111, 1120, 1121, 1200, 1201, 1210, 1211, 1220, 1221, 1300, 1301, 1310, 1311, 1320, 1321, 2000, 2001, 2010

EXAMPLE
a(47) = 1321 because 47 = 1*4! + 3*3! + 2*2! + 1*1!

Grade 10

A069283

a(n) = -1 + number of odd divisors of n.
0, 0, 1, 0, 1, 1, 1, 0, 2, 1, 1, 1, 1, 1, 3, 0, 1, 2, 1, 1, 3, 1, 1, 1, 2, 1, 3, 1, 1, 3, 1, 0, 3, 1, 3, 2, 1, 1, 3, 1, 1, 3, 1, 1, 5, 1, 1, 1, 2, 2, 3, 1, 1, 3, 3, 1, 3, 1, 1, 3, 1, 1, 5, 0, 3, 3, 1, 1, 3, 3, 1, 2, 1, 1, 5, 1, 3, 3, 1, 1, 4, 1, 1, 3, 3, 1, 3, 1, 1, 5, 3, 1, 3, 1, 3, 1, 1, 2, 5, 2

This sequence comes up in a fantastically curricular activity I call “Staircase numbers”: how many ways can you write n as a sum of consecutive positive integers?

Finding how many ways it can be done is more difficult and time-consuming, but accessible in middle or high school. Curricular in high school as an intro to arithmetic series.

****

A003278

Szekeres's sequence: a(n)-1 in ternary = n-1 in binary; also: a(1) = 1, a(2) = 2, a(n) is smallest number k which avoids any 3-term arithmetic progression in a(1), a(2), ..., a(n-1), k.
1, 2, 4, 5, 10, 11, 13, 14, 28, 29, 31, 32, 37, 38, 40, 41, 82, 83, 85, 86, 91, 92, 94, 95, 109, 110, 112, 113, 118, 119, 121, 122, 244, 245, 247, 248, 253, 254, 256, 257, 271, 272, 274, 275, 280, 281, 283, 284, 325, 326, 328, 329, 334, 335, 337, 338, 352, 353

****

A101856

Number of non-intersecting polygons that it is possible for an accelerating ant to produce with n steps (rotations & reflections not included). On step 1 the ant moves forward 1 unit, then turns left or right and proceeds 2 units, then turns left or right until at the end of its n-th step it arrives back at its starting place.

0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 3, 0, 0, 0, 0, 0, 0, 25, 67, 0, 0, 0, 0, 0, 0(list; graph; refs; listen; history; edit; text; internal format)

Grade 11

Richard Guy’s “The Strong Law of Small Numbers” needs to be experienced by students. I like contrasting A000127 (maximal number of regions obtained by joining n points around a circle by straight lines) to A000079 (the powers of 2.) Here is Richard Guy’s manuscript: http://www.ime.usp.br/~rbrito/docs/2322249.pdf

[I have used some of those in grades 11-12 to help students see why proof is needed. For many years, we tell them to look for patterns, which is a good thing, but by grade 11 we need to get across that “coincidences cause careless conjectures”, to quote Richard Guy. You can see the worksheet I use here: http://www.mathedpage.org/infinity/infinity-3.pdf . (Henri Picciotto)

****

A060843

Busy Beaver problem: a(n) = maximal number of steps that an n-state Turing machine can make on an initially blank tape before eventually halting.

1, 6, 21, 107

Grade 12

A000108

The Catalan numbers

We want to set-up bijections for students to discover.

****

A005130

Robbins numbers: a(n) = Product_{k=0..n-1} (3k+1)!/(n+k)!; also the number of descending plane partitions whose parts do not exceed n; also the number of n X n alternating sign matrices (ASM's).
(Formerly M1808)

1, 1, 2, 7, 42, 429, 7436, 218348, 10850216, 911835460, 129534272700, 31095744852375, 12611311859677500, 8639383518297652500, 9995541355448167482000, 19529076234661277104897200, 64427185703425689356896743840, 35886920191613760144748615641 7296 (list; graph; refs; listen; history; edit;text; internal format)

COMMENTS
Also known as the Andrews-Mills-Robbins-Rumsey numbers. - N. J. A. Sloane, May 24 2013
An alternating sign matrix is a matrix of 0's, 1's and -1's such that (a) the sum of each row and column is 1; (b) the nonzero entries in each row and column alternate in sign.

For more advanced pupils: but the story of the ASM conjecture and its resolution is really inspiring, and one can have a lot of fun with combinatorics, trying to get them to work out the lowest terms by hand (3x3, 4x4 probably the limit); one can go via factorials and Stirling's approximation along the way.

Proved in 1995 to be $A_n = \prod_{j=0}^{n-1} \frac{(3j+1)!}{(n+j)!}$

****

A205807 List of values of n for which there exist block designs with n blocks.
7, 12, 13, 20, 21, 26, 30, 31, 35, 50, 56, 57, 63, 70, 72, 73, 82, 90, 91, 99, 100, 111, 117, 130, 132, 133, 143, 155, 176, 182, 183, 190, 195, 196, 208, 221, 222, 247, 255, 272, 273, 301, 304, 305, 306, 307, 324, 330, 336, 357, 371, 380, 381, 392, 407, 425, 438, 475, 484

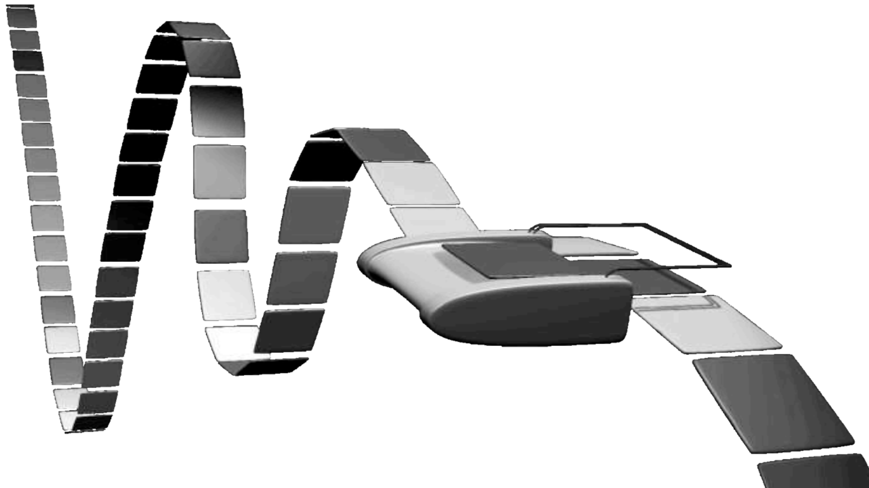
example: The points in the set {1,2,3,4,5,6,7} can be put into 7 blocks with all blocks have 3 points; and every pair of points existing in exactly 1 block. That’s a (7,3,1) Balanced Incomplete Block Design.
123, 145, 167, 246, 257, 347, 356

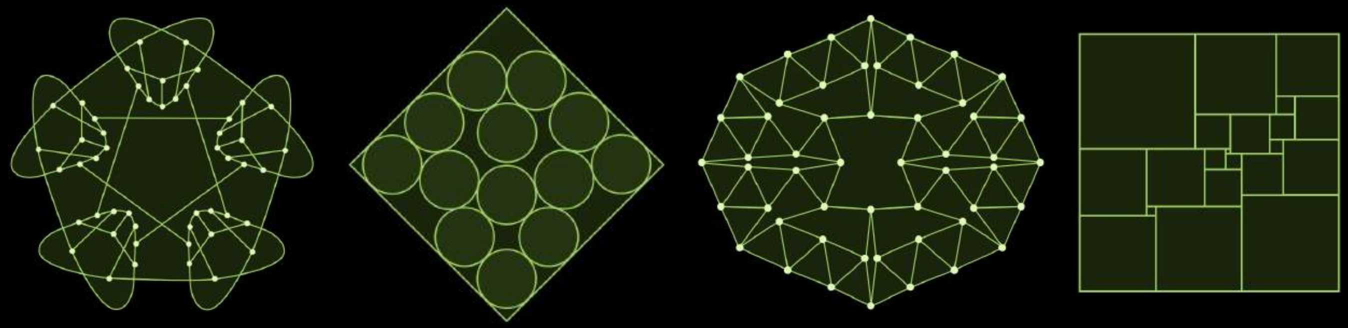
Here is a (10,4,2) Balanced Incomplete Block Design:

0123, 0145, 0246, 0378, 0579, 0689, 1278, 1369, 1479, 1568, 2359, 2489, 2567, 3458, 3467

Here is a (7,3,2) Balanced Incomplete Block Design:

123, 145, 167, 246, 257, 347, 356,
123, 147, 156, 245, 267, 346, 357





Unsolved K-12

A conference for mathematicians and educators

Nov 15-17 2013
Banff International Research Station

The objective of the 2013 K-12 conference was to select one unsolved problem for each grade K-12. Some of these may be inspiring for this conference. In the following 4 pages I'll present the 13 winners - then a selection of other unsolved problems. Finally, I'll show a page from the next K-12 conference which will be dedicated to Mathematical Games.

Winning Unsolved Problems

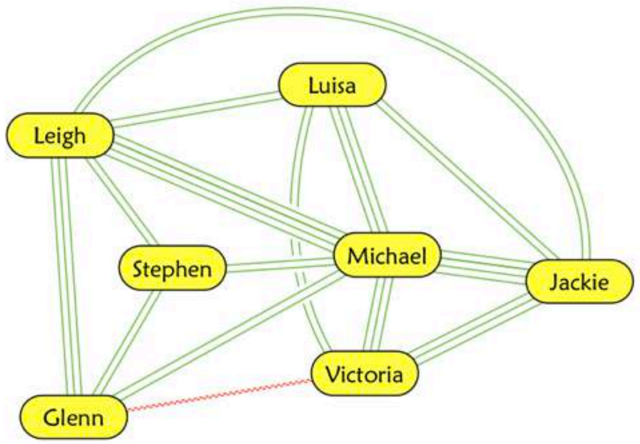
Kindergarten

Maximum Induced Planar Subgraph Problem

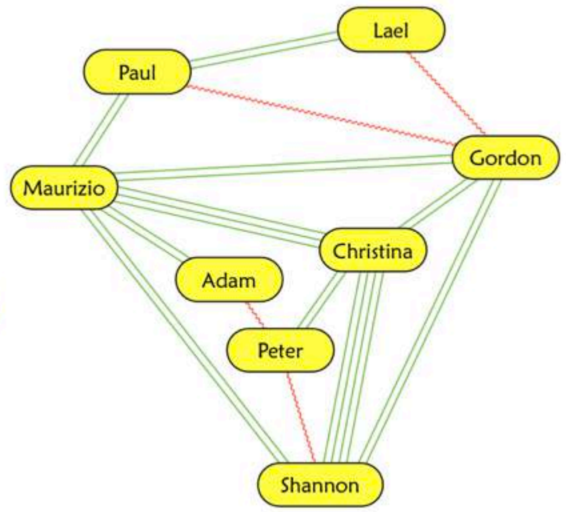
M. Krishnamoorthy & N. Deo, 1977

M. K.

Children write some of their names on the whiteboard. Children are "Friends" if they share at least two letters. Connect them by green lines. Children are "Enemies" if they share no letters. Connect these chilldren with a jagged red line. Laugh about what a ridiculous way this is for you to make friends and enemies. Tell the children that they win by getting the most people on the board with no overlapping connections.



Failure. Victoria and Luisa are connected by an overlapped line.



Success. This is a planar graph, but is it as large as possible?

Unsolved Problem: Find the most efficient algorithm to solve these problems. Currently the record holder is the paper on the next page.

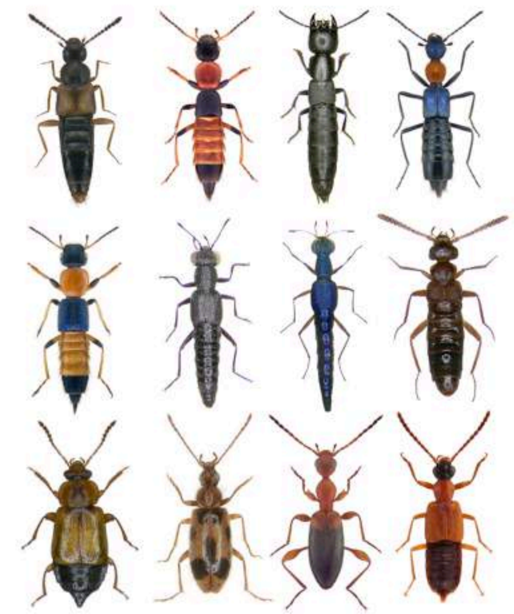
Grade 1

Optimal Neural Network

ImageNet, 2013

ImageNet hosts annual competitions to solve practical unsolved problems about pattern recognition. Neural networks are trained on one set of categorized ImageNet photographs and are then tested on a new set of images to see if they can guess the correct category. See image-net.org for details. Kindergarten students should also be engaged in a rich discussion of sorting. These photographs from retired beetle collector Udo Schmidt are an ideal data set.

Collect beetles from the Staphylinidae family.



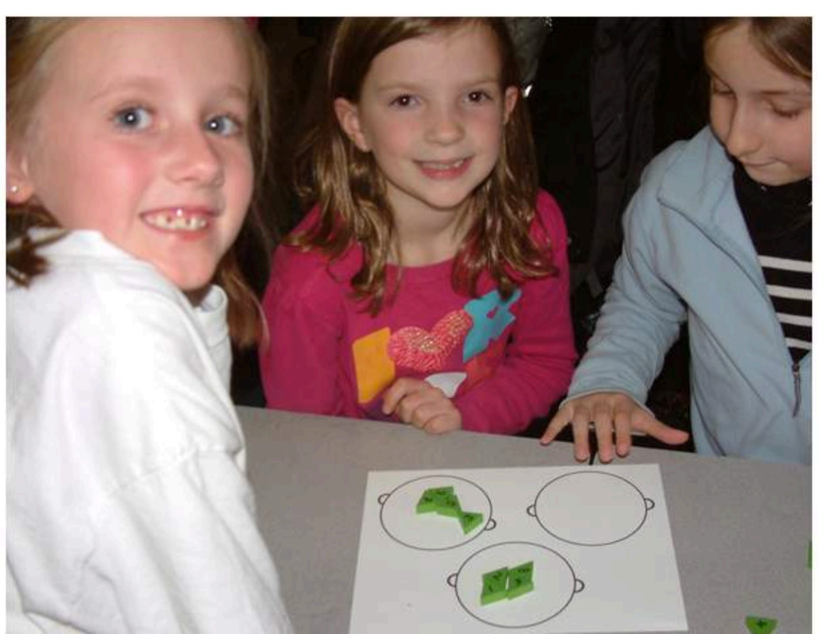
There are 2 impostors from the Anthicidae family.

