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a(n,k) tabf head (staircase) for A048996

M0 (or M_0) multinomial numbers for partitions of n in Abramowitz-Stegun (A-St) order.

M0([a_1,...,a_n]) = sum(a_j,j=1..n)!/product((a_j)!,j=1..n) = m!/product((a_j)!,j=1..n).

The row number is n, and m is the number of parts of a partition of n.

k numbers the partitions in A-ST order from 1 to p(n) = A000041(n).

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n\k 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42
1:  1
2:  1 1
3:  1 2 1
4:  1 2 1 3 1
5:  1 2 2 3 3 4 1
6:  1 2 2 1 3 6 1 4 6  5  1
7:  1 2 2 2 3 6 3 3 4 12  4  5 10  6  1
8:  1 2 2 2 1 3 6 6 3  3  4 12  6 12  1  5 20 10  6 15  7  1
9:  1 2 2 2 2 3 6 6 3  3 6 1 4 12 12 12 12  4  5 20 10 30  5  6 30 20  7  21  8  1
10: 1 2 2 2 2 1 3 6 6  6  3  6  3  3..4 12 12  6 12 24  4  4  6  5 20 20 30 30 20  1  6 30 15 60 15..7 42 35  8 28  9  1
 .
 .
 .
n\k 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42
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The sequence of row lengths is A000041: [1, 2, 3, 5, 7, 11, 15, 22, 30, 42,...] (partition numbers).

One could add the row for n=0 with a 1, if the part 0 is considered for n=0, and only for this n.

For the ordering of this tabf array a(n,k) see the Abramowitz-Stegun ref., pp. 831-2.

E.g. a(5,4) refers to the fourth partition of n=5 in this ordering, namely (1^2,3^1) = [1,1,3], whence a(5,4)=3

because (2+1)!/(2!*1!)=3. Or a(6,5) = 3 for the partition (1^2,4^1)=[1,1,4] of n=6 from the same computation.

a(7,10) = 12 from the 10th partition of n=7 which his (1^2,2^1,3^1)=[1,1,2,3] with (2+1+1)!/(2!*1!*1!) = 12

Changed May 03 2007: a(n,m) into a(n,k).

Added May 03 2007:
The coefficients a(n,k) of row n appear in the calculation of the (1+sum(f[j]*x^j))^p as coefficients of x^n as follows:
[x^n]((1+sum(f[j]*x^j))^p) = sum(sum(binomial(p,m)*M0(n,a_1,..,a_n)*product(f[j]^a_j,j=1..n),(a_1,..,a_n) from
Pa(n,m)),m=1..min{n,p}) with m:=sum(a_j,j=1..n) and Pa(n,m) the set of partitions of n with m parts written in
exponential form (1^a_1,...,n^a_n). If a_j=0 then j is not recorded. M0(n,a_1,...,a_n):=m!/product(a_j,j=1..n) are
the numbers given as a(n,k) above if k is the k-th partition in A-St order.
Example: n=4, p=2: partitions m=1: (4^1), m=2: (1^1,3^1) and (2^2).
[x^4]((1+sum(f[j]*x^j))^2) = binomial(2,1)*(1!/1!)*f_4 +binomial(2,2)((2!/(1!*1!))*f_1*f_3 + (2!/2!)*(f_2)^2) =
2*f_4 + 2*f_1*f_3 + (f_2)^2.
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Added Oct 12 2007:
These M0 (or M_0) numbers for partitions (in short M0 partition numbers) are identical with the M_1 (or M1) partition
numbers for the exponents of the partitions of n, read as partitions of m (the part number).
See A036038 for the M_1 number array.
The M0 numbers enter in the calculation of [x^n] A(x)^m, with an o.g.f. A(x) (ordinary convolutions).
For A(x)=1/(1-x) the M0 numbers appear directly, and the sum over all M0 numbers for fixed part number m
is binomial(n-1,m-1) (Pascal triangle).
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Added May 30 2018:
The M0 numbers a(n, k) give the number of compositions with parts corresponding to those of the k-th partition of n
in A-St order. It is clear that only the exponents of the partitions (also called signature of the partitions) are
important for a(n, k). These exponents are given in A115621 in A-St order.
E.g., n = 5, k = 4, for the partition (1^2,3^1) = [1,1,3] of 5 with a(n, k) = 3!/(2!*1!) = 3, corresponding to the
three compositions: 1,1,3; 1,3,1; and 3,1,1. Therefore one could call these multinomial numbers also composition
numbers.

There is a bijection between compositions and set partitions with blocks of consecutive numbers.
The compositions of n give the cutting prescription for the partition of the set [n] :={1,2, ..., n}; and vice versa,
a given set partition with blocks of consecutive numbers give the composition by recording the number of elements.
E.g., n = 5. k = 4: the three compositions from above yield the consecutive 3-set partition of [5], namely
{1},{2},{3,4,5} and {1},{2,3,4},{5} and {1,2,3},{4},{5}, respectively. Conversely, these set partitions give the

three compositions by recording the number of elements of the subsets of [5] respecting the order.

The partition n=5, k = 5, (1^1,2^2) = [1,2,2] has the same exponents (signature), hence a(5, 5) = a(5, 4) = 3,
and the compositions are 1,2,2; 2,1,2; and 2,2,1. The consecutive 3-subset partition of [5] is {1},{2,3},{4,5},
and {1,2},{3},{4,5} and {1,2},{3.4},{5}, respectively. Conversely, these subsets give directly the compositions.
e.o.f.##