

FACTORIZATIONS OF INTEGERS INTO FACTORS WITH DISTINCT BASES AND EXPONENTS

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ABSTRACT. The integer sequence [A255231](#) counts the number of representations of positive integers n as a product of integer powers such that all bases of the representation and all the exponents are distinct. The manuscript shows the number of representations as a function of the prime signature of n for $\Omega(n) \leq 9$.

1. NOTATION AND SCOPE

Sequence [A255231](#) counts the number of representations of n as a product of $f \geq 1$ factors,

$$(1) \quad n = \prod_{i=1}^f b_i^{e_i}$$

for $n \geq 1$ with the constraint that all the b_i are different and ≥ 2 , and that all the e_i are different and ≥ 1 , which means, the size of the set of the b_i and the size of the set of the e_i are each f .

Representations of that kind could be invented having a visual obstruction in mind which does not reveal at first sight how the representation could be simplified; one cannot combine any two same bases by just adding their exponents, or any two exponents by multiplying bases, because the representations do not have such pairs. The fundamental rules

$$(2) \quad b^{e_i} b^{e_j} = b^{e_i + e_j}; \quad b_i^{e_i} b_j^{e_j} = (b_i b_j)^{e_i + e_j}$$

appear to be useless, so to speak.

Given the unique prime number decomposition

$$(3) \quad n = \prod_{i=1}^{\omega(n)} p_i^{s_i}$$

one realises that the number of admitted representations (1) is a function of the prime signature, a function of the multiset $[s_1 s_2 s_3 \dots]$ of exponents: If the representations are known for some set $\{p_i\}$ of distinct primes, one can swap in each representation another set of distinct primes with the same cardinality at the corresponding prime factorizations of the bases b_i , and the requirements of distinction of the b_i and e_i are still met for the other set of primes and the other n .

There is always at least the (trivial) factorization with a single factor, where $b_1 = n$ and $e_1 = 1$, all prime powers of n packed into the same factor.

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A generic result is that this trivial factorization is the only one if n is a squarefree product of distinct primes, where all $s_i = 1$. (One cannot regroup the p_i by any means to form somewhere a factor that is a perfect square...). A lookup in the index reveals that this covers all numbers in [A000040](#), [A006881](#), [A007304](#), [A046386](#) and so on.

Another generic observation is that if all $s_i = 1$ but one which is 2, the number of representations is 2: the trivial representation exists and there is another representation where the prime with $s_i = 2$ is in a factor of its own with $e_i = 2$ and all the rest packed into a factor with $e_i = 1$. [Distributing the prime with $s_i = 2$ over two distinct factors is not admitted, because each of these factors would need a representation with a (repeated) $e_i = 1$.]

A similar argument seems to hold if all $s_i = 1$ but one which is 3, restricting the number of representations to 3; this covers the n in [A065036](#), [A189975](#), [A189984](#) and so on.

If n is a prime power, $f = 1$, the number of representations can be taken from [A098859](#).

Conjecture: If the prime signature is $[1s_2]$, the number of representations is [A224956](#).

2. RESULTS

The following table considers prime signatures sorted along increasing partitions of $\Omega(n) = \sum_i s_i$, and for the same Ω along $\omega(n) = \sum_i 1$, see [A036036](#). The list of the s_i is given after a **sig** in brackets. This is followed by admitted factorizations (one per line) and a line with a colon and the number of admitted factorizations (just the number of lines after the **sig**).

Multiplications are represented by stars * and x's. The stars indicate that the associated primes p_i are multiplied to construct a single b_i . The x split factors in (1). Each b_i is surrounded by parentheses. Exponentiation is denoted by the caret. In the prime factorization of b_i exponents that equal 1 are not shown explicitly, but the e_i of the representations are shown in all cases explicitly, even if equal to 1.

The (trivial) factorization with a single factor is shown in the line that immediately follows the **sig**.

```

sig [ 1]
(p1)^1
: 1

sig [ 2]
(p1^2)^1
(p1)^2
: 2

sig [ 1 1]
(p1*p2)^1
: 1

sig [ 3]
(p1^3)^1
(p1)^3

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: 2

sig [ 1 2]
(p1*p2^2)^1
(p1)^1 x (p2)^2
: 2

sig [ 1 1 1]
(p1*p2*p3)^1
: 1

sig [ 4]
(p1^4)^1
(p1^2)^2
(p1)^4
(p1)^2 x (p1^2)^1
: 4

sig [ 1 3]
(p1*p2^3)^1
(p1)^1 x (p2)^3
(p1*p2)^1 x (p2)^2
: 3

sig [ 2 2]
(p1^2*p2^2)^1
(p1*p2)^2
(p1^2)^1 x (p2)^2
(p1)^2 x (p2^2)^1
: 4

sig [ 1 1 2]
(p1*p2*p3^2)^1
(p1*p2)^1 x (p3)^2
: 2

sig [ 1 1 1 1]
(p1*p2*p3*p4)^1
: 1

sig [ 5]
(p1^5)^1
(p1)^5
(p1)^1 x (p1^2)^2
(p1^2)^1 x (p1)^3
(p1)^2 x (p1^3)^1
: 5

sig [ 1 4]
(p1*p2^4)^1
(p1)^1 x (p2^2)^2
(p1)^1 x (p2)^4
(p1*p2)^1 x (p2)^3

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(p1*p2^2)^1 x (p2)^2
: 5

sig [ 2 3]
(p1^2*p2^3)^1
(p1^2)^1 x (p2)^3
(p1)^2 x (p2^3)^1
(p1)^2 x (p2)^3
(p1^2*p2)^1 x (p2)^2
(p1*p2)^2 x (p2)^1
: 6

sig [ 1 1 3]
(p1*p2*p3^3)^1
(p1*p2)^1 x (p3)^3
(p1*p2*p3)^1 x (p3)^2
: 3

sig [ 1 2 2]
(p1*p2^2*p3^2)^1
(p1)^1 x (p2*p3)^2
(p1*p3^2)^1 x (p2)^2
(p1*p2^2)^1 x (p3)^2
: 4

sig [ 1 1 1 2]
(p1*p2*p3*p4^2)^1
(p1*p2*p3)^1 x (p4)^2
: 2

sig [ 1 1 1 1 1]
(p1*p2*p3*p4*p5)^1
: 1

sig [ 6]
(p1^6)^1
(p1^3)^2
(p1^2)^3
(p1)^6
(p1^2)^1 x (p1)^4
(p1)^2 x (p1^4)^1
(p1)^3 x (p1^3)^1
: 7

sig [ 1 5]
(p1*p2^5)^1
(p1)^1 x (p2)^5
(p1*p2)^1 x (p2^2)^2
(p1*p2)^1 x (p2)^4
(p1*p2^2)^1 x (p2)^3
(p1*p2^3)^1 x (p2)^2
: 6