Grade 2

Sum-Free Partitions

Issai Schur, 1916

The witch adds frog #1 to a bubbling cauldron? Then frog #2. Then frog #3… How high can the witch go without getting gooped… without adding a number to a cauldron that has two numbers which add to it. For example, 24 cannot be added to the blue cauldron because 2+22 = 24. Grade 2 students start with two cauldrons and are asked to use eight frogs. Groups that are successful are quietly asked to get as high as possible with three cauldrons. An unsolved problem is to find the highest number that can be placed in 5 cauldrons.

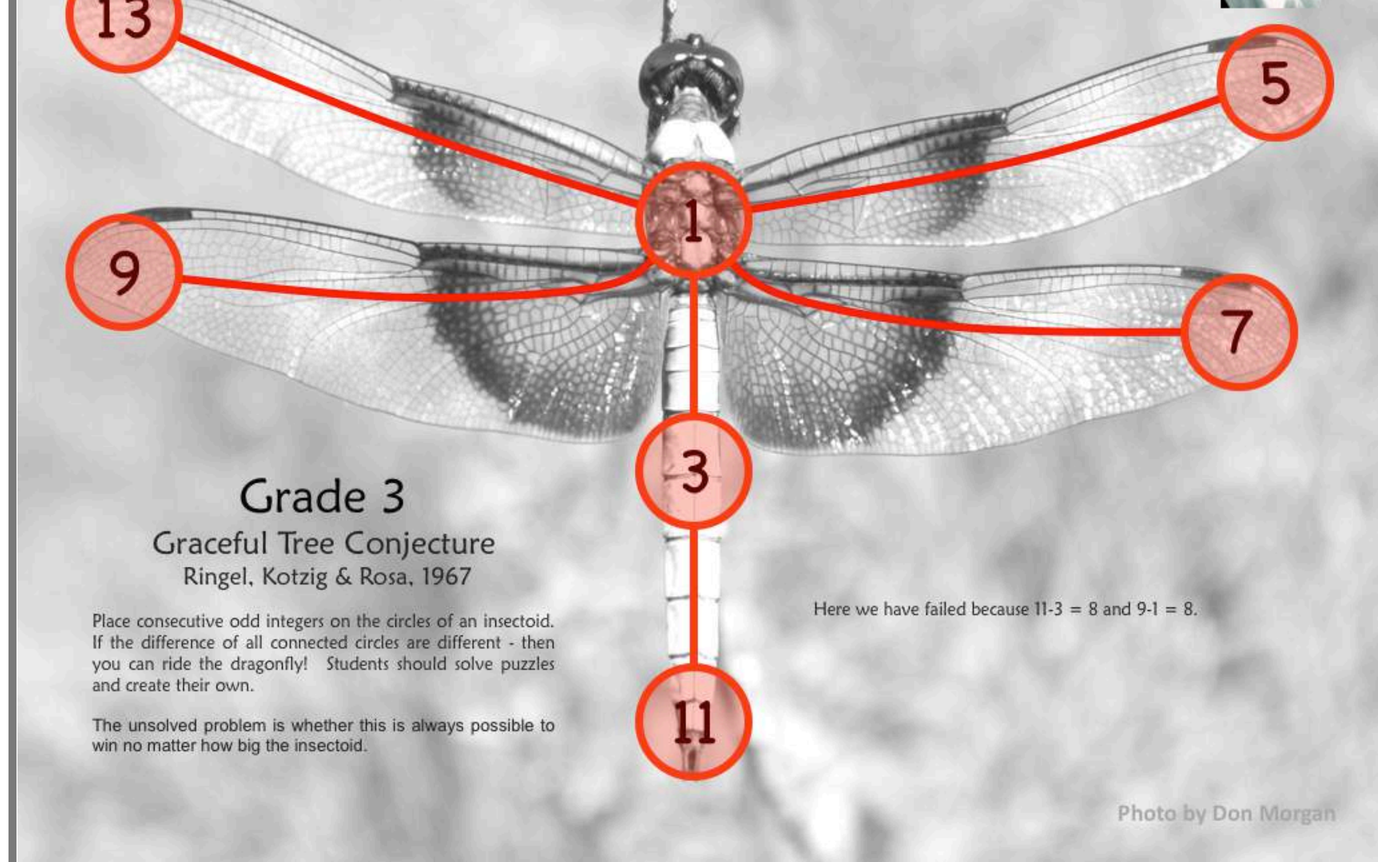


Grade 3

Graceful Tree Conjecture

Ringel, Kotzig & Rosa, 1967

G. Ringel



Place consecutive odd integers on the circles of an insectoid. If the difference of all connected circles are different - then you can ride the dragonfly! Students should solve puzzles and create their own.

The unsolved problem is whether this is always possible to win no matter how big the insectoid.

Here we have failed because 11-3 = 8 and 9-1 = 8.

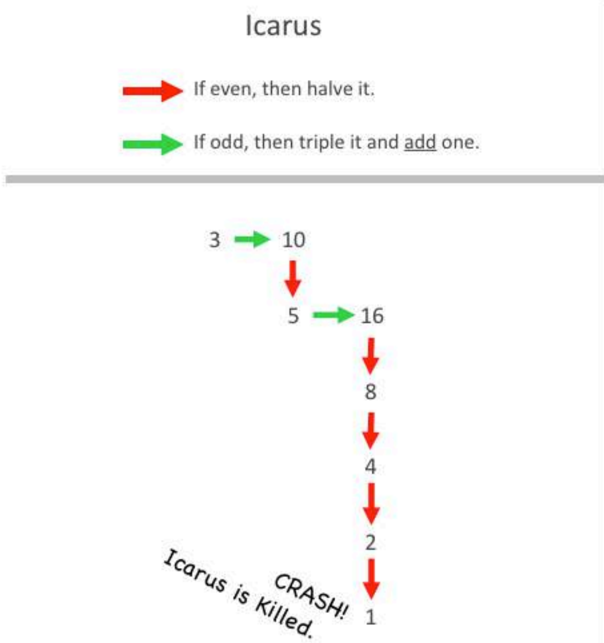
Photo by Don Morgan

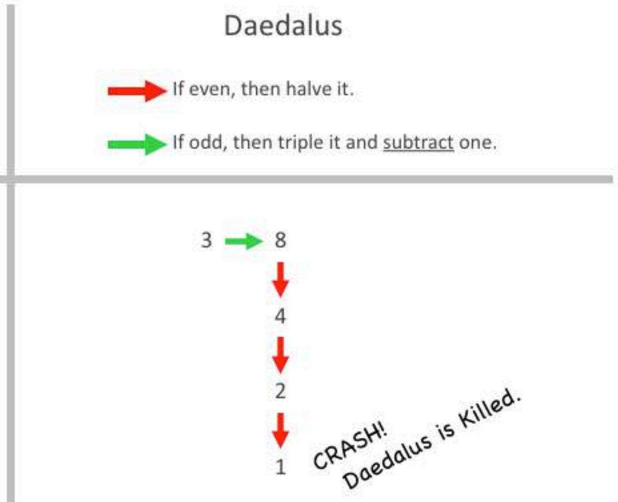
Grade 4

Collatz Conjecture

Lothar Collatz, 1937

On the day before their fateful flight Icarus and Daedalus both have dreams. In Icarus' dream, he writes a number on a rock and hurls it off the tower where they have been imprisoned. If the number is even, it is halved. If it is odd, the number is tripled and one added to the result. For example if Icarus writes a 3 on the rock… you can follow the sequence of numbers until CRASH - he falls into the sea and is killed. This dream has turned into a nightmare. But Icarus knows that if he can just find a number to write on that rock so that he doesn't end up crashing into the sea… so that it doesn't end up at 1… then he will be all right. Daedalus has a similar dream. Your job is to help save the lives of Daedalus and Icarus.





The unsolved problem is to find a way to save both lives. All grade 4 classes will discover a way to save Daedalus' life. How many numbers 20 and under have Daedalus surviving?

Grade 5 (grade 11 in US?)

Non-transitive dice

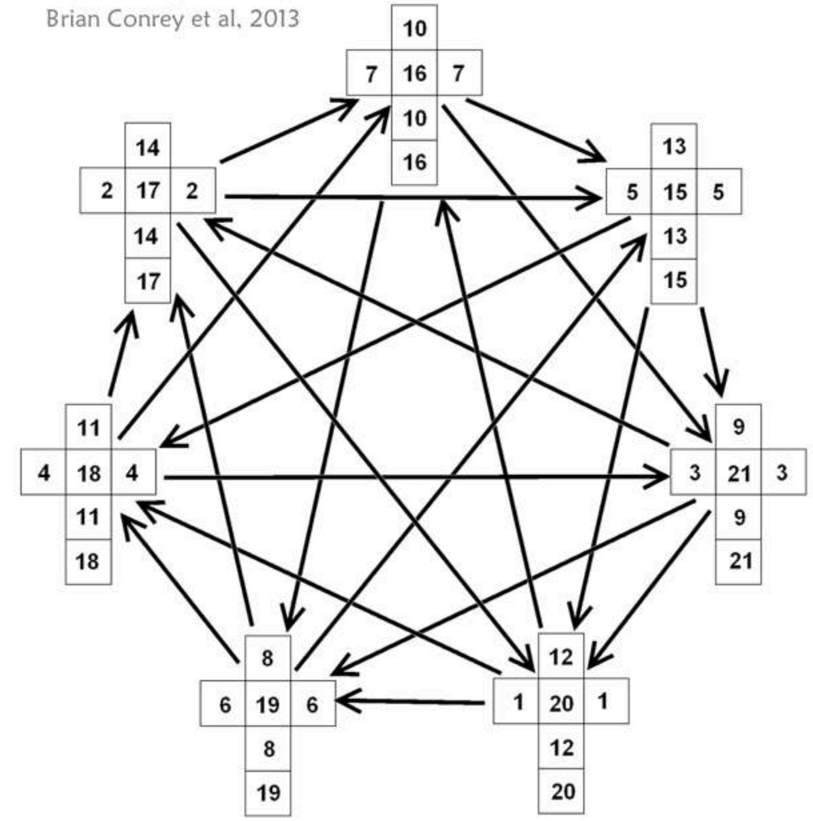
Brian Conrey et al, 2013

What fraction of 3 randomly generated n-sided dice with △n pips are non-transitive?

For six sided dice with 21 pips, this is close to 1/4. This fraction seems to be approached for large n. How intriguing!

Joshua Zucker recommended using 4 dice in the classroom just to ensure that the probability is reasonably high of an AHA! experience. In the elementary school classroom I give students a set of non-transitive dice and ask them to play against one-another. The loser after 7 rolls may opt to change dice with the winner. After 5 minutes I ask them for their opinion about which dice is best.

This problem can be adapted for the US market by asking what FRACTION of time Dice A beats Dice B. This is answered by the standard 6x6 probability grid. In many other countries probability is taught earlier than in the US.



Grade 6

Twin Prime Conjecture

Euclid, 250 BCE

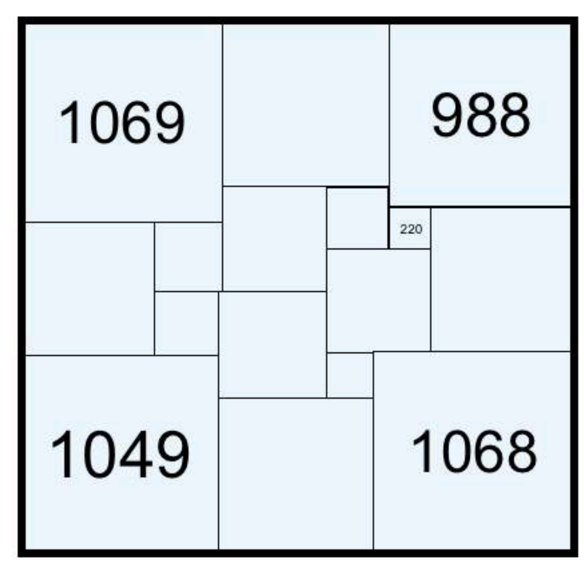
The twin prime conjecture states that there exist an infinite number of prime numbers, p, such that p+2 is prime. The lesson is based on Polya's paper: Heuristic Reasoning in the Theory of Numbers.

