Approximating Polygonal Objects by Deformable Smooth Surfaces

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Abstract. We propose a method to approximate a polygonal object by a deformable smooth surface, namely the t-skin defined by Edelsbrunner for all $0 < t < 1$. We guarantee that they are homeomorphic and their Hausdorff distance is at most $\epsilon > 0$. Such construction makes it possible for fully automatic, smooth and robust deformation between two polygonal objects with different topologies. En route to our results, we also give an approximation of a polygonal object with a union of balls.

1 Introduction

Geometric deformation is a heavily studied topic in disciplines such as computer animation and physical simulation. One of the main challenges is to perform deformation between objects with different topologies, while at the same time maintaining a good quality mesh approximation of the deforming surface.

Edelsbrunner defines a new paradigm for the surface representation to solve these problems, namely the *skin surface* [5] which is a smooth surface based on a finite set of balls. It provides a robust way of deforming one shape to another without any constraints on features such as topologies [2]. Moreover, the skin surfaces possess nice properties such as curvature continuity which provides quality mesh approximation of the surface [3].

However, most of the skin surface applications are still mainly on molecular modeling. The surface is not widely used in other fields because general geometric objects cannot be represented by the skin surfaces easily. This leaves a big gap between the nicely defined surfaces and its potential applications. We are trying to fill this gap in this paper.

1.1 Motivation and Related Works

One of the main goals of the work by Amenta et. al in [1] is to convert a polygonal object into a skin surface. We can view our work here as achieving this goal and the purpose of doing so is to perform deformation between polygonal objects. As noted earlier in some previous works [2,5], deformation can be performed robustly and efficiently if the object is represented by the skin surface.

Moreover, our work here can also be viewed as a step toward converting an arbitrary smooth object into a provably accurate skin surface. In this regard,

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previous work has been done by Kruithof and Vegter [8]. For input their method requires a so-called r-admissible set of balls B which approximate the object well. Then, it expands all the weights of the balls by a carefully computed constant t, before taking the $\frac{1}{t}$-skin of the expanded balls to approximate the smooth object.

However, we observe that there are at least two difficulties likely to occur in such approach. First, such an r-admissible balls are not trivial to obtain. Furthermore, when the computed factor t is closed to 1, the skin surface is almost the same as the union of balls, thus, does not give much improvement from the union of balls. On the other hand, the approach discussed here allows the freedom to choose any constant $0 < t < 1$ for defining the skin surface.

On top of the skin approximation, we also give an approximation of a polygonal object with a union of balls. Such approximation has potential applications in computer graphics such as collision detection and deformation [7,9,10]. Ranjan and Fournier [9] proposed using a union of balls for object interpolation. Sharf and Shamir [10] also proposed using the same representation for shape matching. Those algorithms require a union of balls which accurately approximate the object as an input and to provide such a good set of balls at the beginning is still not trivial.

A comparison with our Previous Work. In [4], we proposed a method to construct a set of weighted points whose alpha shape is the same as the input simplicial complex in $\mathbb{R}^d$, which we call the *subdividing alpha complex.* Given such alpha complex it is quite straightforward to obtain a set of balls which can be used to approximate the object. However, to construct the subdividing alpha complex, we need to make the assumption that the constrained triangulation of the input is given too.

In this paper the input is a piecewise linear complex which constitutes the boundary of the object. To avoid assuming we are given the constrained triangulation, we make use of the notion of *local gap size*(lgs) in the construction of the subdividing alpha complex.

1.2 Approach and Outline

The first step is to construct a set of balls whose alpha shape is the same as the boundary of the polygonal object, namely, the subdividing alpha complex. The radii of the balls constructed are at most ϵ, for a given real number $\epsilon > 0$.

In the second step we fill the interior of the object with balls according to the Voronoi complex of the balls constructed in the first step, namely, the balls that make up the subdividing alpha complex. Specifically, we consider all the Voronoi vertices which are inside the object. Each Voronoi vertex determines an orthogonal ball. We use the set of all such orthogonal balls to approximate the object. It is shown that that the union of such balls is homeomorphic to the object and furthermore, the Hausdorff distance between them is at most ϵ.

To obtain the skin approximation, we invert the weights of the balls that make up the subdividing alpha complex of the boundary. Those inverted balls,