This paper contradicts and provides an alternative result for the problem and solution from the following pages on ProjectP and the Online Encyclopedia of Integer Sequences respectively.

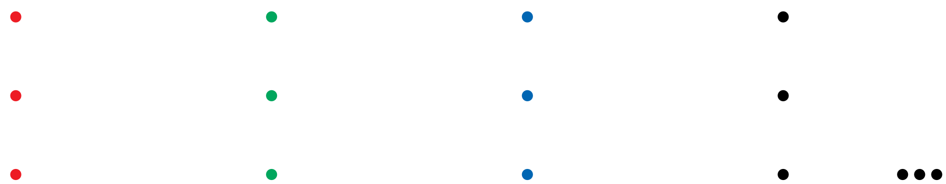
http://www.math.ucsd.edu/~projectp/warmups/eT.html
https://oeis.org/A262402

Formula for number of triangles in a 3 x n grid

The method we will use is to first count the number of combinations of three dots within a 3 x n grid and then subtract the number of three collinear points.

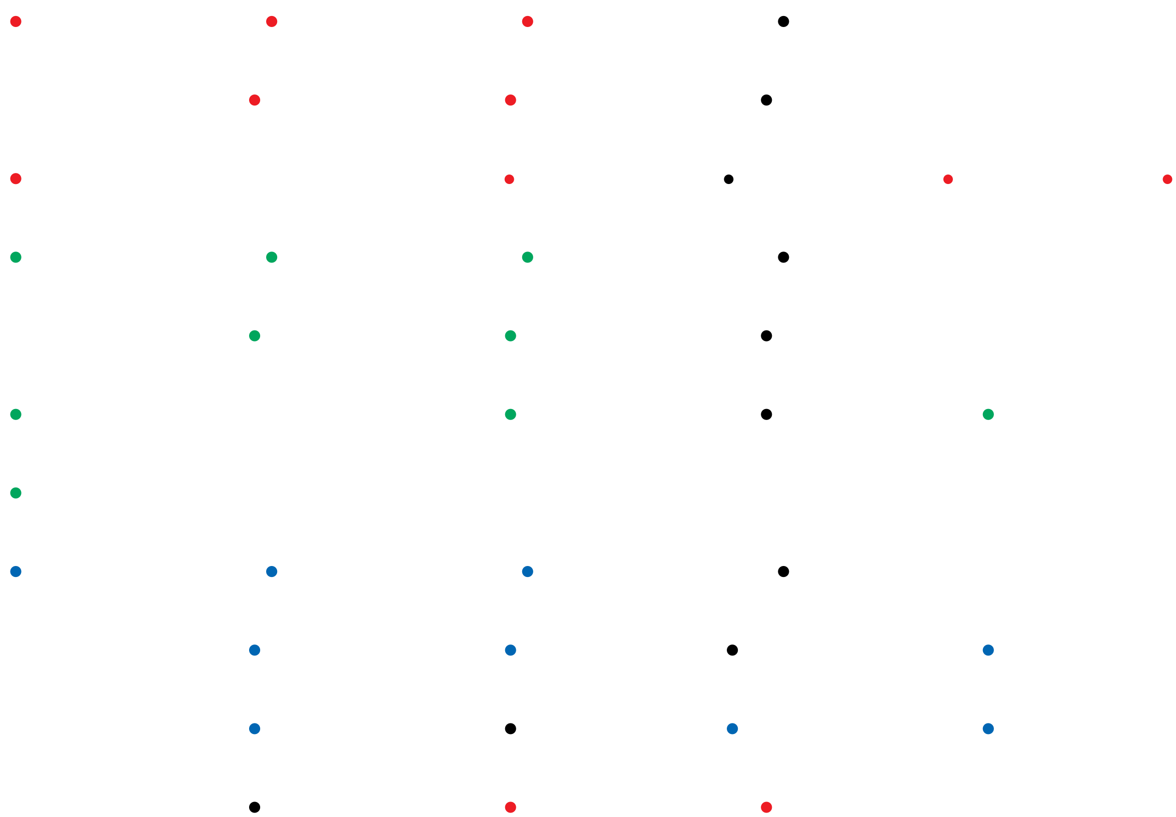
So, we get: 3*n choose 3 - # of collinear points