```

```

sig [ 2 4]
(p1^2*p2^4)^1
(p1*p2^2)^2
(p1^2)^1 x (p2^2)^2
(p1^2)^1 x (p2)^4
(p1)^2 x (p2^4)^1
(p1)^2 x (p2)^4
(p1^2*p2)^1 x (p2)^3
(p1^2*p2^2)^1 x (p2)^2
(p1*p2)^2 x (p2^2)^1
: 9

sig [ 3 3]
(p1^3*p2^3)^1
(p1*p2)^3
(p1*p2^3)^1 x (p1)^2
(p1^3)^1 x (p2)^3
(p1)^3 x (p2^3)^1
(p1^3*p2)^1 x (p2)^2
: 6

sig [ 1 1 4]
(p1*p2*p3^4)^1
(p1*p2)^1 x (p3^2)^2
(p1*p2)^1 x (p3)^4
(p1*p2*p3)^1 x (p3)^3
(p1*p2*p3^2)^1 x (p3)^2
: 5

sig [ 1 2 3]
(p1*p2^2*p3^3)^1
(p1*p3)^1 x (p2*p3)^2
(p1*p3^3)^1 x (p2)^2
(p1*p2^2)^1 x (p3)^3
(p1*p2^2*p3)^1 x (p3)^2
(p1)^1 x (p2)^2 x (p3)^3
: 6

sig [ 2 2 2]
(p1^2*p2^2*p3^2)^1
(p1*p2*p3)^2
(p1^2)^1 x (p2*p3)^2
(p1)^2 x (p2^2*p3^2)^1
(p1^2*p3^2)^1 x (p2)^2
(p1*p3)^2 x (p2^2)^1
(p1^2*p2^2)^1 x (p3)^2
(p1*p2)^2 x (p3^2)^1
: 8

sig [ 1 1 1 3]
(p1*p2*p3*p4^3)^1
(p1*p2*p3)^1 x (p4)^3
(p1*p2*p3*p4)^1 x (p4)^2

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: 3

sig [ 1 1 2 2]
(p1*p2*p3^2*p4^2)^1
(p1*p2)^1 x (p3*p4)^2
(p1*p2*p4^2)^1 x (p3)^2
(p1*p2*p3^2)^1 x (p4)^2
: 4

sig [ 1 1 1 1 2]
(p1*p2*p3*p4*p5^2)^1
(p1*p2*p3*p4)^1 x (p5)^2
: 2

sig [ 1 1 1 1 1 1]
(p1*p2*p3*p4*p5*p6)^1
: 1

sig [ 7]
(p1^7)^1
(p1)^7
(p1)^1 x (p1^3)^2
(p1)^1 x (p1^2)^3
(p1^2)^1 x (p1)^5
(p1)^2 x (p1^5)^1
(p1^3)^1 x (p1^2)^2
(p1^3)^1 x (p1)^4
(p1)^3 x (p1^4)^1
(p1)^3 x (p1^2)^2
: 10

sig [ 1 6]
(p1*p2^6)^1
(p1)^1 x (p2^3)^2
(p1)^1 x (p2^2)^3
(p1)^1 x (p2)^6
(p1*p2)^1 x (p2)^5
(p1*p2^2)^1 x (p2^2)^2
(p1*p2^2)^1 x (p2)^4
(p1*p2^3)^1 x (p2)^3
(p1*p2^4)^1 x (p2)^2
: 9

sig [ 2 5]
(p1^2*p2^5)^1
(p1^2)^1 x (p2)^5
(p1)^2 x (p2^5)^1
(p1)^2 x (p2)^5
(p1^2*p2)^1 x (p2^2)^2
(p1^2*p2)^1 x (p2)^4
(p1^2*p2^2)^1 x (p2)^3
(p1*p2)^2 x (p2^3)^1
(p1*p2)^2 x (p2)^3

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(p1^2*p2^3)^1 x (p2)^2
(p1*p2^2)^2 x (p2)^1
(p1)^2 x (p2^2)^1 x (p2)^3
: 12

sig [ 3 4]
(p1^3*p2^4)^1
(p1)^1 x (p1*p2^2)^2
(p1*p2^2)^1 x (p1*p2)^2
(p1*p2^4)^1 x (p1)^2
(p1^3)^1 x (p2^2)^2
(p1^3)^1 x (p2)^4
(p1)^3 x (p2^4)^1
(p1)^3 x (p2^2)^2
(p1)^3 x (p2)^4
(p1^3*p2)^1 x (p2)^3
(p1^3*p2^2)^1 x (p2)^2
(p1*p2)^3 x (p2)^1
(p1*p2)^1 x (p1)^2 x (p2)^3
(p1)^3 x (p2)^2 x (p2^2)^1
: 14

sig [ 1 1 5]
(p1*p2*p3^5)^1
(p1*p2)^1 x (p3)^5
(p1*p2*p3)^1 x (p3^2)^2
(p1*p2*p3)^1 x (p3)^4
(p1*p2*p3^2)^1 x (p3)^3
(p1*p2*p3^3)^1 x (p3)^2
: 6

sig [ 1 2 4]
(p1*p2^2*p3^4)^1
(p1)^1 x (p2*p3^2)^2
(p1*p3^2)^1 x (p2*p3)^2
(p1*p3^4)^1 x (p2)^2
(p1*p2^2)^1 x (p3^2)^2
(p1*p2^2)^1 x (p3)^4
(p1*p2^2*p3)^1 x (p3)^3
(p1*p2^2*p3^2)^1 x (p3)^2
(p1)^1 x (p2)^2 x (p3)^4
(p1*p3)^1 x (p2)^2 x (p3)^3
: 10

sig [ 1 3 3]
(p1*p2^3*p3^3)^1
(p1)^1 x (p2*p3)^3
(p1*p3^3)^1 x (p2)^3
(p1*p2*p3)^1 x (p2*p3)^2
(p1*p2*p3^3)^1 x (p2)^2
(p1*p2^3)^1 x (p3)^3
(p1*p2^3*p3)^1 x (p3)^2
(p1*p3)^1 x (p2)^3 x (p3)^2

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(p1*p2)^1 x (p2)^2 x (p3)^3
: 9

sig [ 2 2 3]
(p1^2*p2^2*p3^3)^1
(p1)^2 x (p2^2*p3^3)^1
(p1^2*p3)^1 x (p2*p3)^2
(p1*p3)^2 x (p2^2*p3)^1
(p1^2*p3^3)^1 x (p2)^2
(p1^2*p2^2)^1 x (p3)^3
(p1*p2)^2 x (p3^3)^1
(p1*p2)^2 x (p3)^3
(p1^2*p2^2*p3)^1 x (p3)^2
(p1*p2*p3)^2 x (p3)^1
(p1^2)^1 x (p2)^2 x (p3)^3
(p1)^2 x (p2^2)^1 x (p3)^3
: 12

sig [ 1 1 1 4]
(p1*p2*p3*p4^4)^1
(p1*p2*p3)^1 x (p4^2)^2
(p1*p2*p3)^1 x (p4)^4
(p1*p2*p3*p4)^1 x (p4)^3
(p1*p2*p3*p4^2)^1 x (p4)^2
: 5

sig [ 1 1 2 3]
(p1*p2*p3^2*p4^3)^1
(p1*p2*p4)^1 x (p3*p4)^2
(p1*p2*p4^3)^1 x (p3)^2
(p1*p2*p3^2)^1 x (p4)^3
(p1*p2*p3^2*p4)^1 x (p4)^2
(p1*p2)^1 x (p3)^2 x (p4)^3
: 6

sig [ 1 2 2 2]
(p1*p2^2*p3^2*p4^2)^1
(p1)^1 x (p2*p3*p4)^2
(p1*p4^2)^1 x (p2*p3)^2
(p1*p3^2)^1 x (p2*p4)^2
(p1*p3^2*p4^2)^1 x (p2)^2
(p1*p2^2)^1 x (p3*p4)^2
(p1*p2^2*p4^2)^1 x (p3)^2
(p1*p2^2*p3^2)^1 x (p4)^2
: 8

sig [ 1 1 1 1 3]
(p1*p2*p3*p4*p5^3)^1
(p1*p2*p3*p4)^1 x (p5)^3
(p1*p2*p3*p4*p5)^1 x (p5)^2
: 3

sig [ 1 1 1 2 2]

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(p1*p2*p3*p4^2*p5^2)^1
(p1*p2*p3)^1 x (p4*p5)^2
(p1*p2*p3*p5^2)^1 x (p4)^2
(p1*p2*p3*p4^2)^1 x (p5)^2
: 4

sig [ 1 1 1 1 1 2]
(p1*p2*p3*p4*p5*p6^2)^1
(p1*p2*p3*p4*p5)^1 x (p6)^2
: 2

sig [ 1 1 1 1 1 1 1]
(p1*p2*p3*p4*p5*p6*p7)^1
: 1

sig [ 8]
(p1^8)^1
(p1^4)^2
(p1^2)^4
(p1)^8
(p1^2)^1 x (p1^3)^2
(p1^2)^1 x (p1)^6
(p1)^2 x (p1^6)^1
(p1)^2 x (p1^2)^3
(p1^3)^1 x (p1)^5
(p1)^3 x (p1^5)^1
(p1^2)^2 x (p1^4)^1
(p1)^4 x (p1^4)^1
(p1)^4 x (p1^2)^2
: 13

sig [ 1 7]
(p1*p2^7)^1
(p1)^1 x (p2)^7
(p1*p2)^1 x (p2^3)^2
(p1*p2)^1 x (p2^2)^3
(p1*p2)^1 x (p2)^6
(p1*p2^2)^1 x (p2)^5
(p1*p2^3)^1 x (p2^2)^2
(p1*p2^3)^1 x (p2)^4
(p1*p2^4)^1 x (p2)^3
(p1*p2^5)^1 x (p2)^2
(p1)^1 x (p2)^3 x (p2^2)^2
: 11

sig [ 2 6]
(p1^2*p2^6)^1
(p1*p2^3)^2
(p1^2)^1 x (p2^3)^2
(p1^2)^1 x (p2^2)^3
(p1^2)^1 x (p2)^6
(p1)^2 x (p2^6)^1
(p1)^2 x (p2^2)^3

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(p1)^2 x (p2)^6
(p1^2*p2)^1 x (p2)^5
(p1^2*p2^2)^1 x (p2^2)^2
(p1^2*p2^2)^1 x (p2)^4
(p1*p2)^2 x (p2^4)^1
(p1*p2)^2 x (p2)^4
(p1^2*p2^3)^1 x (p2)^3
(p1^2*p2^4)^1 x (p2)^2
(p1*p2^2)^2 x (p2^2)^1
(p1)^2 x (p2^2)^1 x (p2)^4
(p1)^2 x (p2)^3 x (p2^3)^1
: 18

sig [ 3 5]
(p1^3*p2^5)^1
(p1*p2)^1 x (p1*p2^2)^2
(p1*p2^3)^1 x (p1*p2)^2
(p1*p2^5)^1 x (p1)^2
(p1^3)^1 x (p2)^5
(p1)^3 x (p2^5)^1
(p1)^3 x (p2)^5
(p1^3*p2)^1 x (p2^2)^2
(p1^3*p2)^1 x (p2)^4
(p1^3*p2^2)^1 x (p2)^3
(p1^3*p2^3)^1 x (p2)^2
(p1*p2)^3 x (p2^2)^1
(p1*p2)^3 x (p2)^2
(p1)^1 x (p1*p2)^2 x (p2)^3
(p1*p2)^1 x (p1)^2 x (p2)^4
(p1*p2^2)^1 x (p1)^2 x (p2)^3
(p1)^3 x (p2)^1 x (p2^2)^2
(p1)^3 x (p2)^2 x (p2^3)^1
: 18

sig [ 4 4]
(p1^4*p2^4)^1
(p1^2*p2^2)^2
(p1*p2)^4
(p1*p2^4)^1 x (p1)^3
(p1^2)^1 x (p1*p2^2)^2
(p1)^2 x (p1^2*p2^4)^1
(p1*p2)^2 x (p1^2*p2^2)^1
(p1^4)^1 x (p2^2)^2
(p1^4)^1 x (p2)^4
(p1^2)^2 x (p2^4)^1
(p1^2)^2 x (p2)^4
(p1)^4 x (p2^4)^1
(p1)^4 x (p2^2)^2
(p1^4*p2)^1 x (p2)^3
(p1^4*p2^2)^1 x (p2)^2
(p1^2*p2)^2 x (p2^2)^1
(p1*p2^2)^1 x (p1)^3 x (p2)^2
(p1)^2 x (p1^2)^1 x (p2)^4

