



UNIVERSITY OF  
TECHNOLOGY SYDNEY

**Qualitative Spatial and Temporal  
Representation and Reasoning:  
Efficiency in Time and Space**

*by*

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*for the degree of*

**Doctor of Philosophy**

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Faculty of Engineering and Information Technology

University of Technology Sydney (UTS)

January, 2017



# **CERTIFICATE OF ORIGINAL AUTHORSHIP**

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

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# ACKNOWLEDGEMENT

First of all, I would like to gratefully thank my supervisor, Prof. Sanjiang Li, for his patient guidance, inspiring discussions, and firm support in financial and research matters, during my four years' PhD candidature. His insistence on mathematical accuracy and emphasis on independence of research have influenced me very much.

I would also like to thank Dr. Steven Schockaert, who has given me many invaluable suggestions and inspirations on research. He is also a great friend in life who is always there to listen and to help. I have learned a lot from his enthusiasm for finding solutions to problems.

Prof. Matt Duckham is another person to whom I would like to express my sincere special thank. He always did his best to help me, without any complaint.

I would like to thank Prof. Jean-François Condotta from Artois University, France, for his kind and diligent review of an early version of this thesis.

In the early stage of my PhD candidature, Dr. Weiming Liu and Dr. Hua Meng provided me with the most patient help in research and life. From them, I learned how to think mathematically, not just to calculate mathematically.

My special thanks also belong to Michael Sioutis and Dr. Jae Hee Lee, for being great collaborators in research and friends in life. Every time I talk with them I will learn interesting things.

I am very grateful to have my family, especially my parents and my wife

supporting me all these years. They are always the source of joy, understanding, and comfort.

I also want to thank Prof. Hui Kou, who recommended me to study under the supervision of Prof. Sanjiang Li at the University of Technology Sydney.

Finally, I want to thank all my friends in Australia, China, and the UK, and colleagues at QCIS, UTS. The friendship with them is one of the most valuable things in my life. With them, I had wonderful memory of playing soccer, tasting delicious food, travelling around, and many more. Among them, I would like to specially thank Dr. Shoaib Jameel for his intriguing introduction of Linux to me and Prof. Xueying Zhang for her advice on life.

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$a^\circ$	Interior of a point set $a$	21
$\bar{a}$	Closure of a point set $a$	21
$a_1 R a_2,$ $(a_1, a_2) \in R$	$(a_1, a_2)$ satisfies the relation $R$	16
$\text{adj}(v)$	Adjacency set of vertex $v$ in a graph	28
$\alpha, \alpha_i, \beta, \beta_j$	some basic relations	18
$\alpha_{i_1} \cup \dots \cup \alpha_{i_k}$	A relation that is the union of some basic relations	18
$\alpha \diamond \beta, R \diamond S$	Weak composition of two relations	31
$\alpha \otimes \beta, R \otimes S$	Cartesian product of two relations	19
$\mathcal{R} \otimes \mathcal{S}$	Cartesian product of two subclasses	20
$\alpha \in R$	A basic relation $\alpha$ is contained in a relation $R$	18
$S \subseteq R$	A relation $S$ is contained in a relation $R$	18
$\text{BA}(n, m)$	Barabási-Albert model with preferential attachment value $m$ and $n$ vertices	89
$\text{id}_U$	Identity relation	16
$\mathbb{B}_{\mathcal{M}}$	The set of basic relation in a qualitative calculus $\mathcal{M}$	17
$\mathcal{C}$	A set of constraint	25
$c_i$	A constraint	69
$\bar{d}$	Average intersection degree	133
$\delta(a, b)$	A CDC relation	23

