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Change the Dime not the Dollar!

Change. Not evolution, but change as in coinage. Do you like carrying around lots of coins? Ever bought something dreading the burden of more coins? Starting to jingle like Santa? Oh, how I hate having to lean on my left hip socket counterbalancing the weight in my right hip pocket.

A Congressional proposal suggests that we make change with a dollar coin rather than a dollar bill. Ugh! More coins in my pocket! Hollywood will have to make a sequel to "Three Coins in the Fountain" aptly named "**Four** Coins in the Fountain". Perhaps you realize I am not a fan of this idea.

To further illustrate my feelings about coins, consider what happened recently when I went to my local Five and Dime and bought a 95 cent pen. With tax, the pen cost $1.01. I gained all the weight I lost on my diet when the clerk counted back 99 cents change. I was unhappy as I received the first quarter, turned sad as I got the next, grew morose as I got the third, was disheartened when the first dime dropped into my palm clanging with the quarters, became depressed when the second dime came my way, got mad when I saw the penny, bellowed at the second offensive penny, ranted about the devils at the mint when I felt the third, and raged at those founders of our current coinage when the final penny fell to the floor, falling off the pile of money lying in my hand. Has this ever happened to you?

Maybe this transaction got me thinking or maybe it was the full moon, but I wondered - is there a better coinage system? Is there a configuration of 4 coins which minimizes the amount of change I get back from my purchases?

To explore that idea, I made a table of our current coinage system, listing the number of coins I get back when I receive a certain amount of change.

Here is that table:

Amount of Change	Number of Coins	Amount of Change	Number of Coins	Amount of Change	Number of Coins	Amount of Change	Number of Coins	Amount of Change	Number of Coins
1 cent	1	21 cents	3	41 cents	4	61 cents	4	81 cents	5
2 cents	2	22 cents	4	42 cents	5	62 cents	5	82 cents	6
3 cents	3	23 cents	5	43 cents	6	63 cents	6	83 cents	7
4 cents	4	24 cents	6	44 cents	7	64 cents	7	84 cents	8
5 cents	1	25 cents	1	45 cents	3	65 cents	4	85 cents	4
6 cents	2	26 cents	2	46 cents	4	66 cents	5	86 cents	5
7 cents	3	27 cents	3	47 cents	5	67 cents	6	87 cents	6
8 cents	4	28 cents	4	48 cents	6	68 cents	7	88 cents	7
9 cents	5	29 cents	5	49 cents	7	69 cents	8	89 cents	8
10 cents	1	30 cents	2	50 cents	2	70 cents	4	90 cents	5
11 cents	2	31 cents	3	51 cents	3	71 cents	5	91 cents	6
12 cents	3	32 cents	4	52 cents	4	72 cents	6	92 cents	7
13 cents	4	33 cents	5	53 cents	5	73 cents	7	93 cents	8
14 cents	5	34 cents	6	54 cents	6	74 cents	8	94 cents	9
15 cents	2	35 cents	2	55 cents	3	75 cents	3	95 cents	5
16 cents	3	36 cents	3	56 cents	4	76 cents	4	96 cents	6
17 cents	4	37 cents	4	57 cents	5	77 cents	5	97 cents	7
18 cents	5	38 cents	5	58 cents	6	78 cents	6	98 cents	8
19 cents	6	39 cents	6	59 cents	7	79 cents	7	99 cents	9
20 cents	2	40 cents	3	60 cents	3	80 cents	4		

Total number of coins = 470

Making this table was easy because of the repeating pattern which occurs. Also, our current coinage system makes counting change simple because you always use the largest coin to exhaustion before turning to the next smallest coin. Exhausting that coin you keep turning to the next and to the next if possible. For example, when counting out 66 cents change, you use a quarter first. Realizing that there is 41 cents remaining, you give another quarter. Now that 16 cents remains you turn to a dime. Down to 6 cents you give a nickel and finally a penny for a total of 5 coins returned. Could I improve on that?

I made a conjecture. Suppose our coins were 1 cent, 7 cents, 15 cents, and 30 cents. Let's try to make 66 cents change. Using two 30 cent pieces gets you to 60 cents and since the 15 and 7 cent pieces are too big, you have to return 6 pennies. A total of 8 coins. This isn't better! However, there is a problem with the method of exhaustion! The best way to make 66 cents change with this set of four coins is to use one 30 cent piece, one 15 cent piece and three 7 cent pieces. A total of 5 coins. Still not better than our current system, but an enlightening thought. With certain sets of 4 coins, the method of exhaustion doesn't always produce the optimal coin return combination for a given amount of change.

This particular change amount (66 cents) didn't deflate my hope that the 1, 7, 15, 30 cent coinage set would still be better than our current system. So I made another chart.