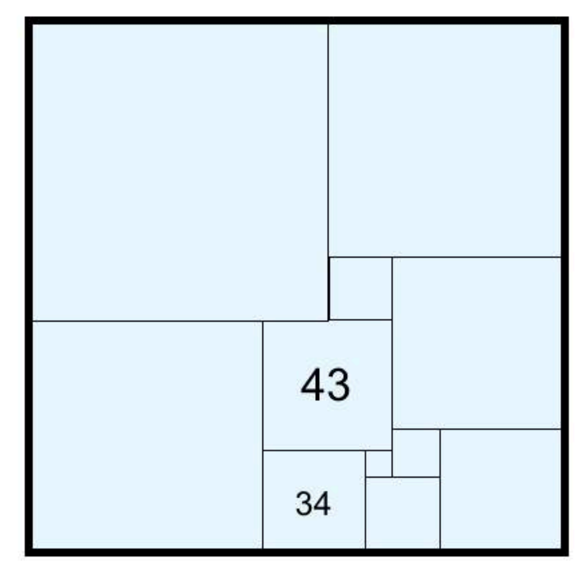
p	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53
p+2	5	7		13		19			31			43			
p+6		11	13	17	19	23		29		37	43	47		53	59

Grade 7

Tiling Rectangles with Squares of Different Sizes

Stuart Anderson, 2013

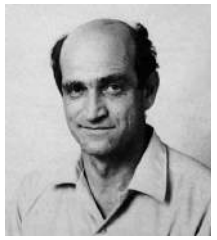




A number in a square represents its edge length. Give students a few numbers in squares and let them complete the rest. Give fewer clues to older students and they will need to use algebra. Youngeer students will be practicing addition and subtraction. The old unsolved problem from the 1930s was whether a square could be tiled using smaller squares of all different sizes (R. L. Brooks, C. A. B. Smith, A. H. Stone and W. T. Tutte). One new unsolved problem (not rigorously investigated) is what is the largest possible fraction: smallest square divided by largest square in any of these rectangles. The current record holder is the solution on the left which boasts an impressively large min/max = 220/1069.

http://www.squaring.net/sq/sr/spsr/spsr_minmax.html

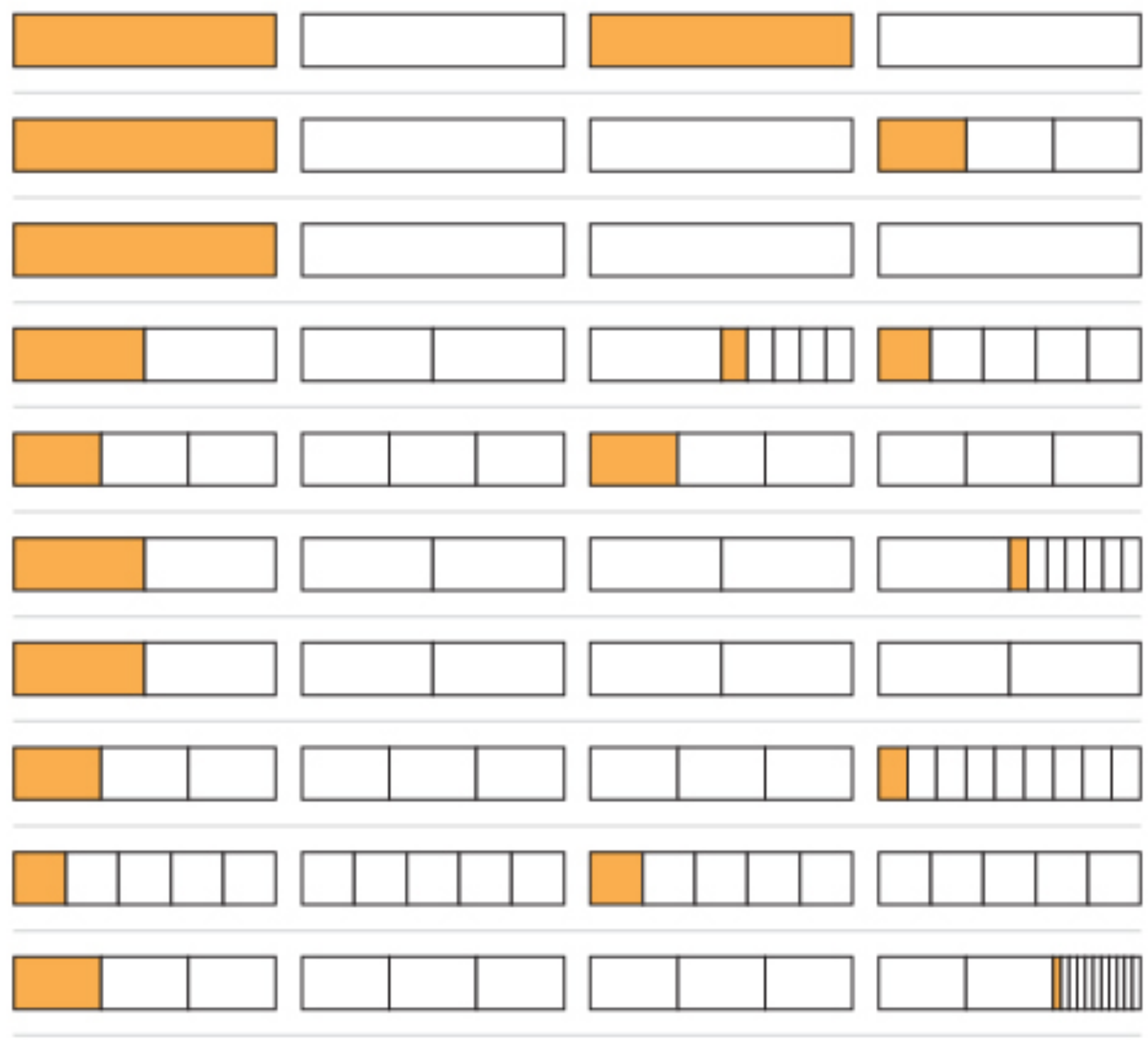
Grade 8



Erdős-Straus Conjecture

Paul Erdős & Ernst Straus, 1948

Andrzej Schinzel conjectures that for any positive k there exists a number N such that, for all n ≥ N, there exists a solution in positive integers to $k/n = 1/x + 1/y + 1/z$. The version of this conjecture for $k = 4$ was made by Erdős and Straus and for $k = 5$ was made by Wacław Sierpiński.

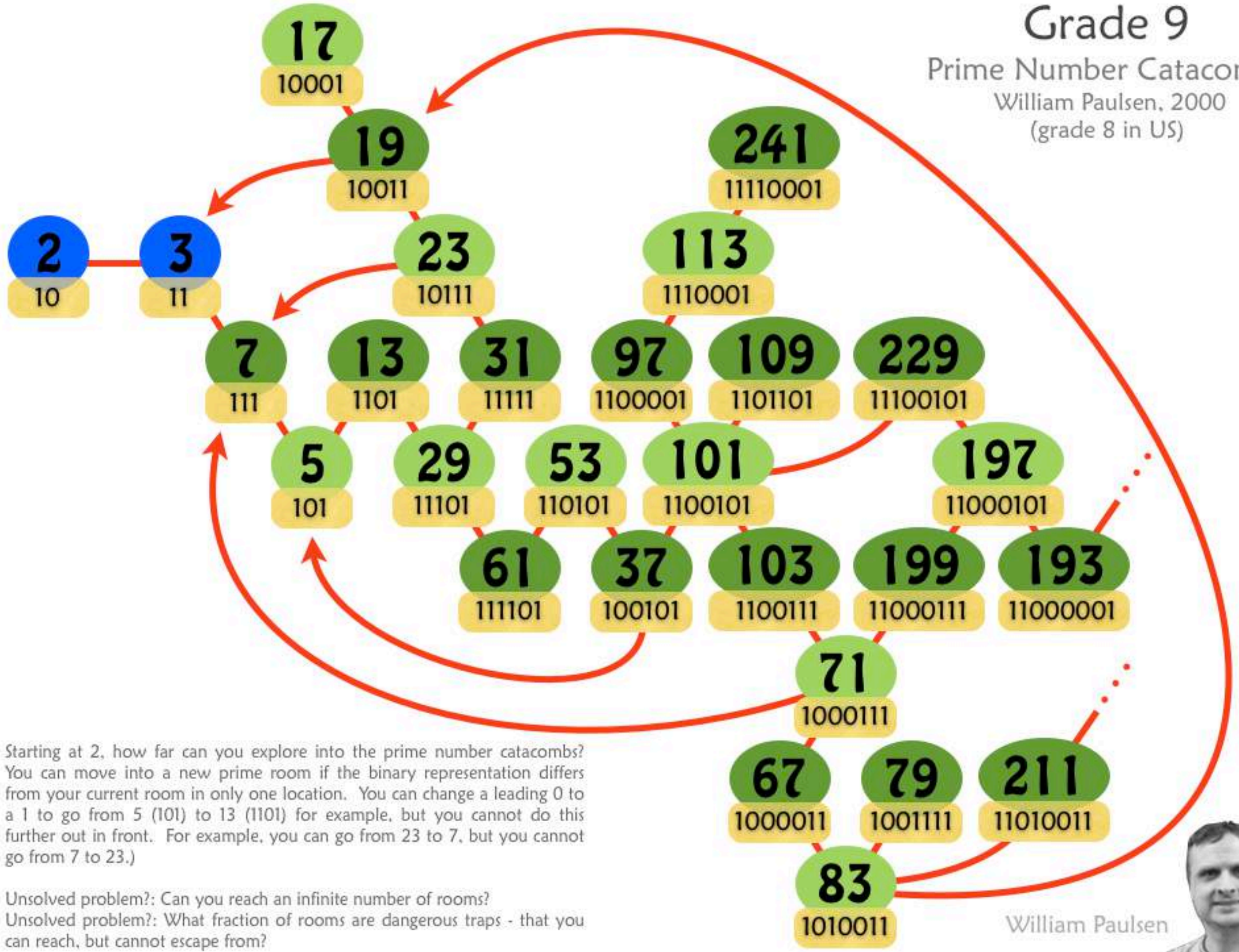


Puzzle by Joshua Zucker

Grade 9

Prime Number Catacombs

William Paulsen, 2000
(grade 8 in US)



Starting at 2, how far can you explore into the prime number catacombs? You can move into a new prime room if the binary representation differs from your current room in only one location. You can change a leading 0 to a 1 to go from 5 (101) to 13 (1101) for example, but you cannot do this further out in front. For example, you can go from 23 to 7, but you cannot go from 7 to 23.)

Unsolved problem?: Can you reach an infinite number of rooms?
Unsolved problem?: What fraction of rooms are dangerous traps - that you can reach, but cannot escape from?

William Paulsen

Grade 10

Imbedded Square

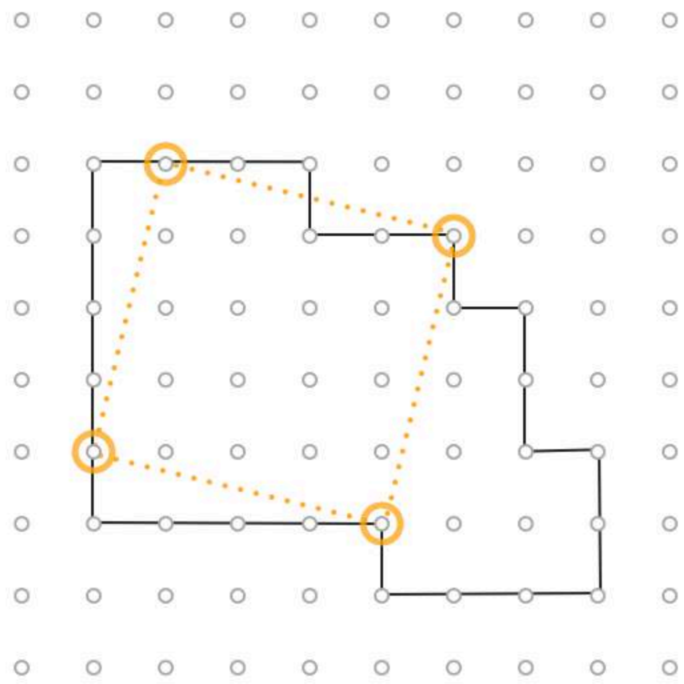
Otto Toeplitz, 1911

The class is told how to tickle their pet amoeba by placing a square blanket on it so that all four corners of the square blanket touch the perimeter of the amoeba. Is this always possible no matter what 2D shape the amoeba forms? This unsolved problem was posed in 1911.

The square blanket above fails because only three of its four corners tickle the perimeter of the amoeba.

A new problem: In taxi cab geometry the imbedded square problem becomes a curricular exercise to give students practice with perpendicular and parallel line segments in a Cartesian coordinate system. The challenge is for them to find some taxi cab loop which does not have an imbedded square (all four corners of the square on the intersections.) I could not find such a loop, but this problem has not been rigorously investigated.

Is there a taxi cab loop (other than the unit square) which does not support two imbedded squares? This has been solved - there do exist such loops and it is a fun challenge for students to find one of them.



Grade 11

Markov Numbers

Andrei Markov, 1879

There are an infinite number of Markov triples {x,y,z} with x≤y≤z that are solutions to the diophantine equation: $x^2 + y^2 + z^2 = 3xyz$. Is there a unique triple associated with each z? That's unsolved. This problem gives students great practice with the quadratic equation.

$$x^2 + y^2 + z^2 = 3xyz$$

{x,y,z} = {1,1,1}

↓

{1,1,2}

↓

{1,2,5}

↙ ↘

{1,5,13} {2,5,29}

{1,13,34} {5,13,194} {5,29,433} {2,29,169}

Grade 12

P=NP?

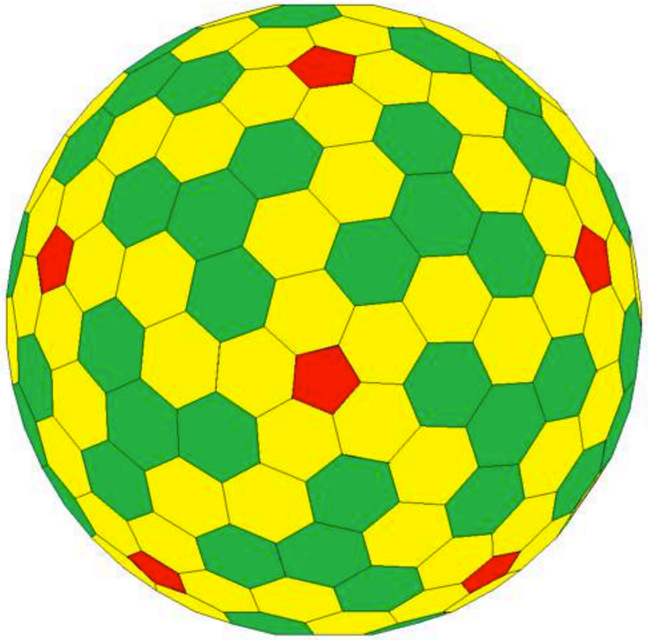
Stephen Cook, 1971

Let's duplicate this most important millennium problem. Grade 12 students are capable of understanding this even if this understanding is not curricular world wide.

Get students to try to solve certain classes of problems with varying n and then plot the time taken to solve versus n:

Finding if an integer, n, is odd or even.
Finding the number of digits of an integer n.
Finding the smallest element of a set.
Finding the number of vertices on a polygon.
Finding the number of vertices on a polyhedron.
Finding the median of a set of n numbers.
Adding two numbers with n digits.
Multiplying two numbers with n digits.
Determining if a graph of n vertices is bipartite.
Determining if a graph of n vertices is tripartite.
Finding the factors of a number n.
Finding a Hamiltonian cycle.

Students will get the idea of exponential versus polynomial times by painful experience! It is also nice for them to experience that while multiplying 5 digit numbers is more difficult than finding a hamiltonian cycle through 5 vertices, by the time you get to 100, the first is boring while the latter is potentially very, very difficult!