```

$(p_1)^2 \times (p_1^2 * p_2)^1 \times (p_2)^3$
 $(p_1)^4 \times (p_2)^2 \times (p_2^2)^1$
: 20

sig [1 1 6]
 $(p_1 * p_2 * p_3^6)^1$
 $(p_1 * p_2)^1 \times (p_3^3)^2$
 $(p_1 * p_2)^1 \times (p_3^2)^3$
 $(p_1 * p_2)^1 \times (p_3)^6$
 $(p_1 * p_2 * p_3)^1 \times (p_3)^5$
 $(p_1 * p_2 * p_3^2)^1 \times (p_3^2)^2$
 $(p_1 * p_2 * p_3^2)^1 \times (p_3)^4$
 $(p_1 * p_2 * p_3^3)^1 \times (p_3)^3$
 $(p_1 * p_2 * p_3^4)^1 \times (p_3)^2$
: 9

sig [1 2 5]
 $(p_1 * p_2^2 * p_3^5)^1$
 $(p_1 * p_3)^1 \times (p_2 * p_3^2)^2$
 $(p_1 * p_3^3)^1 \times (p_2 * p_3)^2$
 $(p_1 * p_3^5)^1 \times (p_2)^2$
 $(p_1 * p_2^2)^1 \times (p_3)^5$
 $(p_1 * p_2^2 * p_3)^1 \times (p_3^2)^2$
 $(p_1 * p_2^2 * p_3)^1 \times (p_3)^4$
 $(p_1 * p_2^2 * p_3^2)^1 \times (p_3)^3$
 $(p_1 * p_2^2 * p_3^3)^1 \times (p_3)^2$
 $(p_1)^1 \times (p_2)^2 \times (p_3)^5$
 $(p_1)^1 \times (p_2 * p_3)^2 \times (p_3)^3$
 $(p_1 * p_3)^1 \times (p_2)^2 \times (p_3)^4$
 $(p_1 * p_3^2)^1 \times (p_2)^2 \times (p_3)^3$
: 13

sig [1 3 4]
 $(p_1 * p_2^3 * p_3^4)^1$
 $(p_1 * p_3)^1 \times (p_2 * p_3)^3$
 $(p_1 * p_3^4)^1 \times (p_2)^3$
 $(p_1 * p_2)^1 \times (p_2 * p_3^2)^2$
 $(p_1 * p_2 * p_3^2)^1 \times (p_2 * p_3)^2$
 $(p_1 * p_2 * p_3^4)^1 \times (p_2)^2$
 $(p_1 * p_2^3)^1 \times (p_3^2)^2$
 $(p_1 * p_2^3)^1 \times (p_3)^4$
 $(p_1 * p_2^3 * p_3)^1 \times (p_3)^3$
 $(p_1 * p_2^3 * p_3^2)^1 \times (p_3)^2$
 $(p_1)^1 \times (p_2)^3 \times (p_3^2)^2$
 $(p_1)^1 \times (p_2)^3 \times (p_3)^4$
 $(p_1 * p_3^2)^1 \times (p_2)^3 \times (p_3)^2$
 $(p_1 * p_2)^1 \times (p_2)^2 \times (p_3)^4$
 $(p_1 * p_2 * p_3)^1 \times (p_2)^2 \times (p_3)^3$
: 15