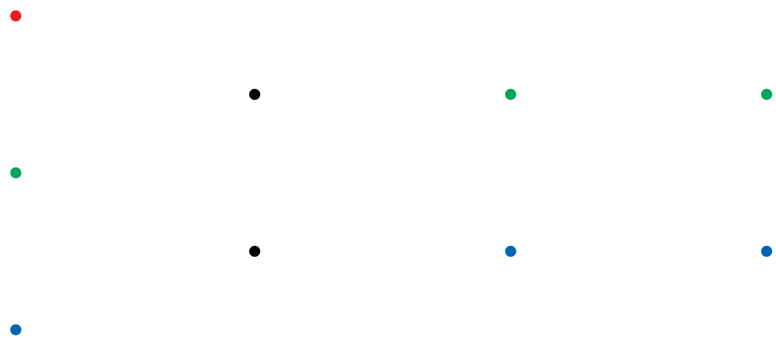
First, we can subtract the number of collinear vertical lines, which is n.



Then, we can subtract the number of horizontal lines. From each row we will have n choose 3 (for n > 2) combinations of lines and since we have three rows, it is 3*(n choose 3).

For example, the horizontal 3x4 collinear points would be



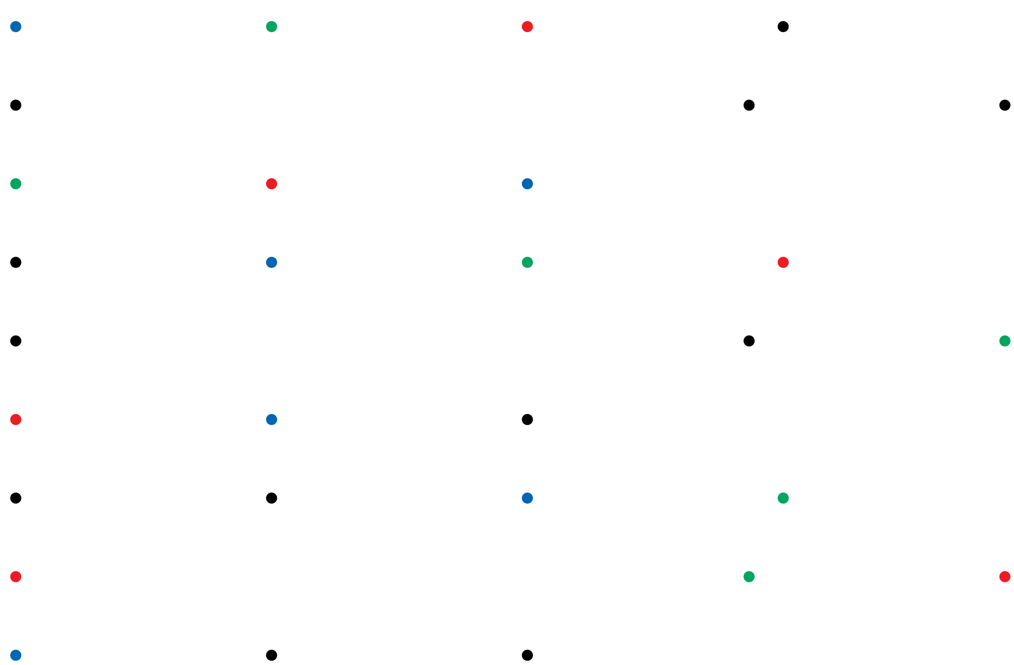


So, now we are at 3*n choose 3 – n – 3*(n choose 3) – collinear points not horizontal or vertical