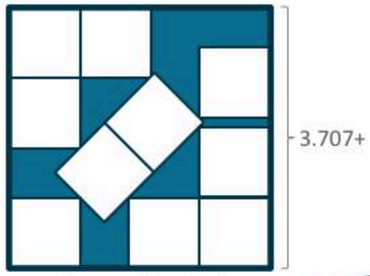
Other Unsolved Problems

Kindergarten - Grade 1

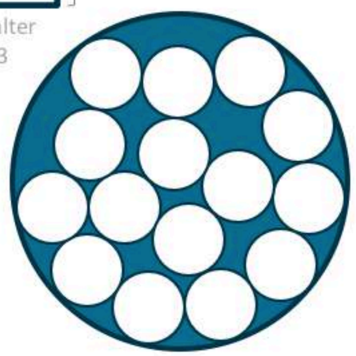
Packing Squares

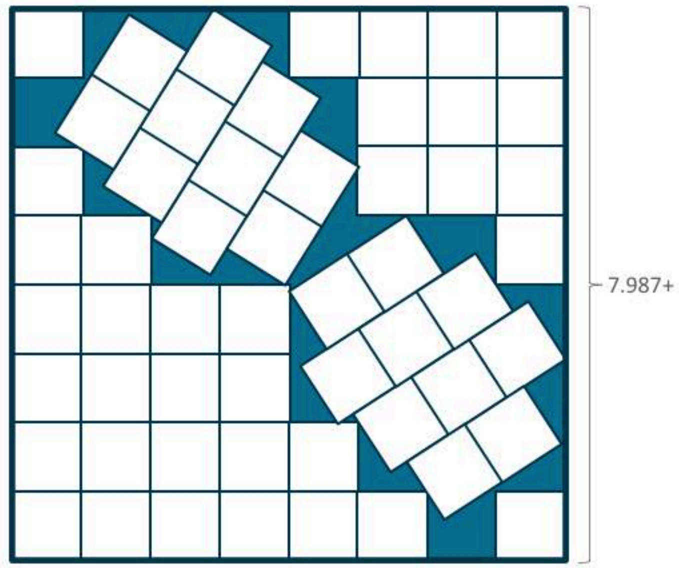
Erdős & Graham, 1975

What is the smallest big square required to hold n unit squares? In general - how do you clean up and pack things away efficiently? The square in square problem has the advantage of being unsolved for n=11 unit squares and having a periodic structure - alternating between orthogonal and complex packing. Ed Pegg prefers the circle problem because coins are readily available to teachers.



n=10 proved by Walter Stromquist, 2003





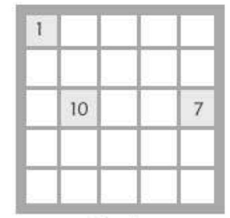
n=55 found (but not proved) by David Cantrell, 2005

Erich Friedman's Packing Centre: www2.stetson.edu/~efriedma/packing.html

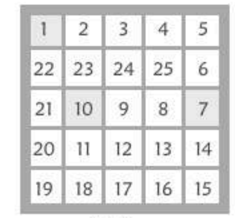
Willie Wiggle Wiggle Worm

Willie Wiggle Wiggle Worm got lost after a thunder shower one day so I made a little home for him out of a milk carton. He didn't know how to count, so I wrote the numbers 1 (head) to 25 (tail) on him. Each day I gave him a puzzle to help him practice counting and writing numbers.

The puzzles work like this: Some number hints are put in the carton. Willie Wiggle Wiggle Worm has to wind his way around in the carton so that the numbers on the carton are the same as the numbers on his body.

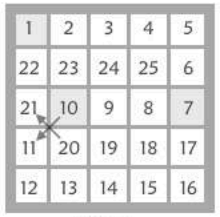


Puzzle



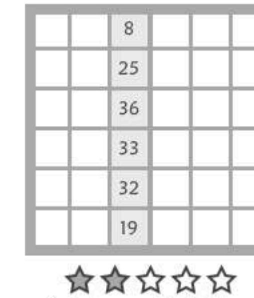
Solution

Willie Wiggle Wiggle Worm does contort into fantastic shapes, but he can't go diagonally.

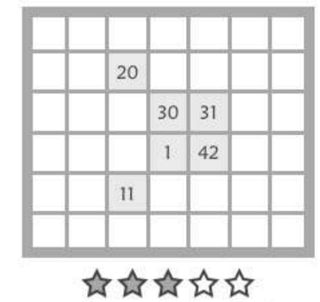


Wrong

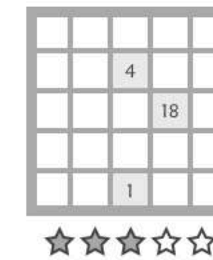
by Lauren



based on a puzzle by Isla, a kindergarten student



by Kindergarten students at River Valley School



by David, a kindergarten student

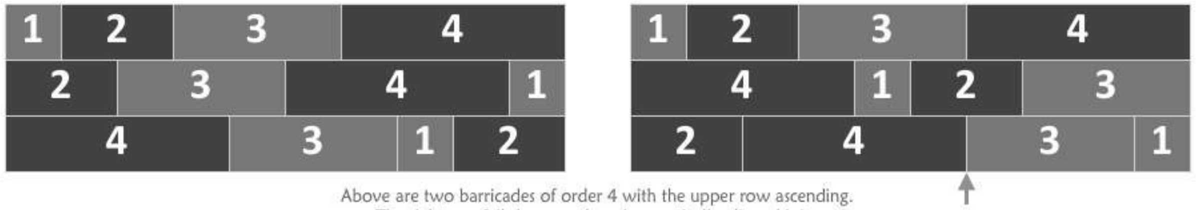
What are the minimum and maximum number of hints required to have a unique solution.

Kindergarten - Grade 9

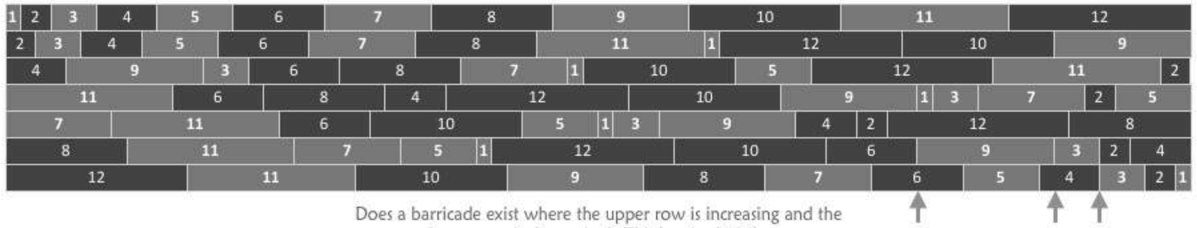
Building (Lego) Barricades

Barry Cipra, 2006

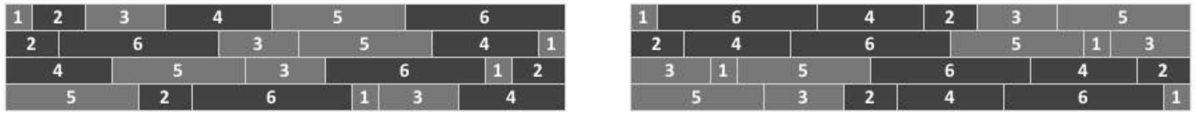
Barricades are constructed using logs of lengths 1 through n in each of (n+2)/2 rows. To make them strong enough to withstand projectiles, a barricade can not have two or more joins vertically aligned. There is some elegant algebra surrounding their construction which makes them a good choice for junior high. They are also great for grade 2s learning that addition is commutative and for younger students learning rules that apply within a system.



Above are two barricades of order 4 with the upper row ascending.
The right one fails because there is a vertically aligned join.



Does a barricade exist where the upper row is increasing and the lower row is decreasing? This barricade fails.



Above left is a barricade of order 6 with the upper row ascending. Above right is a barricade of order 6 that is special because it is rotationally symmetric.

Grade 1

Cookie Monster

Vanderlind, Guy, Larson, 2002

A cookie monster is presented with some jars with cookies in them. He wants to empty the jars in the fewest number of minutes possible. Each minute he may take the same number of cookies out from any number of the jars. What is the best strategy for him to empty a set of jars?

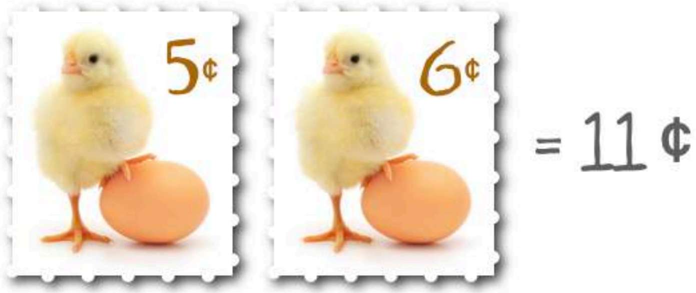
Grade 2

Postage stamp problem

Rohrbach, 1937

Find 6 denominations of stamps so that eggs can be mailed to the museum using at most two stamps. Start by trying to send a hummingbird egg (1¢) and work your way towards Ostrich eggs (50¢). What is the largest egg that you can send?

The unsolved problem is to find a general formula for the largest egg that can be posted as the number of denominations increases. The group of six denominations below can send eggs costing 1¢ through 12¢ - but not 13¢. This is far from optimal. Unsolved Problems in Number Theory (C-12) gives the solution: 1, 2, 5, 8, 9, 10 for six stamps and 1, 2, 3, 7, 11, 15, 19, 21, 22, 24 for 10 stamps.

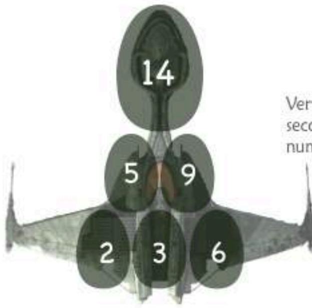


Grade 2

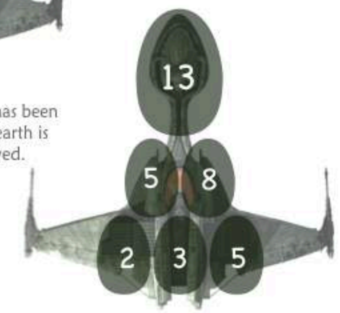
Klingon Attack

Spock, 1966

Very scary - Enemies are attacking earth. You can aim your ion cannon in one place each second. What is the fastest that you can destroy an enemy? A group of enemies? Each number must equal the sum of the two numbers underneath and must be unique.

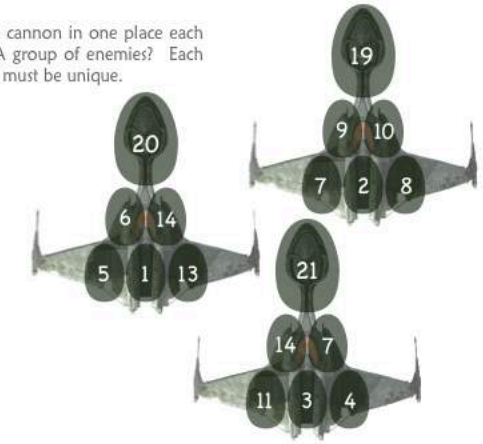


Pseudo-Success! The spaceship has been destroyed in 14 seconds. The earth is saved, but Europe is destroyed.



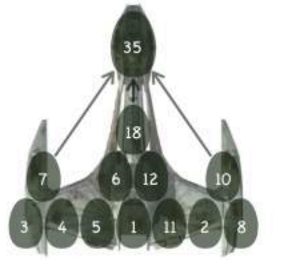
Failure! Duplicate 5s - the earth is destroyed.

Success! 8 seconds is the fastest time that this enemy can be destroyed.



Failure! Duplicate 14s. The earth is destroyed. These three bird-of-prey collectively have 18 targets and actually require 22 seconds to destroy. Try again!

The unsolved problem is which spaceships it is possible to destroy in sufficiently large numbers in n seconds where n is the total number of targets. This problem has not been rigorously investigated.

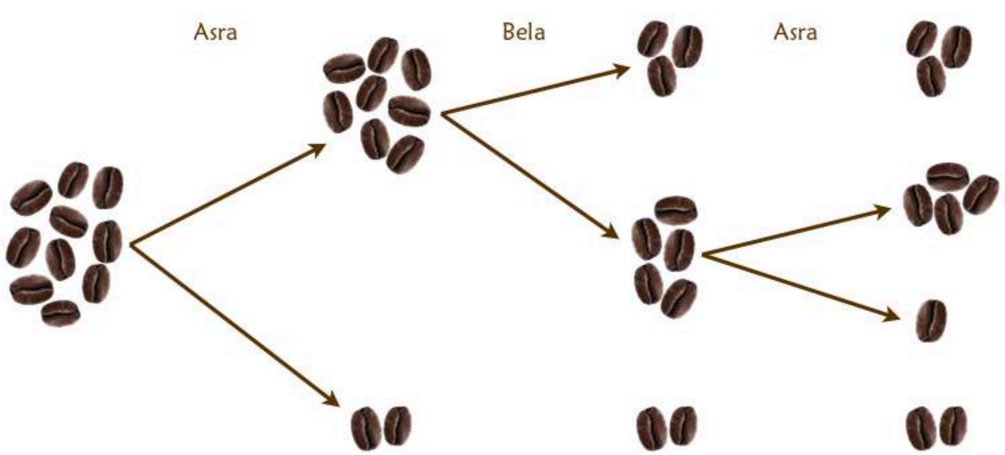


Grade 1

Squirel Nutkin Buries Nuts for Winter

Proposed by Richard Guy

Here is another possible addition to our already deep list of possibilities for grade 1. This is a two player game. Start with a heap of size n. On your turn you may split any heap into two parts... but you must be careful... after you've finished - no two heaps can be the same size. This is a GREAT game for grade 1. It is superior to all other nim games because the end-state is curricular. Children must compare quantities in different stacks. Apparently the game is also rich because there is a tantalizing link to triangular numbers (yet to be proved.) In the game below, Bela cannot go so Asra wins.



This game is not fun for most adults because it lacks strategy - it is only tactical - and the tactics become more and more mind-numbingly difficult as the number nuts / coffee beans increases. Contrast this to Aggression (grade 3) where players can have a strategic intuition that is accurate even for games on large maps. This criticism is not a negative in the grade 1 classroom.

Squirel Nutkin has not been rigorously investigated, so perhaps another unsolved representative from this group of NIM games should be selected - but the one above is definitely the game to present in the classroom. As a varient - consider the game played on a line. When you split a stack you move one part to the left along the line and one part to the right along the line. No two neighbouring piles may be the same size.