sig [2 2 4]
 $(p_1^2 * p_2^2 * p_3^4)^1$
 $(p_1 * p_2 * p_3^2)^2$

```

(p1^2)^1 x (p2*p3^2)^2
(p1)^2 x (p2^2*p3^4)^1
(p1^2*p3^2)^1 x (p2*p3)^2
(p1*p3)^2 x (p2^2*p3^2)^1
(p1^2*p3^4)^1 x (p2)^2
(p1*p3^2)^2 x (p2^2)^1
(p1^2*p2^2)^1 x (p3^2)^2
(p1^2*p2^2)^1 x (p3)^4
(p1*p2)^2 x (p3^4)^1
(p1*p2)^2 x (p3)^4
(p1^2*p2^2*p3)^1 x (p3)^3
(p1^2*p2^2*p3^2)^1 x (p3)^2
(p1*p2*p3)^2 x (p3^2)^1
(p1^2)^1 x (p2)^2 x (p3)^4
(p1)^2 x (p2^2)^1 x (p3)^4
(p1)^2 x (p2^2*p3)^1 x (p3)^3
(p1^2*p3)^1 x (p2)^2 x (p3)^3
: 19

```

```

sig [ 2 3 3]
(p1^2*p2^3*p3^3)^1
(p1^2)^1 x (p2*p3)^3
(p1)^2 x (p2^3*p3^3)^1
(p1)^2 x (p2*p3)^3
(p1*p3)^2 x (p2^3*p3)^1
(p1^2*p3^3)^1 x (p2)^3
(p1^2*p2*p3)^1 x (p2*p3)^2
(p1^2*p2*p3^3)^1 x (p2)^2
(p1*p2)^2 x (p2*p3^3)^1
(p1*p2*p3)^2 x (p2*p3)^1
(p1^2*p2^3)^1 x (p3)^3
(p1^2 x (p2)^3 x (p3^3)^1
(p1^2*p3)^1 x (p2)^3 x (p3)^2
(p1*p3)^2 x (p2)^3 x (p3)^1
(p1^2*p2)^1 x (p2)^2 x (p3)^3
(p1*p2)^2 x (p2)^1 x (p3)^3
: 18

```

```

sig [ 1 1 1 5]
(p1*p2*p3*p4^5)^1
(p1*p2*p3)^1 x (p4)^5
(p1*p2*p3*p4)^1 x (p4^2)^2
(p1*p2*p3*p4)^1 x (p4)^4
(p1*p2*p3*p4^2)^1 x (p4)^3
(p1*p2*p3*p4^3)^1 x (p4)^2
: 6

```

```

sig [ 1 1 2 4]
(p1*p2*p3^2*p4^4)^1
(p1*p2)^1 x (p3*p4^2)^2
(p1*p2*p4^2)^1 x (p3*p4)^2

```

```

(p1*p2*p4^4)^1 x (p3)^2
(p1*p2*p3^2)^1 x (p4^2)^2
(p1*p2*p3^2)^1 x (p4)^4
(p1*p2*p3^2*p4)^1 x (p4)^3
(p1*p2*p3^2*p4^2)^1 x (p4)^2
(p1*p2)^1 x (p3)^2 x (p4)^4
(p1*p2*p4)^1 x (p3)^2 x (p4)^3
: 10

sig [ 1 1 3 3]
(p1*p2*p3^3*p4^3)^1
(p1*p2)^1 x (p3*p4)^3
(p1*p2*p4^3)^1 x (p3)^3
(p1*p2*p3*p4)^1 x (p3*p4)^2
(p1*p2*p3*p4^3)^1 x (p3)^2
(p1*p2*p3^3)^1 x (p4)^3
(p1*p2*p3^3*p4)^1 x (p4)^2
(p1*p2*p4)^1 x (p3)^3 x (p4)^2
(p1*p2*p3)^1 x (p3)^2 x (p4)^3
: 9

sig [ 1 2 2 3]
(p1*p2^2*p3^2*p4^3)^1
(p1*p4)^1 x (p2*p3*p4)^2
(p1*p4^3)^1 x (p2*p3)^2
(p1*p3^2*p4)^1 x (p2*p4)^2
(p1*p3^2*p4^3)^1 x (p2)^2
(p1*p2^2*p4)^1 x (p3*p4)^2
(p1*p2^2*p4^3)^1 x (p3)^2
(p1*p2^2*p3^2)^1 x (p4)^3
(p1*p2^2*p3^2*p4)^1 x (p4)^2
(p1)^1 x (p2*p3)^2 x (p4)^3
(p1*p3^2)^1 x (p2)^2 x (p4)^3
(p1*p2^2)^1 x (p3)^2 x (p4)^3
: 12

sig [ 2 2 2 2]
(p1^2*p2^2*p3^2*p4^2)^1
(p1*p2*p3*p4)^2
(p1^2)^1 x (p2*p3*p4)^2
(p1)^2 x (p2^2*p3^2*p4^2)^1
(p1^2*p4^2)^1 x (p2*p3)^2
(p1*p4)^2 x (p2^2*p3^2)^1
(p1^2*p3^2)^1 x (p2*p4)^2
(p1*p3)^2 x (p2^2*p4^2)^1
(p1^2*p3^2*p4^2)^1 x (p2)^2
(p1*p3*p4)^2 x (p2^2)^1
(p1^2*p2^2)^1 x (p3*p4)^2
(p1*p2)^2 x (p3^2*p4^2)^1
(p1^2*p2^2*p4^2)^1 x (p3)^2
(p1*p2*p4)^2 x (p3^2)^1
(p1^2*p2^2*p3^2)^1 x (p4)^2
(p1*p2*p3)^2 x (p4^2)^1

```

```

: 16

sig [ 1 1 1 1 4]
(p1*p2*p3*p4*p5^4)^1
(p1*p2*p3*p4)^1 x (p5^2)^2
(p1*p2*p3*p4)^1 x (p5)^4
(p1*p2*p3*p4*p5)^1 x (p5)^3
(p1*p2*p3*p4*p5^2)^1 x (p5)^2
: 5

sig [ 1 1 1 2 3]
(p1*p2*p3*p4^2*p5^3)^1
(p1*p2*p3*p5)^1 x (p4*p5)^2
(p1*p2*p3*p5^3)^1 x (p4)^2
(p1*p2*p3*p4^2)^1 x (p5)^3
(p1*p2*p3*p4^2*p5)^1 x (p5)^2
(p1*p2*p3)^1 x (p4)^2 x (p5)^3
: 6

sig [ 1 1 2 2 2]
(p1*p2*p3^2*p4^2*p5^2)^1
(p1*p2)^1 x (p3*p4*p5)^2
(p1*p2*p5^2)^1 x (p3*p4)^2
(p1*p2*p4^2)^1 x (p3*p5)^2
(p1*p2*p4^2*p5^2)^1 x (p3)^2
(p1*p2*p3^2)^1 x (p4*p5)^2
(p1*p2*p3^2*p5^2)^1 x (p4)^2
(p1*p2*p3^2*p4^2)^1 x (p5)^2
: 8

sig [ 1 1 1 1 1 3]
(p1*p2*p3*p4*p5*p6^3)^1
(p1*p2*p3*p4*p5)^1 x (p6)^3
(p1*p2*p3*p4*p5*p6)^1 x (p6)^2
: 3

sig [ 1 1 1 1 2 2]
(p1*p2*p3*p4*p5^2*p6^2)^1
(p1*p2*p3*p4)^1 x (p5*p6)^2
(p1*p2*p3*p4*p6^2)^1 x (p5)^2
(p1*p2*p3*p4*p5^2)^1 x (p6)^2
: 4

sig [ 1 1 1 1 1 1 2]
(p1*p2*p3*p4*p5*p6*p7^2)^1
(p1*p2*p3*p4*p5*p6)^1 x (p7)^2
: 2

sig [ 1 1 1 1 1 1 1 1]
(p1*p2*p3*p4*p5*p6*p7*p8)^1
: 1

sig [ 9]

