## A309873 Period-Doubling Turn Sequence Curves

Turn sequence, for n = 1 to 16 :

A309873 = 1, -1, 1, 1, 1, -1, 1, -1, 1, -1, 1, 1, 1, -1, 1, 1, ... Partial sums are net direction of segment, for n = 0 to 16:

 $A068639 = 0, 1, 0, 1, 2, 3, 2, 3, 2, 3, 2, 3, 4, 5, 4, 5, 6, 7, \ldots$ 

The turn sequence for given turn angle  $\alpha$  is followed by drawing a unit segment forward then turning  $\alpha . a(n)$ , and repeat.

Davis and Knuth take A309873 and similar as folds in a long strip of a paper which is unfolded to a "bending" angle  $\theta$  at each fold. Bending, or unfolding, by  $\theta$  corresponds to turns by  $\alpha = 180^{\circ} - \theta$ .



Davis and Knuth draw turns by  $90^{\circ}$  (= bend by  $90^{\circ}$ ) which is merely 4 unit squares repeatedly re-traversed.



The period doubling sequence is a morphism (see A096268) and can be thought of by an expansion "level" k comprising  $2^k$  segments. Of course drawing any number of segments is possible.

Davis and Knuth draw an example  $120^{\circ}$  turns (= bend by  $60^{\circ}$ ) of 4096 points which is k = 12 and which they call "Fido". The resemblance to something canine is slightly improved in their drawing where segments traversed multiple times are darker. These are in the middle something like a nose (not done here).



Davis and Knuth draw 60° turns too (= bend by 120°) which is a similar Fido shape but opened up more. Curve overlapping begins in level k=5 for this angle.