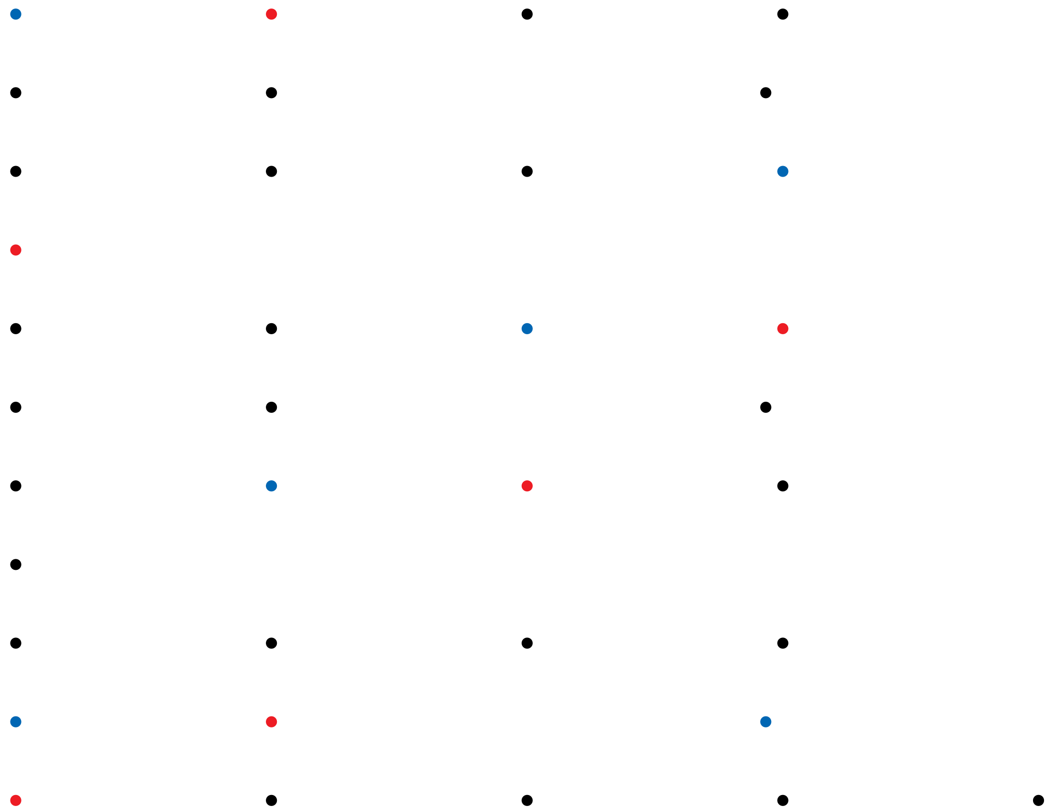
Simplified, this is 4n^3 – 3n^2 - n

The simplest case is 3 x 2, which is not difficult to see that there are no non horizontal or vertical collinear points.

However, when n > 2, we need to consider slopes +-1 (if we view this grid as regular x, y axes). Since for slopes of +-1, you are moving two spots horizontally, there will be n-2 lines for both upwards and downwards diagonals as seen below in the 3 x 5 grid. So we can subtract 2(n-2) for n>2.



Likewise, when $n>4$, we need to consider slopes $\pm 1/2$. Similar to the previous case, we count the number of lines by $n-4$ since we are moving 4 spaces horizontally. So we can subtract $2(n-4)$ for $n>4$.



We can see inductively, that we will be subtracting $2(n-k)$ for $n>k$, where k is even. So, our final result is:

n is even case:

We have $-2(n - 2) - 2(n - 4) - \dots - 2(n - k) = -2((n - 2) + (n - 4) + \dots + 4 + 2)$
$= -4((n - 2)/2 + (n - 4)/2 + \dots + 3 + 2 + 1) = -(n^2)/2 + n$

n is odd case:

We have $-2(n - 2) - 2(n - 4) - \dots - 2(n - k) = -2((n - 2) + (n - 4) + \dots + 3 + 1)$
$= -(n^2)/2 + n - 1/2$

Putting it all together, we get two cases:

n is even:

$4n^3 - (7/2)n^2$

n is odd:

$4n^3 - (7/2)n^2 - \frac{1}{2}$

**Note that we ignored the case where n=2, however it can be checked to see that 2 is still correct for the even formula.

■