```

```

(p1^9)^1
(p1^3)^3
(p1)^9
(p1)^1 x (p1^4)^2
(p1)^1 x (p1^2)^4
(p1^2)^1 x (p1)^7
(p1)^2 x (p1^7)^1
(p1^3)^1 x (p1^2)^3
(p1^3)^1 x (p1)^6
(p1)^3 x (p1^6)^1
(p1)^3 x (p1^3)^2
(p1^4)^1 x (p1)^5
(p1^2)^2 x (p1^5)^1
(p1^2)^2 x (p1)^5
(p1)^4 x (p1^5)^1
: 15

sig [ 1 8]
(p1*p2^8)^1
(p1)^1 x (p2^4)^2
(p1)^1 x (p2^2)^4
(p1)^1 x (p2)^8
(p1*p2)^1 x (p2)^7
(p1*p2^2)^1 x (p2^3)^2
(p1*p2^2)^1 x (p2^2)^3
(p1*p2^2)^1 x (p2)^6
(p1*p2^3)^1 x (p2)^5
(p1*p2^4)^1 x (p2^2)^2
(p1*p2^4)^1 x (p2)^4
(p1*p2^5)^1 x (p2)^3
(p1*p2^6)^1 x (p2)^2
(p1)^1 x (p2)^2 x (p2^2)^3
(p1)^1 x (p2)^4 x (p2^2)^2
(p1*p2)^1 x (p2)^3 x (p2^2)^2
: 16

sig [ 2 7]
(p1^2*p2^7)^1
(p1^2)^1 x (p2)^7
(p1)^2 x (p2^7)^1
(p1)^2 x (p2)^7
(p1^2*p2)^1 x (p2^3)^2
(p1^2*p2)^1 x (p2^2)^3
(p1^2*p2)^1 x (p2)^6
(p1^2*p2^2)^1 x (p2)^5
(p1*p2)^2 x (p2^5)^1
(p1*p2)^2 x (p2)^5
(p1^2*p2^3)^1 x (p2^2)^2
(p1^2*p2^3)^1 x (p2)^4
(p1^2*p2^4)^1 x (p2)^3
(p1*p2^2)^2 x (p2^3)^1
(p1*p2^2)^2 x (p2)^3
(p1^2*p2^5)^1 x (p2)^2