Grade 2

Magic Squares & Cubes

Ed Pegg wonders if there is a place for magic squares and cubes in our set of 13 unsolved problems. He writes: "For centuries, whether an order-five magic cube existed was unknown, until November 14, 2003, when C. Boyer and W. Trump discovered a solution. Note that the numbers 1 to 125 are used in this cube." To see a demonstration of this special cube, go to demonstrations.wolfram.com/MagicCubeOfOrder5

By deleting some entries in a = 4x4 magic square, puzzles can be created to help students practice addition and subtraction.

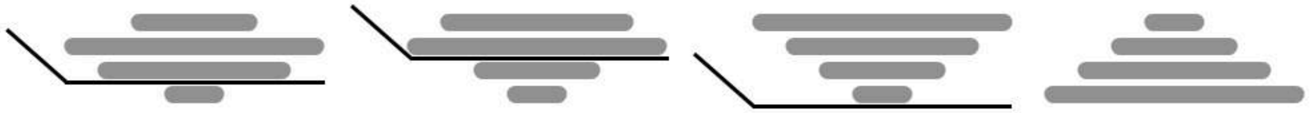
3	9	6	16
14	8	11	1
12	2	13	7
5	15	4	10

Grade 2

Pancake Flipping Problem

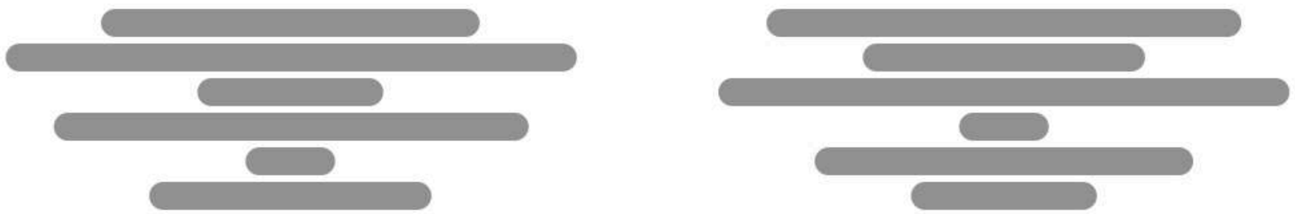
Gates & Papadimitriou, 1979

Find are the fewest number of flips to order pancakes into a beautiful pyramid.
For example, the left stack below takes three flips of the spatula.



Grade 2 students can challenge each other finding the three orderings of four pancakes which take 4 flips.

Another challenge is to ask the fewer number of flips to solve one of the stacks below. It is beyond the ability of most grade 2s to find the optimal solution (seven flips for both), but it is a good competition to establish the importance of creating a system to record results so that students can repeat a solution once discovered.

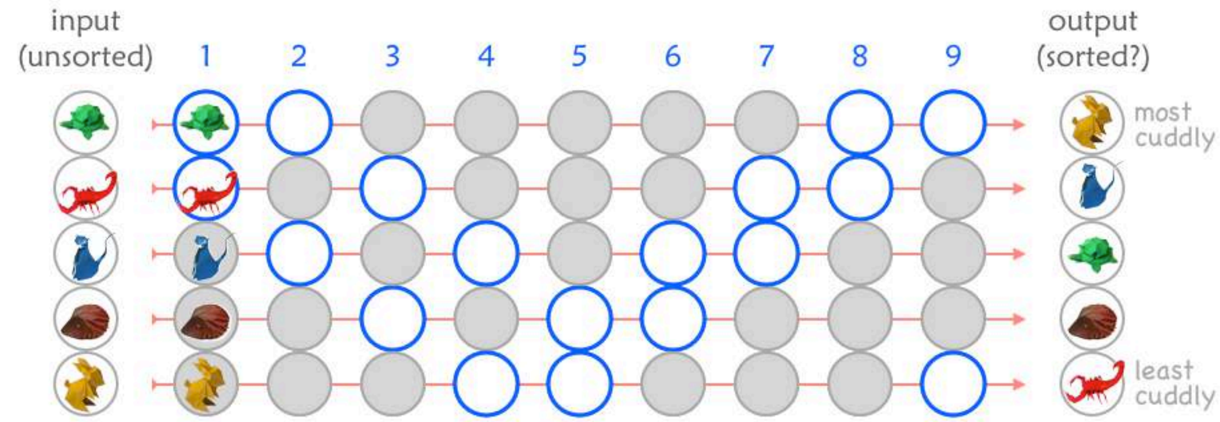


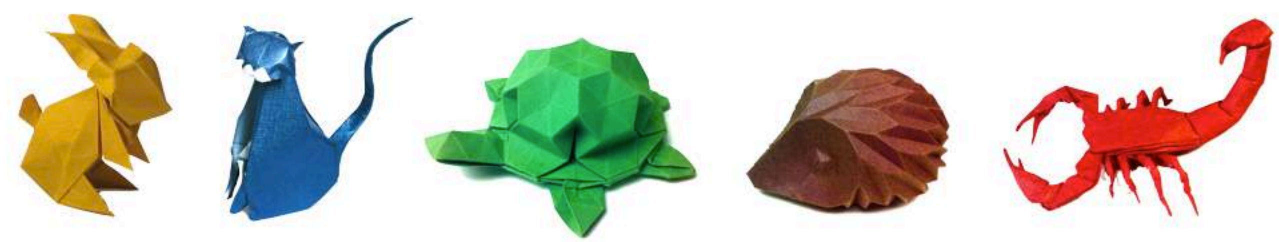
Grade 2

Sorting with fewest comparisons

Knuth, 1973

Lets say that you wanted to sort a bunch of origami animals on a cuddly scale from most cuddly to least cuddly. What are the fewest number of comparisons that need to be made to sort data? Students experiment with algorithms that work, but are too long, algorithms that don't work because they fail to sort all possible inputs, and then try to design their own optimal algorithm.





Grade 2

196 Palidromic Number Generator?

Gruenberger, 1984

Take any positive integer with two or more digits. Add it to the number obtained by reversing its digits. Continue until a palidromic number is obtained. Most small numbers terminate quite quickly. 89 is the first number that poses a real challenge… terminating with 88132000231188. Some numbers like 196 and 295 in base 10 may never become palindromic, but this has yet to be proved.

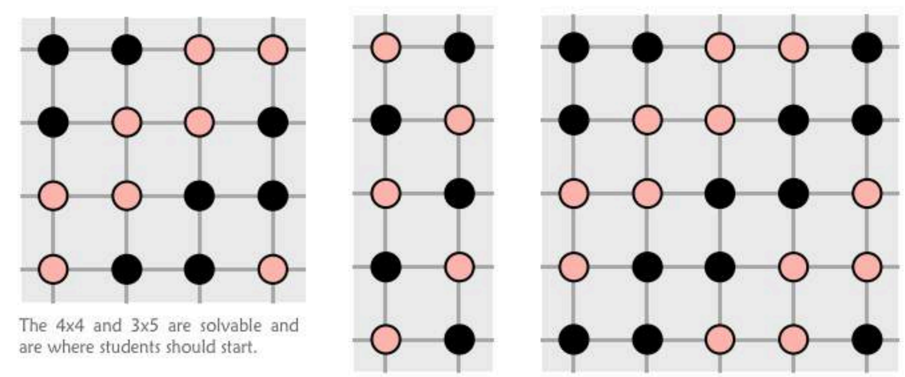
280	285	290	295	300	305	310	315	320
362	867	382	887	303	808	323	823	343
625	1635	665	1675					
1151	6996	1231	7436					
2662		2552	13783					
			52514					
			94039					
			187088					
			1067869					
			10755470					
			18211171					
			35322452					
			60744805					
			111589511					
			227574622					
			454050344					
			897100798					
			1794102596					

Grade 1-4

No Single-Colour Rectangle

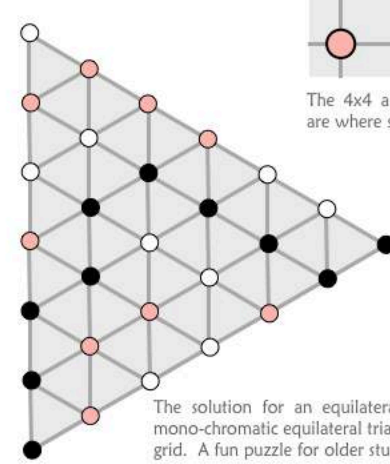
Bill Gasarch, 2009

Add a dark or light drop to each intersection. Can it be done so that no rectangle is created with all vertices the same colour? (Gasarch only considers rectangles with sides horizontal and vertical.)



The 4x4 and 3x5 are solvable and are where students should start.

An alternating pattern shows that any grid of size 2 x n is solvable. Grade 1 students are often capable of making this breakthrough. The 5x5 and 3x7 are both solvable, but too difficult for most grade 1 & 2 students. Here the 5x5 solution fails because there is a rectangle whose vertices are all the same colour.



The solution for an equilateral triangle of side-length 7 avoids all mono-chromatic equilateral triangles including those not parallel to the grid. A fun puzzle for older students, but too difficult for grade 1.

Gasarch has found most of the set of rectangles for which 4 colours are permitted. This set includes the 17x17 square. Partial solutions of these rectangles can be given - the students asked to fill in the remaining colours. The set of rectangles which can be solved with 5 colours is unknown.

Mutant Fibonacci Bunny Sequence

What is the fastest way to get to a target number (in this case 13) by repeatedly either adding the lst three terms or finding the difference between the last two.

