

A triangle for calculating A126156.

Peter Bala, April 24, 2017

Oste and van der Jeugt [1, Section 7] show that a continued fraction of the form

$$\frac{1}{1 - xd_0 - \frac{xh_1}{1 - xd_1 - \frac{xh_2}{1 - xd_2 - \frac{xh_3}{1 - xd_3 - \dots}}}} \quad (1)$$

is the generating function for 2-Motzkin paths weighted by the integers d_i and h_i . This combinatorial interpretation allows one to rapidly calculate the terms of a sequence whose generating function can be expressed as a continued fraction of the form (1). The results are conveniently displayed in the form of a lower triangular array, where the d 's occur as multiplication factors along diagonals of the array and the h 's as horizontal multiplication factors along rows of the array. In the particular case of A126156, Gladkovskii states that the generating function can be expressed as the continued fraction

$$1/(1 - x/(1 - 6x/(1 - 15x/(1 - 28x/(1 - \dots - n(2n - 1)x/(1 - \dots)))))).$$

So in this case the d 's are all zero and the horizontal multiplication factors are given by the formula $h_n = n(2n - 1)$. A126156 will then be the leading diagonal of the following lower triangular array:

1									
↓									
1	— x1 —>	1							
↓		↓							
1	— x6 —>	7	— x1 —>	7					
↓		↓		↓					
1	— x15 —>	22	— x6 —>	139	— x1 —>	139			
↓		↓		↓		↓			
1	— x28 —>	50	— x15 —>	889	— x6 —>	5473	— x1 —>	5473	
↓		↓		↓		↓		↓	⋮

References

- [1] R. Oste and J. Van der Jeugt, Motzkin paths, Motzkin polynomials and recurrence relations, *Electronic Journal of Combinatorics* 22(2) (2015), #P2.8. Section 7