```

```
(p1*p2^3)^2 x (p2)^1
(p1)^2 x (p2)^1 x (p2^2)^3
(p1)^2 x (p2^2)^1 x (p2)^5
(p1^2)^1 x (p2)^3 x (p2^2)^2
(p1)^2 x (p2^3)^1 x (p2)^4
(p1)^2 x (p2)^3 x (p2^4)^1
(p1*p2)^2 x (p2^2)^1 x (p2)^3
: 23
```

```
sig [ 3 6]
(p1^3*p2^6)^1
(p1*p2^2)^3
(p1)^1 x (p1*p2^3)^2
(p1*p2^4)^1 x (p1*p2)^2
(p1*p2^6)^1 x (p1)^2
(p1^3)^1 x (p2^3)^2
(p1^3)^1 x (p2^2)^3
(p1^3)^1 x (p2)^6
(p1)^3 x (p2^6)^1
(p1)^3 x (p2^3)^2
(p1)^3 x (p2)^6
(p1^3*p2)^1 x (p2)^5
(p1^3*p2^2)^1 x (p2^2)^2
(p1^3*p2^2)^1 x (p2)^4
(p1^3*p2^3)^1 x (p2)^3
(p1*p2)^3 x (p2^3)^1
(p1^3*p2^4)^1 x (p2)^2
(p1)^1 x (p1*p2)^2 x (p2)^4
(p1*p2)^1 x (p1)^2 x (p2)^5
(p1*p2^2)^1 x (p1)^2 x (p2)^4
(p1*p2^3)^1 x (p1)^2 x (p2)^3
(p1)^3 x (p2^2)^1 x (p2)^4
(p1)^3 x (p2)^2 x (p2^4)^1
: 23
```

```
sig [ 4 5]
(p1^4*p2^5)^1
(p1*p2^2)^1 x (p1*p2)^3
(p1*p2^5)^1 x (p1)^3
(p1)^2 x (p1^2*p2^5)^1
(p1^2*p2)^1 x (p1*p2^2)^2
(p1*p2)^2 x (p1^2*p2^3)^1
(p1^4)^1 x (p2)^5
(p1^2)^2 x (p2^5)^1
(p1^2)^2 x (p2)^5
(p1)^4 x (p2^5)^1
(p1)^4 x (p2)^5
(p1^4*p2)^1 x (p2^2)^2
(p1^4*p2)^1 x (p2)^4
(p1^4*p2^2)^1 x (p2)^3
(p1^2*p2)^2 x (p2^3)^1
(p1^2*p2)^2 x (p2)^3
(p1^4*p2^3)^1 x (p2)^2
```

```
(p1^2*p2^2)^2 x (p2)^1
(p1*p2)^4 x (p2)^1
(p1)^1 x (p1*p2)^3 x (p2)^2
(p1*p2)^1 x (p1)^3 x (p2^2)^2
(p1*p2)^1 x (p1)^3 x (p2)^4
(p1*p2^3)^1 x (p1)^3 x (p2)^2
(p1)^2 x (p1^2)^1 x (p2)^5
(p1)^2 x (p1^2*p2)^1 x (p2)^4
(p1^2)^1 x (p1*p2)^2 x (p2)^3
(p1)^2 x (p1^2*p2^2)^1 x (p2)^3
(p1)^4 x (p2)^1 x (p2^2)^2
(p1^2)^2 x (p2^2)^1 x (p2)^3
(p1)^4 x (p2^2)^1 x (p2)^3
(p1)^4 x (p2)^2 x (p2^3)^1
: 31
```

```
sig [ 1 1 7]
(p1*p2*p3^7)^1
(p1*p2)^1 x (p3)^7
(p1*p2*p3)^1 x (p3^3)^2
(p1*p2*p3)^1 x (p3^2)^3
(p1*p2*p3)^1 x (p3)^6
(p1*p2*p3^2)^1 x (p3)^5
(p1*p2*p3^3)^1 x (p3^2)^2
(p1*p2*p3^3)^1 x (p3)^4
(p1*p2*p3^4)^1 x (p3)^3
(p1*p2*p3^5)^1 x (p3)^2
(p1*p2)^1 x (p3)^3 x (p3^2)^2
: 11
```

```
sig [ 1 2 6]
(p1*p2^2*p3^6)^1
(p1)^1 x (p2*p3^3)^2
(p1*p3^2)^1 x (p2*p3^2)^2
(p1*p3^4)^1 x (p2*p3)^2
(p1*p3^6)^1 x (p2)^2
(p1*p2^2)^1 x (p3^3)^2
(p1*p2^2)^1 x (p3^2)^3
(p1*p2^2)^1 x (p3)^6
(p1*p2^2*p3)^1 x (p3)^5
(p1*p2^2*p3^2)^1 x (p3^2)^2
(p1*p2^2*p3^2)^1 x (p3)^4
(p1*p2^2*p3^3)^1 x (p3)^3
(p1*p2^2*p3^4)^1 x (p3)^2
(p1)^1 x (p2)^2 x (p3^2)^3
(p1)^1 x (p2)^2 x (p3)^6
(p1)^1 x (p2*p3)^2 x (p3)^4
(p1*p3)^1 x (p2)^2 x (p3)^5
(p1*p3)^1 x (p2*p3)^2 x (p3)^3
(p1*p3^2)^1 x (p2)^2 x (p3)^4
(p1*p3^3)^1 x (p2)^2 x (p3)^3
: 20
```

```

sig [ 1 3 5]
(p1*p2^3*p3^5)^1
(p1*p3^2)^1 x (p2*p3)^3
(p1*p3^5)^1 x (p2)^3
(p1*p2*p3)^1 x (p2*p3^2)^2
(p1*p2*p3^3)^1 x (p2*p3)^2
(p1*p2*p3^5)^1 x (p2)^2
(p1*p2^3)^1 x (p3)^5
(p1*p2^3*p3)^1 x (p3^2)^2
(p1*p2^3*p3)^1 x (p3)^4
(p1*p2^3*p3^2)^1 x (p3)^3
(p1*p2^3*p3^3)^1 x (p3)^2
(p1)^1 x (p2)^3 x (p3)^5
(p1)^1 x (p2*p3)^3 x (p3)^2
(p1*p3)^1 x (p2)^3 x (p3^2)^2
(p1*p3)^1 x (p2)^3 x (p3)^4
(p1*p3^3)^1 x (p2)^3 x (p3)^2
(p1*p2)^1 x (p2)^2 x (p3)^5
(p1*p2)^1 x (p2*p3)^2 x (p3)^3
(p1*p2*p3)^1 x (p2)^2 x (p3)^4
(p1*p2*p3^2)^1 x (p2)^2 x (p3)^3
: 20

sig [ 1 4 4]
(p1*p2^4*p3^4)^1
(p1)^1 x (p2^2*p3^2)^2
(p1)^1 x (p2*p3)^4
(p1*p3^2)^1 x (p2^2*p3)^2
(p1*p3^4)^1 x (p2^2)^2
(p1*p3^4)^1 x (p2)^4
(p1*p2*p3)^1 x (p2*p3)^3
(p1*p2*p3^4)^1 x (p2)^3
(p1*p2^2)^1 x (p2*p3^2)^2
(p1*p2^2*p3^2)^1 x (p2*p3)^2
(p1*p2^2*p3^4)^1 x (p2)^2
(p1*p2^4)^1 x (p3^2)^2
(p1*p2^4)^1 x (p3)^4
(p1*p2^4*p3)^1 x (p3)^3
(p1*p2^4*p3^2)^1 x (p3)^2
(p1)^1 x (p2^2)^2 x (p3)^4
(p1)^1 x (p2)^4 x (p3^2)^2
(p1*p3)^1 x (p2^2)^2 x (p3)^3
(p1*p3)^1 x (p2)^4 x (p3)^3
(p1*p3^2)^1 x (p2)^4 x (p3)^2
(p1*p2)^1 x (p2)^3 x (p3^2)^2
(p1*p2)^1 x (p2)^3 x (p3)^4
(p1*p2*p3^2)^1 x (p2)^3 x (p3)^2
(p1*p2^2)^1 x (p2)^2 x (p3)^4
(p1*p2^2*p3)^1 x (p2)^2 x (p3)^3
: 25

sig [ 2 2 5]
(p1^2*p2^2*p3^5)^1

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```

(p1)^2 x (p2^2*p3^5)^1
(p1^2*p3)^1 x (p2*p3^2)^2
(p1*p3)^2 x (p2^2*p3^3)^1
(p1^2*p3^3)^1 x (p2*p3)^2
(p1*p3^2)^2 x (p2^2*p3)^1
(p1^2*p3^5)^1 x (p2)^2
(p1^2*p2^2)^1 x (p3)^5
(p1*p2)^2 x (p3^5)^1
(p1*p2)^2 x (p3)^5
(p1^2*p2^2*p3)^1 x (p3^2)^2
(p1^2*p2^2*p3)^1 x (p3)^4
(p1^2*p2^2*p3^2)^1 x (p3)^3
(p1*p2*p3)^2 x (p3^3)^1
(p1*p2*p3)^2 x (p3)^3
(p1^2*p2^2*p3^3)^1 x (p3)^2
(p1*p2*p3^2)^2 x (p3)^1
(p1^2)^1 x (p2)^2 x (p3)^5
(p1)^2 x (p2^2)^1 x (p3)^5
(p1)^2 x (p2^2*p3)^1 x (p3)^4
(p1^2)^1 x (p2*p3)^2 x (p3)^3
(p1)^2 x (p2^2*p3^2)^1 x (p3)^3
(p1^2*p3)^1 x (p2)^2 x (p3)^4
(p1^2*p3^2)^1 x (p2)^2 x (p3)^3
(p1*p3)^2 x (p2^2)^1 x (p3)^3
(p1*p2)^2 x (p3^2)^1 x (p3)^3
: 26

sig [ 2 3 4]
(p1^2*p2^3*p3^4)^1
(p1)^2 x (p2^3*p3^4)^1
(p1^2*p3)^1 x (p2*p3)^3
(p1*p3)^2 x (p2^3*p3^2)^1
(p1^2*p3^4)^1 x (p2)^3
(p1*p3^2)^2 x (p2^3)^1
(p1*p3^2)^2 x (p2)^3
(p1^2*p2)^1 x (p2*p3^2)^2
(p1^2*p2*p3^2)^1 x (p2*p3)^2
(p1^2*p2*p3^4)^1 x (p2)^2
(p1*p2)^2 x (p2*p3^4)^1
(p1*p2*p3)^2 x (p2*p3^2)^1
(p1*p2*p3^2)^2 x (p2)^1
(p1^2*p2^3)^1 x (p3^2)^2
(p1^2*p2^3)^1 x (p3)^4
(p1^2*p2^3*p3)^1 x (p3)^3
(p1^2*p2^3*p3^2)^1 x (p3)^2
(p1^2)^1 x (p2)^3 x (p3^2)^2
(p1^2)^1 x (p2)^3 x (p3)^4
(p1)^2 x (p2^3)^1 x (p3)^4
(p1)^2 x (p2)^3 x (p3^4)^1
(p1)^2 x (p2)^3 x (p3)^4
(p1)^2 x (p2^3*p3)^1 x (p3)^3
(p1)^2 x (p2*p3)^3 x (p3)^1
(p1^2*p3^2)^1 x (p2)^3 x (p3)^2