17

1 1 1 3 2 1 6 9 16 31 15 16 1 15 14 1 13

16

1 1 1 3 2 6 11 19 36 17 19 2 17 15 2 13

10

1 1 1 3 5 9 4 18 31 13

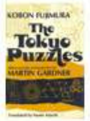
10

1 1 1 0 1 1 2 4 7 13

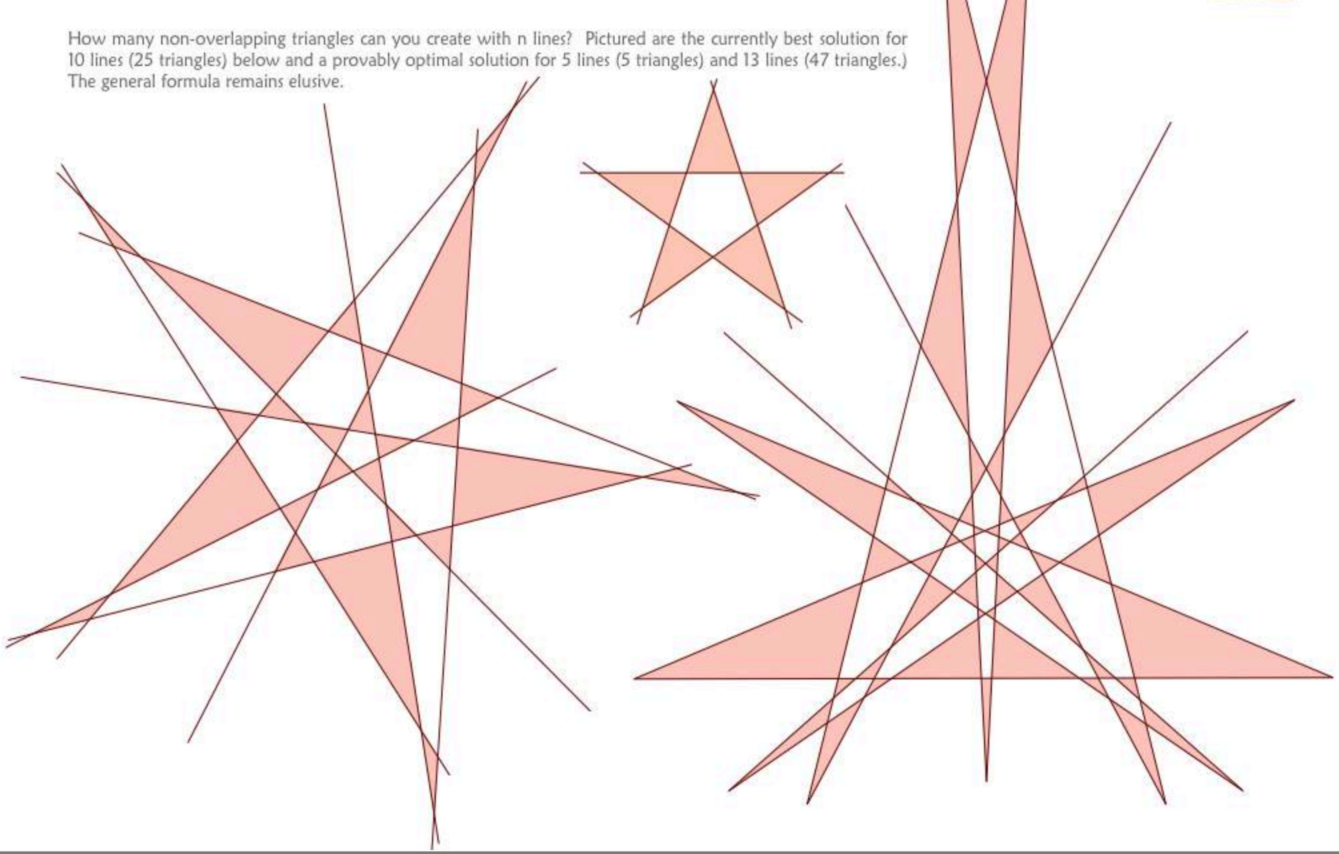
Grade 3

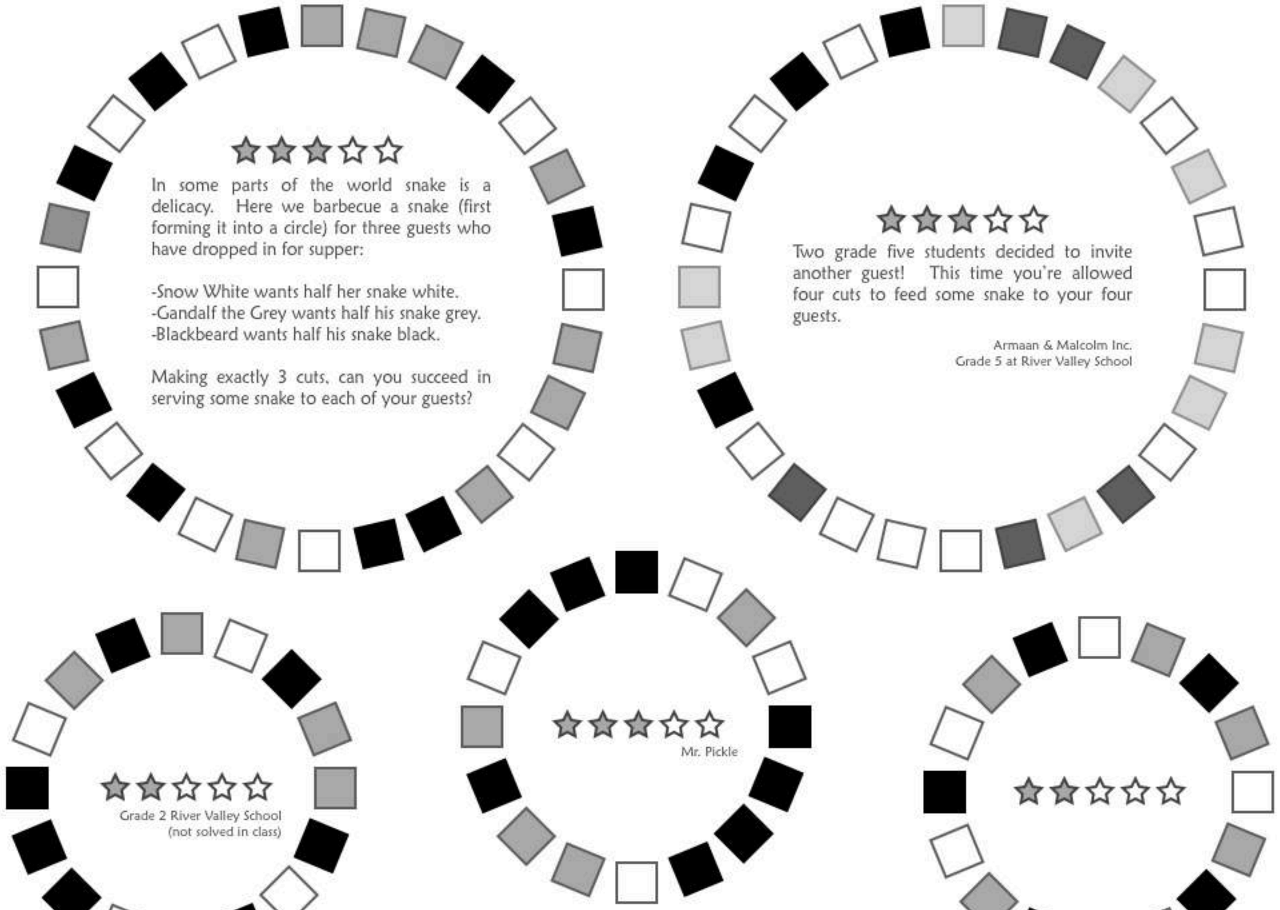
Kobon triangles

Kobon Fujimura, 1979



How many non-overlapping triangles can you create with n lines? Pictured are the currently best solution for 10 lines (25 triangles) below and a provably optimal solution for 5 lines (5 triangles) and 13 lines (47 triangles.) The general formula remains elusive.





In some parts of the world snake is a delicacy. Here we barbecue a snake (first forming it into a circle) for three guests who have dropped in for supper:

-Snow White wants half her snake white.
-Gandalf the Grey wants half his snake grey.
-Blackbeard wants half his snake black.

Making exactly 3 cuts, can you succeed in serving some snake to each of your guests?

Two grade five students decided to invite another guest! This time you're allowed four cuts to feed some snake to your four guests.

Armaan & Malcolm Inc.
Grade 5 at River Valley School

Grade 2 River Valley School
(not solved in class)

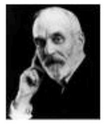
Mr. Pickle

What fraction of snakes of a given length are solvable?

Grade 3

Mrs Perkins' Quilt

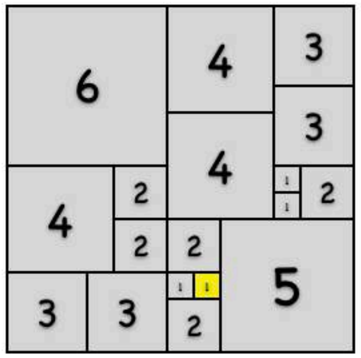
Henry Dudeney, 1917



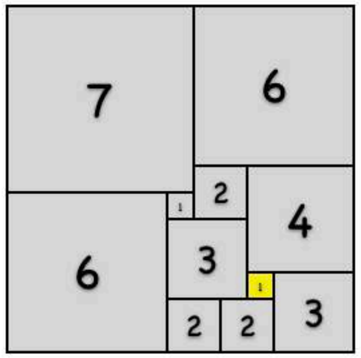
Ed Pegg Jr suggested the the group of square and rectangular problems related to Mrs. Perkins' Quilt. Here is how Henry Dudeney set it up for his readers: "For Christmas, Mrs. Potipher Perkins received a very pretty patchwork quilt constructed of 169 square pieces of silk material. The puzzle is to find the smallest number of square portions of which the quilt could be composed and show how they might be joined together. Or, to put it the reverse way, divide the quilt into as few square portions as possible by merely cutting the stitches."

This 13x13 problem has been generalized to squares of different sizes. The smaller squares need not be all different sizes, but they do need to be **relatively prime** to avoid trivial dissections like the dissection of a 4x4 square on the left.

This problem is great practice for grade 3 students adding and subtracting, but the idea of "relatively prime" is not curricularly appropriate. Therefore, the common generalization of Dudeney's puzzle should be amended so that a 1x1 gold tile MUST be included in every tiling of the teacher's new 5x5 bathroom / 7x7 bedroom / 9x9 kitchen / 11x11 garage / 13x13 living room. Young students understand this formulation and tiling is something more familiar to the bulk of students. Under this formulation the pictured 11x11 solution for the teacher's garage is no longer acceptable because we forgot the gold 1x1 tile.



18 tiles



11 tiles

Grade 4

Multiplicative Persistence

Gottlieb, 1969

Choose a positive integer. Multiply all the digits together. Repeat until you are left with a single digit. The number of steps that you've taken is equal to the multiplicative persistence of the number. What is the largest multiplicative persistence possible? Erdos asked questions when zeros were replaced by 1s… all numbers eventually crash to a single digit, but it can take a long time. If zeros are replaced by n, do numbers always crash. The answer is no for n = 15 for example

59 → (5 × 9) 45 → (4 × 5) 20 → (2 × 0) 0

79 → (7 × 9) 63 → (6 × 3) 18 → (1 × 8) 8

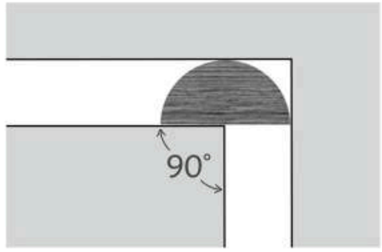
99 → (9 × 9) 81 → (8 × 1) 8

Erdős asked what happens if zeros were always replaced by 1s… all numbers eventually crash to a single digit, but it can take time. If zeros are replaced by n, do numbers always crash? The answer is no for n = 15 (example: 4500 = 4*5*15*15), but seems to be yes for many larger n.

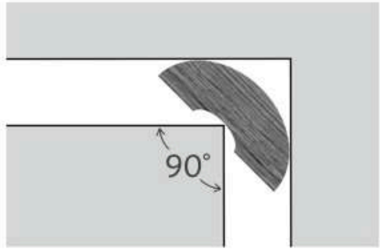
Grade 4,5

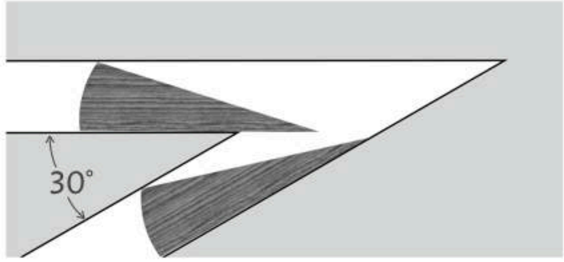
Moving Furniture

Leo Moser, 1966



During moving, you must drag various heavy desks around a ninety degree bend. What is the largest surface area of desk that you can accomplish this feat? Of course it can be done with the half circle desk shown on the left, but that is not optimal as can be seen with the larger desk on the right.





What is the angle for which the surface area of the maximal desk is minimized? At what angle does it become optimal to switch the leading edge to the lagging edge (see above)? These two questions have not been rigorously investigated.

Grade 4

n times Sum equals Product

Trost, 1956

Find a set of positive integers whose sum equals its product. For example, the three solutions for five elements in a set are: {2, 2, 2, 1, 1} which has a product and sum of 8, {3, 3, 1, 1, 1} which has a product and sum of 9, and {5, 2, 1, 1, 1} which gives 10.

Stating with n = 1, the number of different solutions with a set of n positive integers is: 1, 1, 1, 1, 3, 1, 2, 2, 2, 2, 3, 2, 4, 2, 2, 2, 4, 2, 4, 2, 4, 2, 4, 1, 5, 4, 3, 3, 5, 2, 4, 3, 5, 2, 3, 2, 6, 3, 3, 4, 7… (OEIS A033178) Ask students to discover patterns and prove that there is always at least one solution. This problem is D24 in RKG's "Unsolved Problems in Number Theory."

{2, 2, 2, 1, 1}

{3, 3, 1, 1, 1}

{5, 2, 1, 1, 1}

Grade 5

Sums Determining Members of a Set

Leo Moser, 1957

This problem is introduced by getting two children to secretly choose an integer each. They whisper to each other and then announce the result to the class. The class realizes that they cannot know for certain what the two numbers were. (Example: Fiona chooses -10 and Jane chooses +12. They announce "two" to the class who find it impossible to reconstruct the original numbers.)

Next three students are invited up and choose a number between about -5 and +5. Do not try a larger interval the first time playing unless you know your class ability well. All three group-whisper and announce the three sums resulting from adding up all possible pairings of their numbers. This time it is found that the three numbers can be calculated. The bulk of classroom time is spent exploring this three-person problem. (Example: Fiona chooses -2 and Jane and Bob both choose +3. They announce "one, one, six" to the class who can reconstruct the original numbers.)

Say "A group of four announce the following six pair-wise additions. What were their original numbers?" There are actually two possible solutions. The students must find both.

-2, -1, -1, +1, +1, +2

Show your students the following secret numbers that resulted from the game being played three times with eight people. The pair-wise additions are identical!

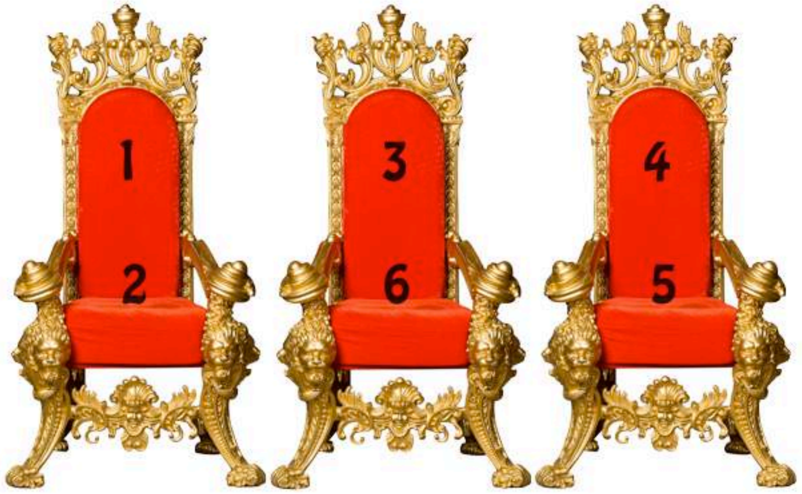
{±1, ±9, ±15, ±19} {±2, ±6, ±12, ±22} {±3, ±7, ±13, ±21}

Grade 7

Unfair Thrones

Royal Empress Menen I of Ethiopia, Empress of Empresses, Queen of Queens, 1903

Create n fractions by placing the integers {1, 2, 3... 2n} in either numerator or denominator. Your success comes by minimizing the difference of the greatest fraction minus the least. The puzzle is introduced by choosing a class empress and announcing that her royal highness has just given birth to twins. Two students are selected and come up to the front of the class to sit on their thrones. When the optimal solution for the twins is discovered... an announcement is made that the Empress has just given birth to a beautiful baby boy. Each new birth makes the problem more difficult.



The three - throne solution is shown above. The chance of civil war is 4/6 (the greatest) minus 1/2 (the least) = 1/6 or 17%.

The mathematician Charles Greathouse emailed me the following in June 2013 when I asked him if the problem is worthy of being selected:

"On one hand, it's in EXP with no obvious reduction ('might be hard'). On the other, I would not be surprised in the theory of Farey series could be brought to bear, so I wouldn't be shocked if it turned out to be easy... On a quick analysis I find

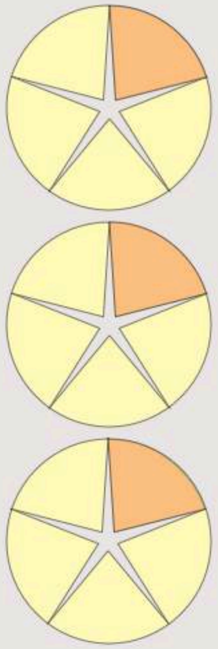
0, 1/6, 1/6, 1/8, 6/35, 5/24, 3/14, 3/16, 1/6
as minimal scores for 1..2, 1..4, and so on."

Grade 7

Cupcake Problem

Alan Frank

It's a birthday party gone awry. You've got 3 cupcakes but 5 children! Divide the cupcakes so that each child gets the same amount and the smallest piece is as large as possible.

