Using GENERATINGFUNCTIONOLOGY to Enumerate Distinct-Multiplicity Partitions

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Last Update of this page (but not article): Sept. 11, 2012 [Thanks to Daniel Kane and Robert Rhoades]

Previous Updates: Feb. 1, 2012 [thanks to Daniel Kane], Feb. 4, 2012 (page and article) [Thanks to Vaclav Kotesovec], Feb. 8, 2012 [Thanks to Mark Ward]

In Fond Memory of

Herbert Saul WILF (June 13, 1931- Jan. 7, 2012),

Who asked so many good questions,

and knew so well what is an ANSWER

לעלוי נשמתו של

הרברט שאול וילף

(כ"ח סיון ה'תרצ"א-י"ב טבת ה'תשע"ב)

זכר גאון לברכה

ששאל שאלות כל כך טובות

וידע היטב מה זאת תשובה

Added Feb. 1, 2012: Daniel Kane seems to have found the true asymptotics of log f(n), see the end of the current version of the article. As soon as Daniel would write it up, I will put a link to it here. Meanwhile, he kindly allowed me to post his Email message .

Added Feb. 4, 2012: According to Vaclav Kotesovec's numerics, Daniel Kane's proposed asymptotics does not seem to quite fit, as stated (but the $n^{1/3}$ part may still be right). See Vaclav Kotesovec's graph .

Added Feb. 8, 2012: Mark Ward, in collaboration with Jim Fill and Svante Janson also found (private communication, see Mark Ward's Email message) the same asymptotics as Daniel Kane (so it must be correct, apparently 508 terms do not suffice to see what is going on exactly) and are working on refined asymptotics. I hope that they would join forces, and as soon as the article is posted in the arxiv (and/or their websites), I will link to it here.

Added Sept. 11, 2012: Daniel Kane and Robert Rhoades found a more refined asymptitics!

Maple Package

- **DMP [Version of Jan. 25, 2012, adding procedure RandDMP] A Maple package for automatically generating the generating function for enumerating**

integer partitions with distinct multiplicities whose parts are drawn from any given finite set of positive integers, and for computing the beginning of the hard-to-count sequence enumerating unrestricted partitions with distinct (non-zero) multiplicity.

Web-books

- **If you want to see the first 250 terms of OEIS sequence A098859, the number of partitions of an integer n with distinct multiplicities, computed from scratch, the input file would yield the output file**

- **If you want to see the generating function, in q, a certain rational function such that when you take its Maclaurin expansion, the coefficient of q^n tells you the number of ways of making change from n cents if you are allowed any number of pennies, nickels, dimes, and quarters, such that you can't have the same number of coins of two different denominations (if they do occur in the change, of course it is OK to have, e.g., no pennies and no quarters), as well as the first 300 terms of this sequence, then the input file would yield the output file**

- **If you want to see the analog of the above but now you are allowed to use half-dollars and dollar-coins in addition to pennies, nickles, dimes, and quarters, and with the first 1000 terms of the sequence input file would yield the output file**

- **If you want to see the (rational, as it turns out, see the paper) generating functions for multiplicity-free partitions whose largest part is m, for each of 1 ≤ m ≤ 8, as well as the first 200 terms for each enumerating sequence the input file would yield the output file**

- **If you want to see the first 508 sequences of the sequence, using Maciej Ireneusz Wilczynsk's precomputed table Maciej Ireneusz Wilczynsk's precomputed table, as well as comparing them to the unrestricted partitions, and a numerical estimate of the asymptotics, the input file would yield the output file**

- **[Added Jan. 25, 2012] If you want to see ten (uniform) random distinct-multiplicity partitions of 100, using the Wilf methodology for random selection of random objects beautifully described in the Nijenhuis-Wilf classic text "Combinatorial Algorithms", the input file would yield the output file**

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