```

```
(p1*p3)^2 x (p2)^3 x (p3^2)^1
(p1^2*p2)^1 x (p2)^2 x (p3)^4
(p1^2*p2*p3)^1 x (p2)^2 x (p3)^3
(p1*p2)^2 x (p2)^1 x (p3)^4
(p1*p2)^2 x (p2*p3)^1 x (p3)^3
: 30
```

```
sig [ 3 3 3]
(p1^3*p2^3*p3^3)^1
(p1*p2*p3)^3
(p1*p2*p3^3)^1 x (p1*p2)^2
(p1*p2^3*p3)^1 x (p1*p3)^2
(p1*p2^3*p3^3)^1 x (p1)^2
(p1^3)^1 x (p2*p3)^3
(p1)^3 x (p2^3*p3^3)^1
(p1^3*p3^3)^1 x (p2)^3
(p1*p3)^3 x (p2^3)^1
(p1^3*p2*p3)^1 x (p2*p3)^2
(p1^3*p2*p3^3)^1 x (p2)^2
(p1^3*p2^3)^1 x (p3)^3
(p1*p2)^3 x (p3^3)^1
(p1^3*p2^3*p3)^1 x (p3)^2
(p1*p3^3)^1 x (p1)^2 x (p2)^3
(p1*p2^3)^1 x (p1)^2 x (p3)^3
(p1)^3 x (p2*p3^3)^1 x (p2)^2
(p1)^3 x (p2^3*p3)^1 x (p3)^2
(p1^3*p3)^1 x (p2)^3 x (p3)^2
(p1^3*p2)^1 x (p2)^2 x (p3)^3
: 20
```

```
sig [ 1 1 1 6]
(p1*p2*p3*p4^6)^1
(p1*p2*p3)^1 x (p4^3)^2
(p1*p2*p3)^1 x (p4^2)^3
(p1*p2*p3)^1 x (p4)^6
(p1*p2*p3*p4)^1 x (p4)^5
(p1*p2*p3*p4^2)^1 x (p4^2)^2
(p1*p2*p3*p4^2)^1 x (p4)^4
(p1*p2*p3*p4^3)^1 x (p4)^3
(p1*p2*p3*p4^4)^1 x (p4)^2
: 9
```

```
sig [ 1 1 2 5]
(p1*p2*p3^2*p4^5)^1
(p1*p2*p4)^1 x (p3*p4^2)^2
(p1*p2*p4^3)^1 x (p3*p4)^2
(p1*p2*p4^5)^1 x (p3)^2
(p1*p2*p3^2)^1 x (p4)^5
(p1*p2*p3^2*p4)^1 x (p4^2)^2
(p1*p2*p3^2*p4)^1 x (p4)^4
(p1*p2*p3^2*p4^2)^1 x (p4)^3
(p1*p2*p3^2*p4^3)^1 x (p4)^2
(p1*p2)^1 x (p3)^2 x (p4)^5
```

```
(p1*p2)^1 x (p3*p4)^2 x (p4)^3
(p1*p2*p4)^1 x (p3)^2 x (p4)^4
(p1*p2*p4^2)^1 x (p3)^2 x (p4)^3
: 13
```

```
sig [ 1 1 3 4]
(p1*p2*p3^3*p4^4)^1
(p1*p2*p4)^1 x (p3*p4)^3
(p1*p2*p4^4)^1 x (p3)^3
(p1*p2*p3)^1 x (p3*p4^2)^2
(p1*p2*p3*p4^2)^1 x (p3*p4)^2
(p1*p2*p3*p4^4)^1 x (p3)^2
(p1*p2*p3^3)^1 x (p4^2)^2
(p1*p2*p3^3)^1 x (p4)^4
(p1*p2*p3^3*p4)^1 x (p4)^3
(p1*p2*p3^3*p4^2)^1 x (p4)^2
(p1*p2)^1 x (p3)^3 x (p4^2)^2
(p1*p2)^1 x (p3)^3 x (p4)^4
(p1*p2*p4^2)^1 x (p3)^3 x (p4)^2
(p1*p2*p3)^1 x (p3)^2 x (p4)^4
(p1*p2*p3*p4)^1 x (p3)^2 x (p4)^3
: 15
```

```
sig [ 1 2 2 4]
(p1*p2^2*p3^2*p4^4)^1
(p1)^1 x (p2*p3*p4^2)^2
(p1*p4^2)^1 x (p2*p3*p4)^2
(p1*p4^4)^1 x (p2*p3)^2
(p1*p3^2)^1 x (p2*p4^2)^2
(p1*p3^2*p4^2)^1 x (p2*p4)^2
(p1*p3^2*p4^4)^1 x (p2)^2
(p1*p2^2)^1 x (p3*p4^2)^2
(p1*p2^2*p4^2)^1 x (p3*p4)^2
(p1*p2^2*p4^4)^1 x (p3)^2
(p1*p2^2*p3^2)^1 x (p4^2)^2
(p1*p2^2*p3^2)^1 x (p4)^4
(p1*p2^2*p3^2*p4)^1 x (p4)^3
(p1*p2^2*p3^2*p4^2)^1 x (p4)^2
(p1)^1 x (p2*p3)^2 x (p4)^4
(p1*p4)^1 x (p2*p3)^2 x (p4)^3
(p1*p3^2)^1 x (p2)^2 x (p4)^4
(p1*p3^2*p4)^1 x (p2)^2 x (p4)^3
(p1*p2^2)^1 x (p3)^2 x (p4)^4
(p1*p2^2*p4)^1 x (p3)^2 x (p4)^3
: 20
```

```
sig [ 1 2 3 3]
(p1*p2^2*p3^3*p4^3)^1
(p1*p3*p4)^1 x (p2*p3*p4)^2
(p1*p3*p4^3)^1 x (p2*p3)^2
(p1*p3^3*p4)^1 x (p2*p4)^2
(p1*p3^3*p4^3)^1 x (p2)^2
(p1*p2^2)^1 x (p3*p4)^3
```

```

(p1*p2^2*p4^3)^1 x (p3)^3
(p1*p2^2*p3*p4)^1 x (p3*p4)^2
(p1*p2^2*p3*p4^3)^1 x (p3)^2
(p1*p2^2*p3^3)^1 x (p4)^3
(p1*p2^2*p3^3*p4)^1 x (p4)^2
(p1)^1 x (p2)^2 x (p3*p4)^3
(p1*p4)^1 x (p2*p4)^2 x (p3)^3
(p1*p4^3)^1 x (p2)^2 x (p3)^3
(p1*p3)^1 x (p2*p3)^2 x (p4)^3
(p1*p3^3)^1 x (p2)^2 x (p4)^3
(p1*p2^2*p4)^1 x (p3)^3 x (p4)^2
(p1*p2^2*p3)^1 x (p3)^2 x (p4)^3
: 18