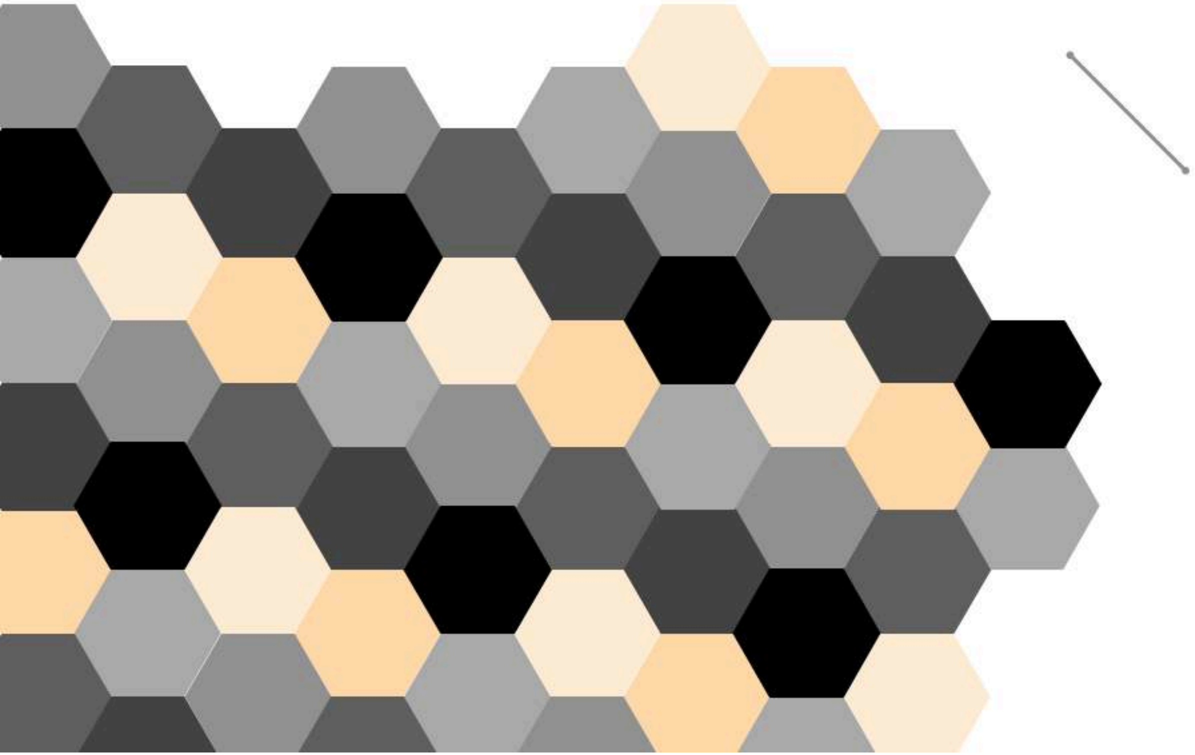


Grade 7

Chromatic Number of the Plane

Hadwiger & Nelson, 1945

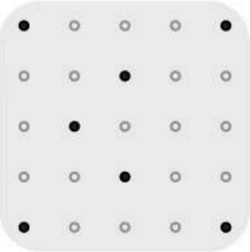
Place a toothpick on a student's page. If its ends are touching different colours, the student wins. One table of students beats another if they can win with fewer colours. This artistic problem also boasts some beautiful proofs to give an upper and lower bound.



Grade 8

Heilbronn Triangles in the Unit Square

Hans Heilbronn, c.1950



Ed Pegg Jr suggested the Heilbronn triangle problem: "For points in a square of side length one, find the three points that make the triangle with minimal area. Finding the placement of points that produces the largest such triangle is known as the Heilbronn triangle problem."

This problem could be developed either for grade 3 (concentrating on area), but it is going to be even more suitable for higher grades (concentrating on the formula for area and Pick's theorem). In either case a discrete version should be presented to the class... For example, to the left is a solution for the 7-point problem in a 5x5 square. How about 9 points in a 9x9 square below? Pick's theorem is marvelous in class - either as an accessible proof, or as an algebraic exercise where the theorem is given with missing variables. This unsolved problem may be just the excuse we need to promote Pick's theorem.

This problem also includes Dudeney's No-three-In a line problem.

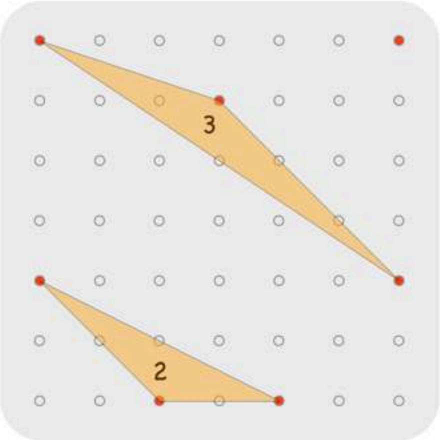
This problem is an ideal vehicle to introduce Pick's theorem. When I introduce Pick's theorem to students learning algebra, I first tell students that their may be a relationship of the form:

Area of quadrilaeral =
a*(# of lattice points on the perimeter) + b*(# of lattice points on the interior) + c.

... and ask them to find a, b and c if this is true. In fact this is true for simple (non-intersecting) polygons so my suggestion was incorrect and purposely misleading. Proving Pick's theorem is not too difficult and probably belongs in the K-12 mathematical experience.

Alternatively...

Area of polygon =
a*(# of lattice points on the perimeter) + b*(# of lattice points on the interior) + c*(# of verticies) + d.

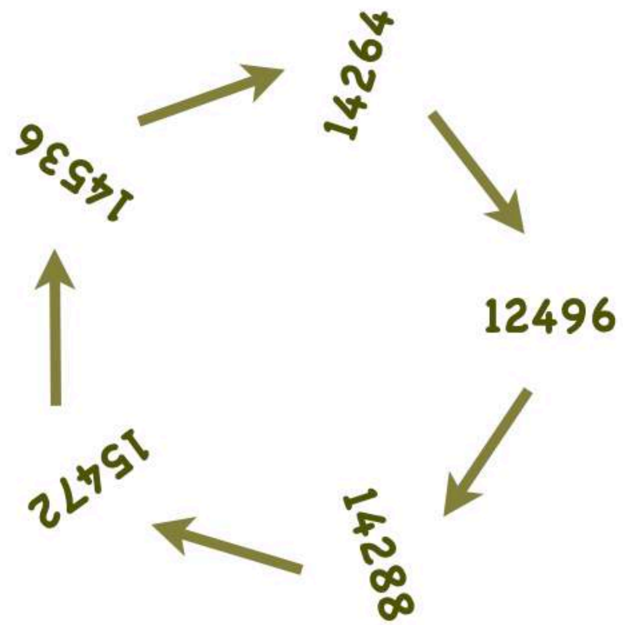


Grade 9

Perfect, Amicable and Sociable Numbers

Poulet, 1918

Perfect and amicable numbers have been known and celebrated since Classical times, but Sociable numbers were only discovered in 1918. Take a number n. Add its proper divisors. Repeat ad infinitum. If this iteration results in a cycle of period 1; you're sitting on a perfect number (6, 28, 496). If a cycle has period 2; you're sitting on amicable numbers ({220,284}, {1184, 1210}, {2620, 2924}). If a cycle has length more than 2; you're sitting on sociable numbers ({12496, 14288, 15472, 14536, 14264}, {14316, 19116, 31704, 47616, 833328, 177792, 295488, 629072, 589786, 294896, 358336, 418904, 366556, 274924,275444, 243760, 376736, 381028, 285778, 152990, 122410, 97946, 48976, 45946, 22976, 22744, 19916, 17716}). RKG Unsolved Problems in Number Theory presents many relevant unsolved problems.



Grade 9

Building with 1s

Richard Guy & John Conway, 1962

The class is split in two. Both halves choose numbers and see which half can reach them faster. One group can use 1s, brackets, addition and multiplication. The other half can use 1s, brackets, addition and powers, but not multiplication.

$$(1+1)(1+1+1+1+1+1+1)^{(1+1)}+1+1$$

$$(1+1+1+1+1+1)^{(1+1)}+(1+1+1+1)^{(1+1+1)}$$

$$((1+1+1)^{(1+1)}+1)^{(1+1)}$$

For example, a group of three students in the powers group tried to reach 100. The upper left one failed because it uses multiplication. The other two succeed, but the one on the bottom left is better because it uses only eight 1s. The multiplication half of the class cannot get to 100 in eight steps, so 100 is won by the powers group. 95 is won by the multiplication group. What is the largest number the class can find that is winnable by the multiplication group?

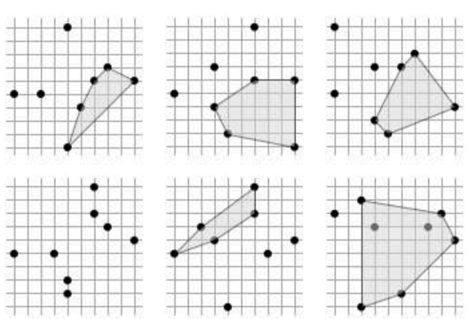
There are unsolved problems pertaining to the half of the class working with multiplication, but the use of using powers and not multiplication has not been rigorously investigated. Some related sequences in the OEIS are A003037, A005421, and A005520.

Grade 10

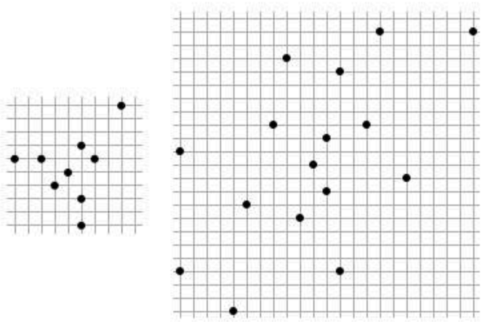
Convex Polygons

Erdös & Szekeres, 1935

What is the fewest number of points in general position (no 3 on a line) so that an n sided convex polygon is guaranteed by choosing a subset of the points as vertices. In 1935 Erdös & Szekeres conjectured that the answer was $2^{n-2}+1$ for $n \geq 3$. This conjecture still holds after Szekeres and Peters proved it correct for n=6 in 2006. It remains unsolved for all higher n.



Students experience this problem by attempting to find the number of points required for n=5. To add a stronger curricular link to Cartesian coordinates, the coordinates of each point must be integral and lie on one of the 100 intersections of a 10 x 10 grid.



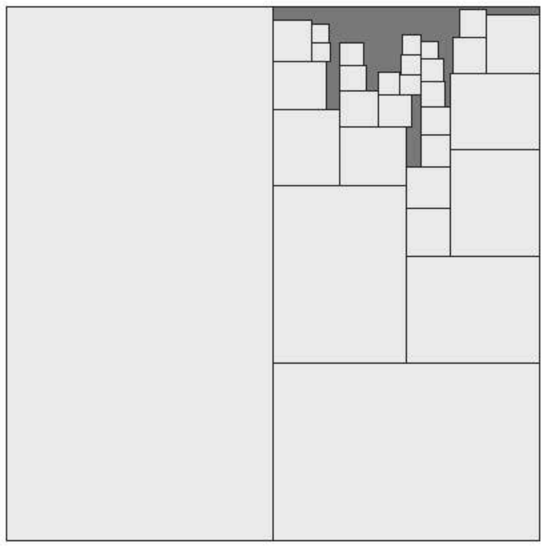
The little square on the left shows that 9 points are insufficient to guarantee that five of them form the vertices of an empty convex pentagon ("empty" means that none of the unused points are inside). The big square on the right show that 16 points are insufficient to guarantee that six among them form the vertices of an empty convex hexagon. It is not yet known if any number is sufficient to guarantee the existence of an empty hexagon.

Grade 10-12

Rectangling the Unit Square

It's beautiful to prove $\sum_{n=1}^{\infty} \frac{1}{n} \cdot \frac{1}{n+1} = 1$

Is it then possible to tile a unit square with an infinite number of rectangles of edge lengths 1/n and 1/(n+1)? If this is impossible - which is the first rectangle that cannot be squeezed in without overlapping a previous rectangle? This problem was suggested by Joshua Zucker.