Amount of Change	Number of Coins	Amount of Change	Number of Coins	Amount of Change	Number of Coins	Amount of Change	Number of Coins	Amount of Change	Number of Coins
1 cent	1	21 cents	3	41 cents	6	61 cents	3	81 cents	5
2 cents	2	22 cents	2	42 cents	6	62 cents	4	82 cents	4
3 cents	3	23 cents	3	43 cents	5	63 cents	5	83 cents	5
4 cents	4	24 cents	4	44 cents	3	64 cents	6	84 cents	6
5 cents	5	25 cents	5	45 cents	2	65 cents	6	85 cents	7
6 cents	6	26 cents	6	46 cents	3	66 cents	5	86 cents	8
7 cents	1	27 cents	7	47 cents	4	67 cents	3	87 cents	8
8 cents	2	28 cents	4	48 cents	5	68 cents	4	88 cents	6
9 cents	3	29 cents	3	49 cents	6	69 cents	5	89 cents	5
10 cents	4	30 cents	1	50 cents	6	70 cents	6	90 cents	3
11 cents	5	31 cents	2	51 cents	4	71 cents	7	91 cents	4
12 cents	6	32 cents	3	52 cents	3	72 cents	7	92 cents	5
13 cents	7	33 cents	4	53 cents	4	73 cents	6	93 cents	6
14 cents	2	34 cents	5	54 cents	5	74 cents	4	94 cents	7
15 cents	1	35 cents	5	55 cents	6	75 cents	3	95 cents	7
16 cents	2	36 cents	4	56 cents	7	76 cents	4	96 cents	6
17 cents	3	37 cents	2	57 cents	7	77 cents	5	97 cents	4
18 cents	4	38 cents	3	58 cents	5	78 cents	6	98 cents	5
19 cents	5	39 cents	4	59 cents	4	79 cents	7	99 cents	6
20 cents	6	40 cents	5	60 cents	2	80 cents	7		

Total number of coins = 450

Surprise! This system is better on the whole than our current system. But is it the best??

Spurred by this success, I started to list all the possible 4 coin systems and tried to figure out the total number of coins returned. Once I started to list all of those 4 coin possibilities (1,2,3,4 - 1,2,3,5 - 1,2,3,6 - 1,2,3,7 - .... 1, 97, 98, 99) I became discouraged by the monumental task.

Then I remembered one of my most valuable resources - my students!! Excited about this problem, I presented seniors Jeff Greenfield, David Raabe and sophomore Joe Culbert with the task of writing a computer program to find the best 4 coin system. Two days later, they had written the following Turbo Pascal program.

```
program coin;

const       highest_number_of_coins_returned = 100;
```

```
var
        coin1, coin2, coin3, coin4 : integer;
        count1,count2,count3,count4 : integer;
        cents : integer;
        i, j, k : integer;
        num1, num2, num3, num4, total, answer, trial, x  : integer;
        try  : array[1..12144] of integer;
        outfile : text;

begin
        assign (outfile,'info.txt');
        rewrite (outfile);

        coin1 := 1; {a 4 coin set always has a penny}

        for coin2 := coin1 + 1 to 97 do      {try all possible 4 coin sets}
                for coin3 := coin2 + 1 to 98 do
                        for coin4 := coin3 + 1 to 99 do

                        begin

                                total := 0;

                                        for cents := 1 to 99 do  {try each change amount}
                                                begin {calculate each possible combination of coins =
cents}
                                                        trial :=0;
                                                        num1 := cents;
                                                        k := num1 div coin4;

                                                        for count1 := k downto 0 do
                                                           begin
                                                            num2 := num1 - (count1*coin4);
                                                            j := num2 div coin3;

                                                        for count2 := j downto 0 do
                                                           begin
                                                            num3 := num2 - (count2*coin3);
                                                            i := num3 div coin2;

                                                        for count3 := i downto 0 do
                                                           begin
                                                            count4 := num3 - (count3*coin2);
                                                            trial := 1 + trial;
                                                            try[trial]
:=count1+count2+count3+count4;{store}

                                                        end;
                                                        end;
                                                        end;

                                                        answer := highest_number_of_coins_returned;

                                                        for x := 1 to trial do {find smallest combination}
                                                                if try[x] < answer then answer := try[x];
                                                        total := total+answer;
                                                end;

                                writeln (outfile,coin1,',',coin2:2,',',coin3:2,',',coin4:2,':',total:6);
                        end;
        close(outfile);
end.
```

The program ran for 4 hours Wednesday morning and we watched with anticipation as the numbers flew by. The best coinage system??

Before the answer - a disclaimer. We think our program has found the best system. However we have not confirmed this with another source. Also, we don't know if this is an original thought or if someone has already made this discovery. We have thought of numerous extensions of this problem (best 3 or 5 coin system, what do the numbers look like when graphed, etc.) but have not explored them.

And now, drum roll please. We think that the best 4 coin system is:

1 cent 5 cents 18 cents 25 cents

Here is the chart for this system:

Amount of Change	Number of Coins	Amount of Change	Number of Coins	Amount of Change	Number of Coins	Amount of Change	Number of Coins	Amount of Change	Number of Coins
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19 cents	2	39 cents	5	59 cents	4	79 cents	4	99 cents	6
20 cents	3	40 cents	4	60 cents	4	80 cents	4		

Total number of coins = 389

Notice that the combination for 66 cents is the best so far! To make change for 66 cents in this system, you use 1 quarter, 2 “dimes”, and 1 nickel. It is a little confusing to figure out the combination that are optimal, but a fun challenge.

So, acknowledging but discounting the upheaval caused by having to make change more creatively, we recommend that the dollar stay a dollar but the dime becomes 18 cents! So Congress, please, if anything, change the dime not the dollar!