```

```

sig [ 2 2 2 3]
(p1^2*p2^2*p3^2*p4^3)^1
(p1)^2 x (p2^2*p3^2*p4^3)^1
(p1^2*p4)^1 x (p2*p3*p4)^2
(p1*p4)^2 x (p2^2*p3^2*p4)^1
(p1^2*p4^3)^1 x (p2*p3)^2
(p1*p3)^2 x (p2^2*p4^3)^1
(p1^2*p3^2*p4)^1 x (p2*p4)^2
(p1*p3*p4)^2 x (p2^2*p4)^1
(p1^2*p3^2*p4^3)^1 x (p2)^2
(p1*p2)^2 x (p3^2*p4^3)^1
(p1^2*p2^2*p4)^1 x (p3*p4)^2
(p1*p2*p4)^2 x (p3^2*p4)^1
(p1^2*p2^2*p4^3)^1 x (p3)^2
(p1^2*p2^2*p3^2)^1 x (p4)^3
(p1*p2*p3)^2 x (p4^3)^1
(p1*p2*p3)^2 x (p4)^3
(p1^2*p2^2*p3^2*p4)^1 x (p4)^2
(p1*p2*p3*p4)^2 x (p4)^1
(p1^2)^1 x (p2*p3)^2 x (p4)^3
(p1)^2 x (p2^2*p3^2)^1 x (p4)^3
(p1^2*p3^2)^1 x (p2)^2 x (p4)^3
(p1*p3)^2 x (p2^2)^1 x (p4)^3
(p1^2*p2^2)^1 x (p3)^2 x (p4)^3
(p1*p2)^2 x (p3^2)^1 x (p4)^3
: 24

```

```

sig [ 1 1 1 1 5]
(p1*p2*p3*p4*p5^5)^1
(p1*p2*p3*p4)^1 x (p5)^5
(p1*p2*p3*p4*p5)^1 x (p5^2)^2
(p1*p2*p3*p4*p5)^1 x (p5)^4
(p1*p2*p3*p4*p5^2)^1 x (p5)^3
(p1*p2*p3*p4*p5^3)^1 x (p5)^2
: 6

```

```

sig [ 1 1 1 2 4]
(p1*p2*p3*p4^2*p5^4)^1
(p1*p2*p3)^1 x (p4*p5^2)^2

```

```

(p1*p2*p3*p5^2)^1 x (p4*p5)^2
(p1*p2*p3*p5^4)^1 x (p4)^2
(p1*p2*p3*p4^2)^1 x (p5^2)^2
(p1*p2*p3*p4^2)^1 x (p5)^4
(p1*p2*p3*p4^2*p5)^1 x (p5)^3
(p1*p2*p3*p4^2*p5^2)^1 x (p5)^2
(p1*p2*p3)^1 x (p4)^2 x (p5)^4
(p1*p2*p3*p5)^1 x (p4)^2 x (p5)^3
: 10

sig [ 1 1 1 3 3]
(p1*p2*p3*p4^3*p5^3)^1
(p1*p2*p3)^1 x (p4*p5)^3
(p1*p2*p3*p5^3)^1 x (p4)^3
(p1*p2*p3*p4*p5)^1 x (p4*p5)^2
(p1*p2*p3*p4*p5^3)^1 x (p4)^2
(p1*p2*p3*p4^3)^1 x (p5)^3
(p1*p2*p3*p4^3*p5)^1 x (p5)^2
(p1*p2*p3*p5)^1 x (p4)^3 x (p5)^2
(p1*p2*p3*p4)^1 x (p4)^2 x (p5)^3
: 9

sig [ 1 1 2 2 3]
(p1*p2*p3^2*p4^2*p5^3)^1
(p1*p2*p5)^1 x (p3*p4*p5)^2
(p1*p2*p5^3)^1 x (p3*p4)^2
(p1*p2*p4^2*p5)^1 x (p3*p5)^2
(p1*p2*p4^2*p5^3)^1 x (p3)^2
(p1*p2*p3^2*p5)^1 x (p4*p5)^2
(p1*p2*p3^2*p5^3)^1 x (p4)^2
(p1*p2*p3^2*p4^2)^1 x (p5)^3
(p1*p2*p3^2*p4^2*p5)^1 x (p5)^2
(p1*p2)^1 x (p3*p4)^2 x (p5)^3
(p1*p2*p4^2)^1 x (p3)^2 x (p5)^3
(p1*p2*p3^2)^1 x (p4)^2 x (p5)^3
: 12

sig [ 1 2 2 2 2]
(p1*p2^2*p3^2*p4^2*p5^2)^1
(p1)^1 x (p2*p3*p4*p5)^2
(p1*p5^2)^1 x (p2*p3*p4)^2
(p1*p4^2)^1 x (p2*p3*p5)^2
(p1*p4^2*p5^2)^1 x (p2*p3)^2
(p1*p3^2)^1 x (p2*p4*p5)^2
(p1*p3^2*p5^2)^1 x (p2*p4)^2
(p1*p3^2*p4^2)^1 x (p2*p5)^2
(p1*p3^2*p4^2*p5^2)^1 x (p2)^2
(p1*p2^2)^1 x (p3*p4*p5)^2
(p1*p2^2*p5^2)^1 x (p3*p4)^2
(p1*p2^2*p4^2)^1 x (p3*p5)^2
(p1*p2^2*p4^2*p5^2)^1 x (p3)^2
(p1*p2^2*p3^2)^1 x (p4*p5)^2
(p1*p2^2*p3^2*p5^2)^1 x (p4)^2

```

```

(p1*p2^2*p3^2*p4^2)^1 x (p5)^2
: 16

sig [ 1 1 1 1 1 4]
(p1*p2*p3*p4*p5*p6^4)^1
(p1*p2*p3*p4*p5)^1 x (p6^2)^2
(p1*p2*p3*p4*p5)^1 x (p6)^4
(p1*p2*p3*p4*p5*p6)^1 x (p6)^3
(p1*p2*p3*p4*p5*p6^2)^1 x (p6)^2
: 5

sig [ 1 1 1 1 2 3]
(p1*p2*p3*p4*p5^2*p6^3)^1
(p1*p2*p3*p4*p6)^1 x (p5*p6)^2
(p1*p2*p3*p4*p6^3)^1 x (p5)^2
(p1*p2*p3*p4*p5^2)^1 x (p6)^3
(p1*p2*p3*p4*p5^2*p6)^1 x (p6)^2
(p1*p2*p3*p4)^1 x (p5)^2 x (p6)^3
: 6

sig [ 1 1 1 2 2 2]
(p1*p2*p3*p4^2*p5^2*p6^2)^1
(p1*p2*p3)^1 x (p4*p5*p6)^2
(p1*p2*p3*p6^2)^1 x (p4*p5)^2
(p1*p2*p3*p5^2)^1 x (p4*p6)^2
(p1*p2*p3*p5^2*p6^2)^1 x (p4)^2
(p1*p2*p3*p4^2)^1 x (p5*p6)^2
(p1*p2*p3*p4^2*p6^2)^1 x (p5)^2
(p1*p2*p3*p4^2*p5^2)^1 x (p6)^2
: 8

sig [ 1 1 1 1 1 1 3]
(p1*p2*p3*p4*p5*p6*p7^3)^1
(p1*p2*p3*p4*p5*p6)^1 x (p7)^3
(p1*p2*p3*p4*p5*p6*p7)^1 x (p7)^2
: 3

sig [ 1 1 1 1 1 2 2]
(p1*p2*p3*p4*p5*p6^2*p7^2)^1
(p1*p2*p3*p4*p5)^1 x (p6*p7)^2
(p1*p2*p3*p4*p5*p7^2)^1 x (p6)^2
(p1*p2*p3*p4*p5*p6^2)^1 x (p7)^2
: 4

sig [ 1 1 1 1 1 1 1 2]
(p1*p2*p3*p4*p5*p6*p7*p8^2)^1
(p1*p2*p3*p4*p5*p6*p7)^1 x (p8)^2
: 2

sig [ 1 1 1 1 1 1 1 1]
(p1*p2*p3*p4*p5*p6*p7*p8*p9)^1
: 1

```

REFERENCES

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