Evaluating Methods for Interpolating Continuous Surfaces from Irregular Data: a Case Study

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Abstract

An artificial and 'real' set of test data are modelled as continuous surfaces by linear interpolators and three different cubic interpolators. Values derived from these surfaces, of both elevation and slope, are compared with analytical values for the artificial surface and a set of independently surveyed values for the real surface. The differences between interpolators are shown with a variety of measures, including visual inspection, global statistics and spatial variation, and the utility of cubic interpolators for representing curved areas of surfaces demonstrated.

1 Introduction

Terrain models and their derivatives are used in a wide range of applications as 'off the shelf' products. However, Schneider (2001b) points out how the representation of surface continuity in many applications is both implicit and contradictory for different products of the terrain model. The use of continuous representations of terrain, which are argued to better represent the real nature of terrain surfaces, is suggested as an important area of research. Furthermore, it is stated that the nature of a representation should also be application specific with, for example, a surface from which avalanche starting zones are to be derived implying a different set of constraints than those required to interpolate temperature. In the latter case it is sufficient to know only elevation at a point, whereas in the former derivatives of the terrain surface such as gradient, aspect and curvature (Maggioni and Gruber, 2003) are all required.

Models of terrain can generally be categorised as either regular or irregular tessellations of point data sometimes with additional ancillary information representing structural features such as breaks in slope or drainage divides (Weibel and Heller, 1991). Regular tessellations have dominated within the modelling and spatial analysis communities, despite the oft-cited advantages of irregular tessellations (e.g. Weibel and Heller, 1991). Within the domain of regular tessellations geomorphologists and GIScientists have combined to examine the robustness of descriptive indices of topography, such as slope and aspect (e.g. Evans, 1980; Skidmore 1989; Corripio, 2003; Schmidt et al., 2003), hypsometry, and hydrological catchment areas (Walker and Willgoose, 1999; Gallant and Wilson, 2000) all derived using a range of data models, resolutions and algorithms.

Irregular tessellations are commonly based upon a triangular irregular network (TIN) which in itself is derived from point or line data (e.g. Peucker et al., 1978). Surfaces interpolated from irregular tessellations may or may not fulfil basic conditions of continuity, with Schneider (2001b) describing a family of techniques derived from Computer-Aided Graphics Design (CAGD) which may be used to attempt to fulfill continuity constraints. Hugentobler (2002) describes one of these techniques, triangular Coons patches, which are applied to the problem of representing a continuous surface.

In comparison to regular tessellations, little work has been carried out to assess the implications of differing surface representations and their resulting products with irregular tessellations, as opposed to comparisons between regular and irregular tessellations (though the work of Kumler (1994) is an exception to this observation). In this paper we introduce a case study where a suite of interpolation methods are applied to a TIN and resulting elevations and first order surface derivatives compared in order to assess the properties of different techniques.

The paper first lists a range of methods for comparing the properties of these representations, mostly derived from the literature on regular tessellations. A methodology for carrying out such comparisons on irregularly tessellated points is then presented and a subset of the resulting values are discussed. The implications of the case study for interpolation of TINs for differing applications are then examined. Finally, some recommendations for further work in this area are made.