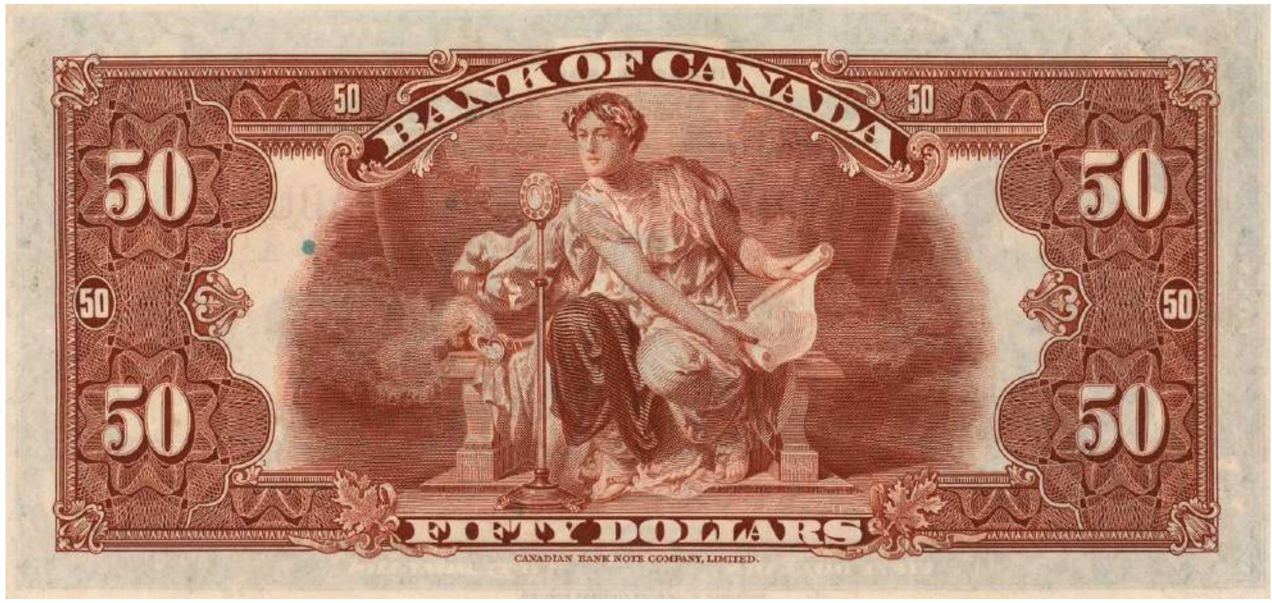


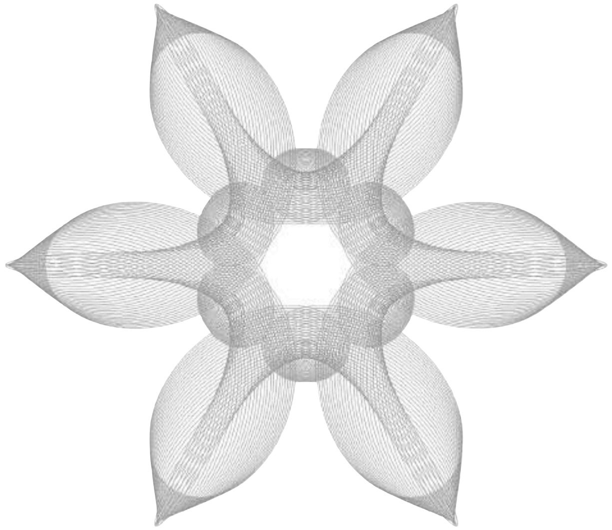
Grade 6-12

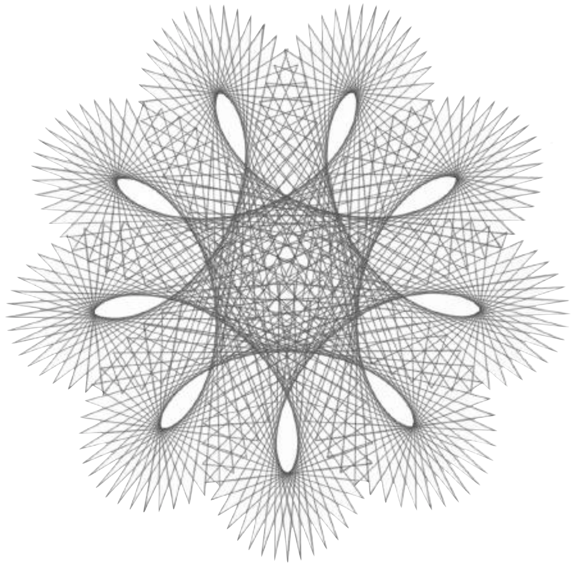
Guilloché Patterns

Guillot, c.1620

Guilloché patterns are patterns created with imbedded cogs. A simplified two-cog version is available commercially under the name “spirograph”. There is no known way to efficiently reconstruct the cogs that were used to create a Guilloché pattern - hence their traditional use on paper money to prevent forgery. This problem could be used with grade 6 students using spirograph and exploring the relationship between the number of teeth on the cogs and the number of revolutions required to complete a pattern. It could also be used with high school students studying polar coordinates. Ed Pegg Jr suggested this unsolved problem.







Mathematical Games K-12 Conference

This game actually won at the conference, but later it was decided to organize a third conference: Math Games K-12. This game is a probable winner.

Grade 3

A little bit of Aggression

Eric Solomon, 1973

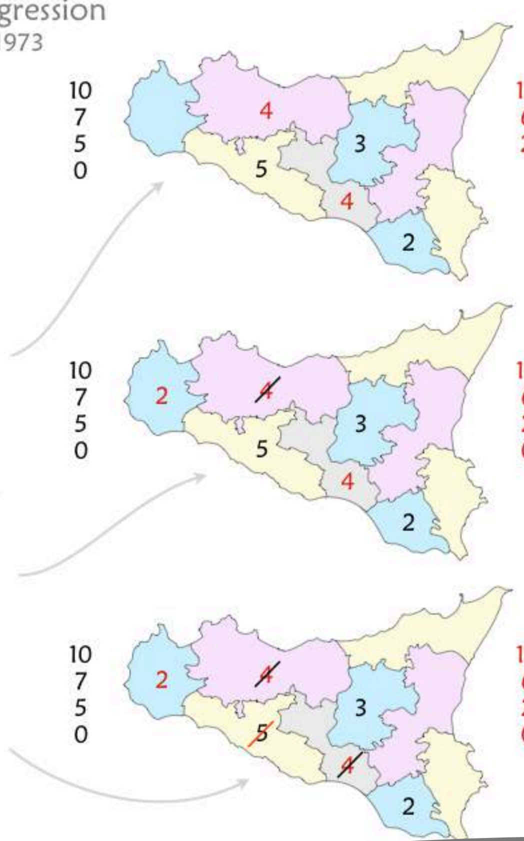
A little bit of aggression is a two player game. These rules are adapted to the grade 3 classroom from Eric Solomon's original game.

Set-up:
Players choose a map and an equal number of armies. For example, we have chosen a map of Sicily with 10 armies each. The youngest player begins.

Placement Phase:
Players take turns choosing an empty region and placing any number of their armies into that region. Armies do not move once assigned to a region. If a player has no armies left or if there are no empty regions on the board - they pass. Placement Phase continues until both players pass. Example: Black has finished placing all his armies and will pass on her next turn. Red has two armies left to place.

Attacking Phase:
The player who passed first in the Placement Phase begins. Players alternate selecting an enemy region and counting all of their neighbouring armies. If their combined strength is greater than the number of armies in the enemy region, the enemy armies are all destroyed. Friendly armies lose nothing. Continue until no more fighting is possible. Example: Black destroys one of the red armies of strength 4 because 3 + 5 > 4. Red will next destroy the black army of strength 5 because 2 + 4 > 5.

Scoring:
The player who controls the most regions wins.
In the case of a tie, count the armies - the winner is the person with the most.
Example: Black controls two regions - Red controls one. Black wins.



This excellent game will be presented at an upcoming conference: Math Games K-12.

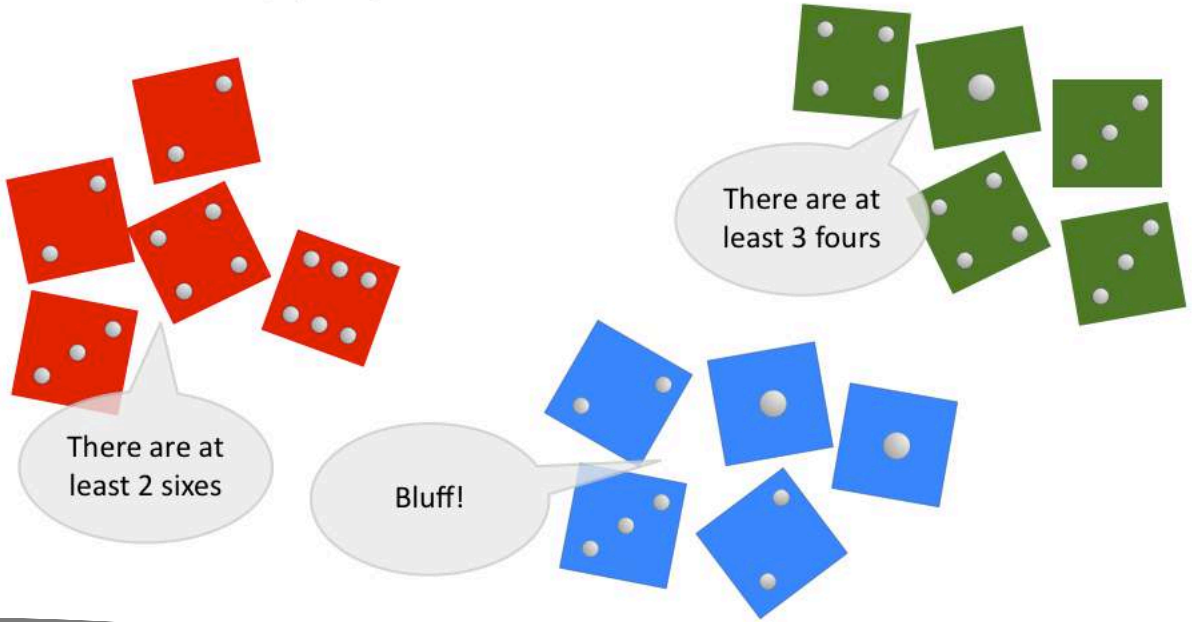
Grade 5

Perudo

Atahualpa and Pizarro, 1530s

The game perudo is one of the classical gems of game design. It is a joyous way to give students an intuition about probability before they study it. Perudo is also a cheap game needing only dice and cupped hands.

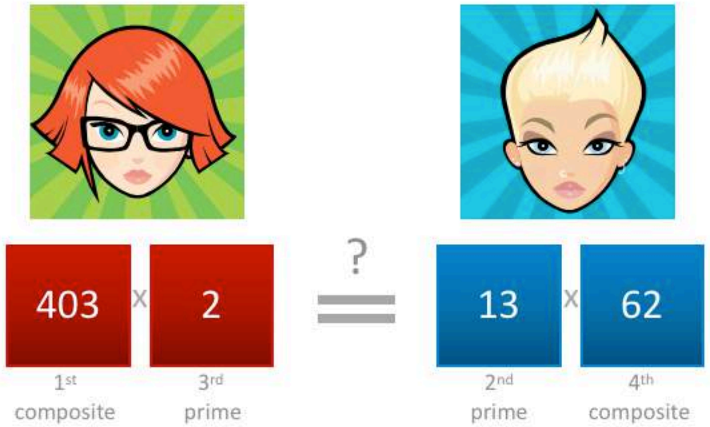
Games are a joyous excuse for problem solving, so they deserve to be featured in the thirteen. Few games are as cheap and indestructible as Perudo. If there were no constraint then Kris Burm's masterpieces Dvonn and Tzaar would deserve consideration along with Go, Reiner Knizia's Lost Cities, Antoine Bauza's Hanabi, and any top ranked game on BoardGameGeek.com



Grade 6

Composite first vs. Prime first

Rivest, Shamir & Aldeman, 1978



Left team must choose a composite number first. Right team then chooses a prime. Left team chooses a prime. Right team chooses a composite. Right team wins if the product of their two numbers equals the product of left's two numbers. For initial games, left team should be limited to choosing numbers less than 12.

If right can reliably beat left with reasonable constraints this is the equivalent of cracking the RSA cipher. Ed Pegg prefers a block of 50 factoring challenges like the following rather than requesting the cracking of RSA:

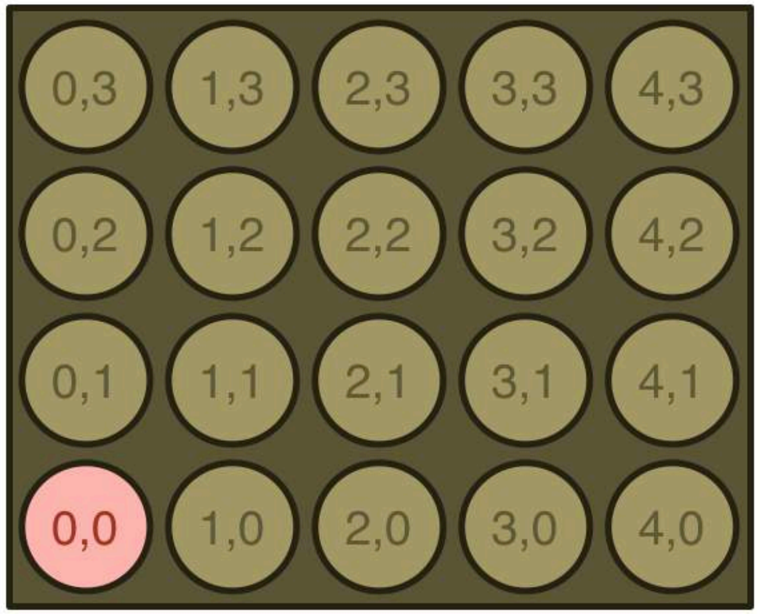
First n digits of Euler's constant for n=226, 268, 288
First n digits of Pi for n=270, 300, 402, 418
First n digits of Golden Ratio for n=240, 262, 269, 278
First n digits of sqrt(2) for n = 298, 299, 301, 308, 347
2^n + 3 for n = 846, 888, 951, 1060, 1072
3^n+2 for n = 554, 571, 590, 638, 655, 674, 686
10^n+3 for n= 200, 201, 270, 281, 287, 290
10^n+7 for n=216, 217, 234, 236, 246, 265, 268, 269

Grade 10

Chomp!

Frederick Schuh, 1952

This two player game is used to introduce Cartesian coordinates. The first player can always win if they play well, but the strategy is unknown except in special circumstances. Players take turns choosing one segment of chocolate and eating all the segments to the North and East. The player who chooses the poisonous red piece loses the game.



Schedule

Friday February 27

19:00 - Informal socializing, games and puzzles in the BIRS lounge.

Saturday February 28

9:00 - 9:20 Presentation by BIRS about housekeeping, checking-out, and videoing instructions.

9:20 - 10:00 introductions and (optional) one favourite integer sequence.

10:00 - 10:25 Max Alekseyev talks about the Online Encyclopedia of Integer Sequences (OEIS) and his own story.

10:25 - 10:45 Coffee Break

10:45 - 11:45 Joshua Zucker leads

11:45 - 12:00 Richard Guy "The leaning tower of Pascal"

12:00 - 13:15 Lunch

13:15 - 14:15 Amanda Serenevy leads

14:15 - 14:30 Henri Picciotto's favourite integer sequences

14:30 - 14:45 Coffee Break

14:45 - 15:30 Paulino Preciado (educator) and Vincent Chan (mathematician) lead

15:30 - 15:45 Richard Guy "Euler and the Law of Small Numbers."

15:45 - 16:45 Lora Saarnio (educator) and Michael Cavers (mathematician) lead

Sunday March 1st

8:30 - 9:30 Zaak Robichaud (educator) Veso Jungic (mathematician) lead

9:30 - 10:45 Rakhee Vijairaghava (educator) and Tom Edgar (mathematician) close the conference

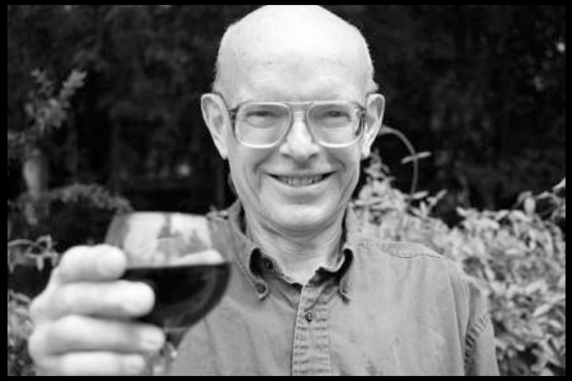
10:45 - 11:30 Coffee Break and getting stuff out of rooms and into cars or the BIRS lounge.

11:30 - 12:00 Farewell lunch in Vistas dining room

Julia Robinson Mathematics Festival

12:10 - those attending the Julia Robinson Mathematics Festival should leave for Calgary.

14:00 - 17:00 Julia Robinson Mathematics Festival is being held at Mount View School: 2004 - 4 Street NE, Calgary AB T2E 3T8



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