

# Euler-Pascal/Cube

- Triangles Series-ly
- Euler-Pascal/Cube
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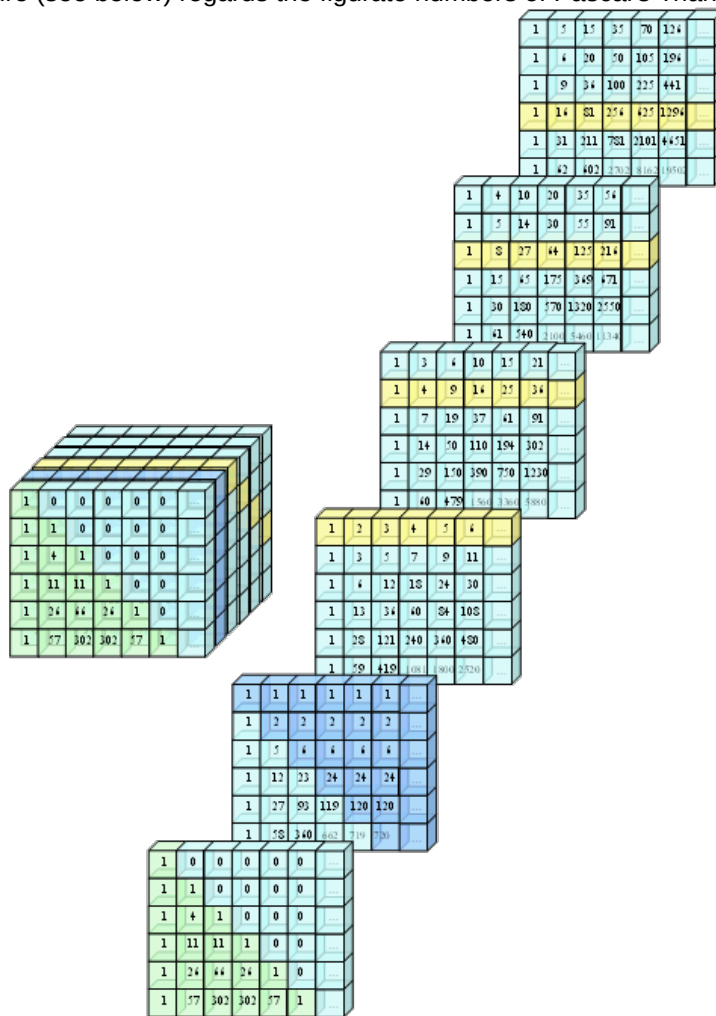
Jacob Bernoulli saw a "peculiar sympathy" in power series' inter-relationships ([Edwards](#) p. 133). The algorithm that generates formulas for power values (that is [Worpitzky's Identity of 1883](#)) logically also generates formulas for values that are summations of powers (where [Bernoulli numbers](#) come into play) as well as shells or nexus number series and more. Sympathy between and among series is 'three dimensional'! Use the [Search Page](#) (which works a little bit) to call up particular A000000 [Sloane series](#) in these pages. Worpitzky's formulaic procedure (see below) regards the figurate numbers of Pascal's Triangle along with values of rows of Euler's Triangle and may be the figurate number relationships that Fermat and Pascal were discussing for sums of powers ([Pengelly](#) p. 4). For more: [Pascal and Fermat, Historically](#).

## Euler/Pascal Cube Pictured (and its formulas)

### The Relatedness of Numbers (including Fermat's Last Theorem)

### The Euler Triangle and Figurate Number Recursion

### The Many Figurate Renditions of a Power Series' Value



SUMMARY OF THE EULER/PASCAL CUBE:

- 1) The (Worpitzky/Euler/Pascal Cube) "SeriesAtLevelR" algorithm is:  

$$\text{Sum} [\text{Eulerian}[n, i - 1] * \text{Binomial}[n + x - i + r, n + r], \{i, 1, n\}]$$
 Offset: (1,1, 0-->relative to the powers) for (x, n, r)  
 FOR POSITIVE INTEGERS

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Due to figurate number content within recursively accumulating series that exist for each power level, Worpitzky's identity of 1883—which is based upon figurate numbers—not only will define values that are power values, but also will logically apply to sequences that are shells (of shells) [i.e., nexus numbers or difference sequences of powers] as well as summations (of summations) of power series.  
 The "SeriesAtLevelR" algorithm is Worpitzky's ID at  $r = 0$  and defines nexus number/shell values when  $r < 0$  and summations (of summations . . .) of powers when  $r > 0$ .

2) The (Worpitzky/Euler/Pascal Cube) "MagicNKZ" algorithm is:  
 Sum  $[(-1)^j \cdot \text{Binomial}[n + 1 - z, j] \cdot (k - j + 1)^n, \{j, 0, k+1\}]$   
 Offset: (0,1,0) for (k, n, z)

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In order to generate sequences that are Euler Triangle rows (see [A008292](#)) or accumulations from Euler Triangle rows, values of Euler/Pascal cube series are defined with the variables of  $n$ th power level,  $k$ th order of occurrence and  $z$ th accumulation level. The generating algorithm is based upon binomial definitions for Euler Triangle values. Rows of Euler's Triangle are given when  $z = 0$ . The number of summations from an initial Euler Triangle row is "enumerated" by the value that is  $z$ .

"SeriesAtLevelR" and "MagicNKZ" produce logically reciprocating formulas when solving for either  $x$  and  $k$  and/or  $r$  and  $z$  variables since the differently generated equations describe the same Euler/Pascal Cube series (and logically inform each other).

## Formulas within the Cube:





### [A Mathematica Notebook of Formulas](#)

#### Two-Variable Equations



-  [SeriesAtLevelR](#)
-  [MagicNKZ](#)

#### Single-Variable Equations in concise equation layouts with depictions of series and with links to Sloane's On-line Encyclopedia of Integer Sequences


Parallel (same) sequences:

-  [MagicNKZ definition of z in tables of equations](#) AND
-  [SeriesAtLevelR definition of r in tables of equations](#)
  
-  [MagicNKZ definition of k in tables of equations](#) AND
-  [SeriesAtLevelR definition of x in tables of equations](#)

Unrelated (different) sequences:

-  [SeriesAtLevelR definition of n in tables of equations](#)
-  [MagicNKZ definition of n in tables of equations](#)

#### Matrices of numbers/values

-  [SeriesAtLevelR](#) (bitmaps)
-  [MagicNKZ](#) (tables of numbers)

[Euler/Pascal Cube Slide Show](#)

More Single-Variable equations  
[SeriesAtLevelR](#)  
[MagicNKZ](#)  
 Yet more single variable equations  
[SeriesAtLevelR](#)  
[MagicNKZ](#)



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