Parallel Pole Assignment of Single-Input Systems*

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Abstract. We present a parallelization of Petkov, Christov, and Konstantinov's algorithm for the pole assignment problem of single-input systems. Our new implementation is specially appropriate for current high performance processors and shared memory multiprocessors and obtains a high performance by reordering the access pattern, while maintaining the same numerical properties.
The experimental results on two different platforms (SGI PowerChallenge and SUN Enterprise) report a higher performance of the new implementation over traditional algorithms.

1 Introduction

Consider the continuous, time-invariant linear system defined by

$$\dot{x}(t) = Ax(t) + Bu(t), \quad x(0) = x_0,$$

with n states, in vector $x(t)$, and m inputs, in vector $y(t)$. Here, A is the $n \times n$ state matrix, and B is the $n \times m$ input matrix.

In the design of linear control systems, $u(t)$ is used to control the behaviour of the system. Specifically, the control

$$u(t) = -Fx(t),$$

where F is an $m \times n$ feedback matrix, is used to modify the properties of the closed-loop system

$$\dot{x}(t) = (A - BF)x(t).$$

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The problem of finding an appropriate feedback F is referred to as the *problem of synthesis of a state regulator* [11]. In some applications, e.g., for asymptotic stability [4,11], F can be chosen so that the eigenvalues of the closed-loop matrix are in the open left-half complex plane.

In this paper we are interested in the *pole assignment problem* of single-input systems ($m = 1$ and $B = b$ is a vector), or PAPSIS, which consists in the determination of a feedback vector $F = f$, such that the poles of the closed-loop system are allocated to a pre-specified set $\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_n\}$ [4]. This problem has a solution (unique in the single-input case) if and only if the system is controllable [15]. We assume hereafter that this condition is satisfied.

A survey of existing algorithms for the pole assignment problem can be found, e.g., in [4,5,6,11,14]. Among these, methods based on the Schur form of the closed-loop state matrix [6,9,10] are numerically stable [3,7].

In [2] we apply block-partitioned techniques to obtain efficient implementations of Miminis and Paige's algorithm for PAPSIS [6]. In this paper we apply similar techniques to obtain LAPACK-like [1] block-partitioned variants and parallel implementations of Petkov, Christov, and Konstantinov's algorithm (hereafter, PCK) [10] for PAPSIS.

We assume the system to be initially in unreduced controller Hessenberg form [13]. This reduction can be carried out by means of efficient blocked algorithms based on (rank-revealing) orthogonal factorizations [12].

Our algorithms are specially designed to provide a better use of the cache memory, while maintaining the same numerical properties. The experimental results on SGI PowerChallenge and SUN Enterprise multiprocessors report the performance of our block-partitioned serial and parallel algorithms.

2 The Sequential PCK Algorithm

Consider the controllable single-input system in controller Hessenberg form defined by (A, b), with real entries,

$$(b|A) = \left[\begin{array}{c|cccc} \beta_1 & \alpha_{11} & \ldots & \alpha_{1,n-1} & \alpha_{1n} \\ & \alpha_{21} & \ldots & \alpha_{2,n-1} & \alpha_{2n} \\ & & \ddots & \vdots & \vdots \\ & & & \alpha_{n,n-1} & \alpha_{nn} \end{array}\right]. \tag{1}$$

As the system is controllable, it can be shown that $\beta_1, \alpha_{21}, \ldots, \alpha_{n,n-1} \neq 0$ [13].

The PCK algorithm is based on orthogonal transformations of the eigenvectors and proceeds as follows. (For simplicity we only describe the algorithm for pole assignment of real eigenvalues.) Let $\lambda \in \mathbb{R}$ and $v \in \mathbb{R}^n$ be, respectively, an eigenvalue and its corresponding eigenvector of the closed-loop matrix $A - bf$. Let Q be an orthogonal matrix such that $Qv = (v_1, 0, \ldots, 0)^T$. This matrix can be constructed so that $Q^T AQ$ and $Q^T (A - bf)Q$ are in Hessenberg form. Furthermore,

$$Q^T (A - bf)Qe_1 = (\lambda, 0, \ldots, 0)^T, \tag{2}$$