Decision Procedures Using Model Building Techniques

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Abstract. Few year ago we have developed an Automated Deduction approach to model building. The method, called RAMC[1] looks simultaneously for inconsistencies and models for a given formula. The capabilities of RAMC have been extended both for model building and for unsatisfiability detection by including in it the use of *semantic strategies*. In the present work we go further in this direction and define more general and powerful semantic rules. These rules are an extension of Slagle's semantic resolution. The robustness of our approach is evidenced by proving that the method is also a decision procedure for a wide range of classes decidable by semantic resolution and in particular by hyper-resolution. Moreover, the method *builds models* for satisfiable formulae in these classes, in particular, for satisfiable formulae that do not have any finite model.

1 Introduction

Model building and model checking are extremely important topics in Logic and Computer Science. Few years ago we have developed an Automated Deduction approach to model building. The method, called RAMC, looks simultaneously for inconsistencies and models for a given formula. It is refutationally complete, builds models incrementally and allows a unified view of model building and model checking [CZ92, CP95b]. A particularly interesting feature of the method is that it allows to build *infinite* models as well as finite ones. This feature is particularly interesting when dealing with decidable — but non finitely controllable[2] — classes.

In order to increase the capabilities of RAMC (both for proof search and model building), we introduced in the method the use of *semantic strategies* [CP95a] by modifying some of its key rules. In the present work we go further in this direction by defining more general and more powerful rules. These rules can be seen as an extension of semantic resolution as defined in [SLA67]. Concerning model building, our method can be seen as a procedure transforming partial

[1] standing for Refutation And Model Construction.

[2] A class is said *finitely controllable* iff any satisfiable formula in the class has a finite model.

(even trivial) models into total ones (the trivial models are the interpretations needed for the application of semantic resolution). This feature is especially important in interactive model building.

The study of decidable classes is also an outstanding problem in Logic and Computer Science. Though decidability is strongly related to the existence of models, standard techniques used in the field of decidability are different from those used in model building. In the study of decidable classes a key technique is to show the existence of bounds (say $n \in \mathbb{N}$) such that it is sufficient to expand a formula schema up to a maximum of n Herbrand instances and then test this finite expansion for (in)consistency to decide the validity of the formula (see for example [DG79, BÖR84, LEW79]). A characteristic of this technique is that there is practically *no uniform* treatment of the different classes (for each class the bound n must be found in an ad hoc manner). A first unified approach to treat decidable class — to the best of our knowledge — was the one by the Russian school (Maslov, Zamov)[3]. The first work in this direction in the West seems to be [JOY76] who used resolution as a decision procedure. More recently Fermüller, Leitsch, Tammet and Zamov went further in this direction using resolution as the base calculus, instead of Maslov's inverse method (see [FL92, FLTZ93, TAM91]). They studied strategies for partitioning the Herbrand universe in a finite number of equivalence classes, allowing to generate finite search spaces. In some cases they are also able to extract models from the set of generated Herbrand instances [FL92, TAM91, FL95].

In principle our model building method was not intended to be a unified approach to treat decidable classes. Now, intuitively, a method that looks simultaneously for complementary goals: refutations and models, should "work" as a decision procedure. *Therefore the main aim of this work is to put links between our work on model building and the unified treatment of decidable classes of first order logic.* More precisely, we show in this paper the robustness of our approach by proving that the method is also a decision procedure for a wide range of classes decidable by semantic resolution, and in particular by hyper-resolution. Therefore we capture all the classes decidable by the Fermüller and Leitsch model building method [FL95]. Besides the method *builds models* for all the satisfiable formulae in these classes.

The paper is divided into 6 sections: Section 2 recalls briefly the ideas underlying our method and the notions necessary for the understanding of the paper (in particular some disinference rules). In section 3 two new rules are introduced: the $\mathcal{I}$-resolution and the $\mathcal{I}$-disresolution. It is proven that they are sound and preserve refutational completeness. In section 4, after recalling a very useful rule: the so-called GMPL rule, we show that it increases strictly the power of the method. In section 5, the limits of the method without strategy are shown. A strategy restricting the application of the disinference rules is proposed. Classes of c-clauses for which the method with the new strategy is a decision procedure and builds models are identified. Section 6 contains concluding remarks and main lines of future work.

[3] [MIN91], page 397: "One of the aims of the inverse method was to give a unified treatment of decidable cases of the predicates calculus"