Locating and Capturing an Evader in a Polygonal Environment

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Abstract. This paper contains two main results: First, we revisit the well-known *visibility based pursuit-evasion* problem and show that, in contrast to deterministic strategies, a single pursuer can locate an unpredictable evader in any simply-connected polygonal environment using a randomized strategy. The evader can be arbitrarily faster than the pursuer and it may know the position of the pursuer at all times but it does not have prior knowledge of the random decisions made by the pursuer. Second, using the randomized algorithm together with the solution of a known lion and man problem [12] as subroutines, we present a strategy for two pursuers (one of which is at least as fast as the evader) to quickly capture an evader in a simply-connected polygonal environment.

1 Introduction

Pursuit-evasion games are among the fundamental problems studied by Robotics researchers. In a pursuit-evasion game, one or more pursuers try to capture an evader who, in turn, tries to avoid capture indefinitely. A typical example is the homicidal chauffeur game where a driver wants to collide with a pedestrian and the goal is to determine conditions under which he can (not) do so. Among the numerous applications of this game are collision avoidance and air traffic control.

Recently, there has been increasing interest in capturing intelligent evaders with certain sensing capabilities who are contaminating a complex environment. Aside from its obvious applications in search-and-rescue and surveillance, this problem provides a natural test-bed for many mobile robot algorithms.

Perhaps the most well-understood game in this context is the *visibility-based pursuit-evasion* where one or more pursuers try to locate an evader in a polygonal environment [13,10,2]. In this game, the evader is very powerful: it has unbounded speed and global visibility, meaning that it knows the location

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of the pursuers at all times. In [15,4] the authors study a similar game in a probabilistic framework and propose a greedy algorithm.

The main ingredient of a pursuit-evasion game is the presence of an adversarial evader who actively avoids capture. Due the this aspect, the solutions to pursuit-evasion problems are game theoretic and distinguish themselves from their target finding counterparts (e.g. [11,14]) where the evader's motion is independent from the pursuer's.

In our present work, we propose randomized pursuer strategies for the visibility based pursuit-evasion problem. Randomization is a powerful technique which allows us to solve many problems that are not solvable by deterministic algorithms and has found wide-spread applications in many areas ranging from computational geometry to cryptography.

As we show in the following sections, it turns out that randomization provides a drastic increase in the power of the pursuers. For example, it is known that there are simply-connected environments where $\Theta(\log n)$ pursuers are required [2] in order to locate the evader with deterministic strategies. In contrast, we show that a single pursuer can locate the evader in any simply-connected environment with high probability, even if the evader knows the pursuer's location at all times and has unbounded speed (Theorem 1). The power of randomized strategies comes from the fact that the evader has no prior knowledge of the random decisions inherent in such strategies. It is worth noting that the randomized strategies work against any evader strategy and require no prior information about the strategy of the evader.

We also address the harder task of capturing the evader. For this problem we present a strategy for two pursuers, one of which is at least as fast as the evader. The strategy is based on the randomized strategy to locate the evader and the solution of a known lion and man problem [12] which is reviewed in Section 3.1. The same strategy can be used to capture the evader while protecting a door. This problem was introduced in [7] to model scenarios where the goal is to locate the evader which may leave the polygonal area through a door and win the game.

The two-pursuer strategy can be modified so that a single pursuer can also capture the evader. However, the expected time to capture in this case, though finite, may be significantly longer than the expected time to capture with two pursuers.

1.1 Randomized strategies

The power of randomization in the context of pursuit-evasion games is nicely illustrated by the example in Figure 1. A similar example can be found in [2].

In this example, a single pursuer can never locate the evader using a deterministic strategy: Let us distinguish four points A, B, C and D as shown in the figure. We will slightly abuse the notation and use these points to refer to the maximal convex region inside the polygon that contains these points as well. Now suppose the pursuer has a deterministic strategy of visiting those