A196050(n) = a(n)

Complete Prime function reduction

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Prime function prime(.) = A000040(.) := {.} (curly bracket notation). E.g,. {4} = 7.

Complete prime function reduction cpfr(n), for n = 2, ..., 100.

E.g., n = 7, 7 = prime(4), 4 = 2^2, 2 = prime(1), and there is no further reduction possible. Hence

cpfr(7) = prime(prime(1)*prime(1)) = prime(prime(1)^2) = {{1}^2} in curly bracket notation.

a(n) = A196050(n) (Matula-Göbel numbers) equals the number of pairs of curly brackets, i.e. the number of operations of the prime function prime(.).

E.g., a(7) = 3.

b(n) = A001222(n), the number of factors, i.e., the number of prime divisors of n counted with multiplicities.

E.g., b(7) = 1.

For the obvious rooted tree interpretation see Figure 2 of the F. Göbel reference [JCT, B29 (1980) 141-143]. There the number of vertices, including the root, are given for n=1..45. These numbers are a(n) + 1.

n	cpfr(n)	a(n)	b(n)
2	{1}	1	1
3	{{1}}	2	1
4	{1}^2	2	2
5	{{{1}}}	3	1
6	{1}*{{1}}	3	2
7	{{1}^2}	3	1
8	{1}^3	3	3
9	{{1}}^2	4	2
10	{1}*{{{1}}}	4	2

11	{{{{1}}}}	4	1
12	{1}^2*{{1}}	4	3
13	{{1}*{{1}}}	4	1
14	{1}*{{1}^2}	4	2
15	{{1}}*{{{1}}}	5	2
16	{1}^4	4	4
17	{{{1}^2}}	4	1
18	{1}*{{1}}^2	5	3
19	{{1}^3}	4	1
20	{1}^2*{{{1}}}	5	3
21	{{1}}*{{1}^2}	5	2
22	{1}*{{{{1}}}}	5	2
23	{{{1}}^2}	5	1
24	{1}^3*{{1}}	5	4
25	{{{1}}}^2	6	2
26	{1}*{{1}*{{1}}}	5	2
27	{{1}}^3	6	3
28	{1}^2*{{1}^2}	5	3
29	{{1}*{{{1}}}}	5	1
30	{1}*{{1}}*{{{1}}}	6	3
31	{{{{{1}}}}}	5	1
32	{1}^5	5	5
33	{{1}}*{{{{1}}}}	6	2
34	{1}*{{{1}^2}}	5	2
35	{{{1}}}*{{1}^2}	6	2
36	{1}^2*{{1}}^2	6	4
37	{{1}^2*{{1}}}	5	1
38	{1}*{{1}^3}	5	2
39	{{1}}*{{1}*{{1}}}	6	2
40	{1}^3*{{{1}}}	6	4
41	{{{1}*{{1}}}}	5	1

42	{1}*{{1}}*{{1}^2}	6	3
43	{{1}*{{1}^2}}	5	1
44	{1}^2*{{{{1}}}}	6	3
45	{{1}}^2*{{{1}}}	7	3
46	{1}*{{{1}}^2}	6	2
47	{{{1}}*{{{1}}}}	6	1
48	{1}^4*{{1}}	6	5
49	{{1}^2}^2	6	2
50	{1}*{{{1}}}^2	7	3
51	{{1}}*{{{1}^2}}	6	2
52	{1}^2*{{1}*{{1}}}	6	3
53	{{1}^4}	5	1
54	{1}*{{1}}^3	7	4
55	{{{1}}}*{{{{1}}}}	7	2
56	{1}^3*{{1}^2}	6	4
57	{{1}}*{{1}^3}	6	2
58	{1}*{{1}*{{{1}}}}	6	2
59	{{{{1}^2}}}	5	1
60	{1}^2*{{1}}*{{{1}}}	7	4
61	{{1}*{{1}}^2}	6	1
62	{1}*{{{{{1}}}}}	6	2
63	{{1}}^2*{{1}^2}	7	3
64	{1}^6	6	6
65	{{{1}}}*{{1}*{{1}}}	7	2
66	{1}*{{1}}*{{{{1}}}}	7	3
67	{{{1}^3}}	5	1
68	{1}^2*{{{1}^2}}	6	3
69	{{1}}*{{{1}}^2}	7	2
70	{1}*{{{1}}}*{{1}^2}	7	3
71	{{1}^2*{{{1}}}}	6	1
72	{1}^3*{{1}}^2	7	5

73	{{{1}}*{{1}^2}}	6	1
74	{1}*{{1}^2*{{1}}}	6	2
75	{{1}}*{{{1}}}^2	8	3
76	{1}^2*{{1}^3}	6	3
77	{{1}^2}*{{{{1}}}}	7	2
78	{1}*{{1}}*{{1}*{{1}}}	7	3
79	{{1}*{{{{1}}}}}	6	1
80	{1}^4*{{{1}}}	7	5
81	{{1}}^4	8	4
82	{1}*{{{1}*{{1}}}}	6	2
83	{{{{1}}^2}}	6	1
84	{1}^2*{{1}}*{{1}^2}	7	4
85	{{{1}}}*{{{1}^2}}	7	2
86	{1}*{{1}*{{1}^2}}	6	2
87	{{1}}*{{1}*{{{1}}}}	7	2
88	{1}^3*{{{{1}}}}	7	4
89	{{1}^3*{{1}}}	6	1
90	{1}*{{1}}^2*{{{1}}}	8	4
91	{{1}^2}*{{1}*{{1}}}	7	3
92	{1}^2*{{{1}}^2}	7	3
93	{{1}}*{{{{{1}}}}}	7	2
94	{1}*{{{1}}*{{{1}}}}	7	3
95	{{{1}}}*{{1}^3}	7	2
96	{1}^5*{{1}}	7	6
97	{{{{1}}}^2}	7	1
98	{1}*{{1}^2}^2	7	3
99	{{1}}^2*{{{{1}}}}	8	3
100	{1}^2*{{{1}}}^2	8	4

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