$F_k$	$\{v_j \in \text{adj}(v_k) : j > k\}$	29
$G = (V, E)$	An undirected graph, with vertices $V$ and edges $E$	27
$G_{\mathcal{N}}$	Constraint graph	27
$\mathcal{M}$	A qualitative calculus	17
$\text{mbr}(a)$	The MBR of $a$	21
$\mathcal{N}, \mathcal{N}'$	Qualitative constraint network	25
$\mathcal{N}_c$	Core of $\mathcal{N}$	103
$\mathcal{N}_m$	Minimal subnetwork of $\mathcal{N}$	42
$\mathcal{N}_p$	A-closure of $\mathcal{N}$	70
$\mathcal{N}_p^G$	partially path consistent subnetwork of $\mathcal{N}$ w.r.t. $G$	35
$\mathcal{N} _{V_0}$	The restriction of $\mathcal{N}$ on $V_0 \subseteq V$	27
$\mathcal{N} \models (uRv)$	$\mathcal{N}$ entails $(uRv)$	47
$o_i$	Spatial object or region	122
$\mathbf{O}_5, \mathbf{O}_8$	Specific sets of relations in RCC5/8	73
$\pi, (c_1, \dots, c_s)$	A path in a QCN	69
$\pi_{<i}, \pi_{>i}$	$(c_1, \dots, c_{i-1})$ and $(c_{i+1}, \dots, c_s)$	69
$\text{CT}(\pi)$	The composition of a path $\pi$ in a QCN	70
$ \pi $	The length of a path $\pi$ in a QCN	69
$\mathcal{P}_{xy}^{\mathcal{N}}$	The set of all paths from $x$ to $y$ in a QCN $\mathcal{N}$	70
$R, S, T$	A relation	16
$R_{ij}, S_{ij}, T_{ij}$	A relation between $v_i$ and $v_j$	16
$R^{-1}, S^{-1}, T^{-1}$	Converse of a relation	16
$\mathcal{R}, \mathcal{S}, \mathcal{T}, \mathcal{X}$	Subclass of relations or subalgebra	18
$\widehat{\mathcal{R}}, \widehat{\mathcal{S}}, \widehat{\mathcal{T}}, \widehat{\mathcal{X}}$	Closure of a subclass of relations	60



$\mathbf{Rel}(\mathcal{U})$	The power set of $\mathcal{U} \times \mathcal{U}$	17
$\sigma$	A solution of a network	37
$t, t_j$	Spatial clustering index tile	122
$\mathcal{U}$	Universe, domain	16
$(uRv)$	A constraint	25
$\star$	Universal relation	16
$V$	A set of variables or vertices	25
$v, w, u$	Variable, vertex	25
$W$	The intersection of the weak compositions of all paths from $x$ to $y$ in $\mathcal{N} \setminus \{(xRy)\}$	74
$x_a^-, y_a^-, x_a^+, y_a^+$	Lower (upper) bound of the projection of point set $a$ on $x/y$ -axis	21
$I_x(a), [x_a^-, x_a^+],$ $I_y(a), [y_a^-, y_a^+]$	The projection of point set $a$ on $x/y$ -axis	21
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# ABSTRACT

Qualitative Spatial and Temporal Reasoning (QSTR) provides a human-friendly abstract way to describe and to interpret spatial and temporal information. To describe the qualitative information, QSTR makes use of qualitative relations between entities and usually stores them in a qualitative constraint network (QCN). The QCNs are then used as the basis to process qualitative spatial and temporal information, including qualitative reasoning and query answering.

Time efficiency of reasoning techniques in QSTR is critical for applications to deal with qualitative spatial and temporal information in large-scale datasets. In this thesis, we present a special family of tractable subclasses of relations, called distributive subalgebras. We show that several efficient algorithms are applicable to the QCNs over distributive subalgebras for solving important reasoning problems. We also identify maximal distributive subalgebras for popular relation models in QSTR and point out their connections with several previously identified important subclasses.

Regarding the network representation in QSTR, there are two important problems, which in turn affect the time efficiency of other applications.

First, the network representation can have redundant relations, which will significantly increase the efforts needed for tasks whose efficiency is strongly related to the number of relations in a network. Fortunately, for any QCN over distributive subalgebras of qualitative calculi PA, RCC5, and RCC8, we show that essentially it has a unique subset consisting of non-redundant relations,

which expresses the same qualitative information as the original QCN. We also devise an efficient algorithm to construct such subsets.

Second, the network representation sometimes requires a large storage space when encoding large-scale data. This could severely limit the ability of relation retrieval for any two given spatial entities. In fact, when the size of a QCN becomes large, it might be too costly or even infeasible to fit the QCN into fast accessible storage and relation retrieval will become inefficient. We propose two alternative representation techniques to compactly encode qualitative spatial relations between regions. For this purpose, the first technique uses minimum bounding rectangles (MBRs) to encode both topological relations and directional relations, while the second technique focuses on encoding topological relations by generating axis-aligned rectangles for spatial entities. We show that for large real-world datasets of regions, these two techniques can significantly reduce the storage size of qualitative spatial information and in the meantime the relations between regions can be efficiently inferred from those simple geometric shapes.