A New Formulation and Solution of the Minimum Energy Control Problem of Positive 2D Continuous-Discrete Linear Systems

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Abstract. A new formulation of the minimum energy control problem for the positive 2D continuous-discrete linear systems is proposed. Necessary and sufficient conditions for the reachability of the systems are established. Conditions for the existence of the solution to the minimum energy control problem and procedures for computation of an input minimizing the given performance index are given. Effectiveness of the procedure is demonstrated on numerical example.

Keywords: 2D continuous-discrete, linear, positive system, reachability, minimum energy control.

1 Introduction

A dynamical system is called positive if its trajectory starting from any nonnegative initial state remains forever in the positive orthant for all nonnegative inputs. An overview of state of the art in positive theory is given in the monographs [5, 10]. Variety of models having positive behavior can be found in engineering, economics, social sciences, biology and medicine, etc.

The positive 2D continuous-discrete linear systems have been introduced in [14], positive hybrid linear systems in [11] and the positive fractional 2D hybrid systems in [13]. Different methods of solvability of 2D hybrid linear systems have been discussed in [25] and the solution to singular 2D hybrids linear systems has been derived in [27]. The realization problem for positive 2D hybrid systems has been addressed in [15]. Some problems of dynamics and control of 2D hybrid systems have been considered in [4, 6]. The problems of stability and robust stability of 2D continuous-discrete linear systems have been investigated in [1–3, 9, 26, 28, 29] and of positive fractional 2D continuous-discrete linear systems in [12]. Recently the stability and robust stability of general model and of Roesser type model of scalar continuous-discrete linear systems have been analyzed by Busłowicz in [2, 3]. Stability of continuous-discrete 2D linear systems has been considered in [17]. The minimum energy control problem for standard linear systems has been formulated and solved by Klamka [21–24] and for 2D linear systems with variable coefficients in

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[20]. The controllability and minimum energy control problem of linear systems with distributed delays has been investigated by Klamka in [24]. The minimum energy control of fractional positive continuous-time linear systems has been addressed in [7] and for descriptor positive discrete-time linear systems in [8].

In this paper a new formulation and solution to the minimum energy control problem for positive 2D continuous-discrete linear systems will be presented.

The paper is organized as follows. In section 2 necessary and sufficient conditions for the positivity of 2D continuous-discrete linear systems are established. The reachability and the problem formulation are given in section 3. Problem solution and a procedure for solving the minimum energy control problem are given in section 4. Concluding remarks are given in section 5.

The following notation will be used: $\Re$ – the set of real numbers, $\Re^{n\times m}$ – the set of $n\times m$ real matrices, $\Re_+^{n\times m}$ – the set of $n\times m$ matrices with nonnegative entries and $\Re_+^n = \Re_+^{n\times 1}$, M_n – the set of $n\times n$ Metzler matrices (real matrices with nonnegative off-diagonal entries), I_n – the $n\times n$ identity matrix.

2 Positivity of 2D Continuous-Discrete Systems

Consider the 2D continuous-discrete linear system

$$\dot{x}(t,i) = Ax(t,i) + Bu(t,i) \tag{1}$$

where $\dot{x}(t,i) = \frac{\partial x(t,i)}{\partial t}$, $x(t,i)\in\Re^n$, $u(t,i)\in\Re^m$ are the state and input vectors and $A\in\Re^{n\times n}$, $B\in\Re^{n\times m}$ $(n\geq m)$ and $t\in\Re_+$ is continuous variable (usually time) and $i\in Z_+$ is discrete variable.

Definition 1. The system (1) is called (internally) positive if $x(t,i)\in\Re_+^n$, $t\in\Re_+$, $i\in Z_+$ for any boundary conditions $x_{0i}\in\Re_+^n$, $x_{t0}\in\Re_+^n$, $\dot{x}_{t0}\in\Re_+^n$ and all inputs $u(t,i)\in\Re_+^m$, $t\in\Re_+$, $i\in Z_+$.

Theorem 1. The system (1) is positive if and only if

$$A\in M_n \text{ and } B\in\Re_+^{n\times m}. \tag{2}$$

Proof. Necessity. Let $u(t,i)=0$, $t\geq 0$ and $x(0,i)=e_i$ (i-th ($i=1,...,n$) column of the identity matrix I_n). The trajectory does not live the orthant $\Re_+^n$ only if the derivative $\dot{x}(0,i) = Ae_i \geq 0$, what implies $a_{ij}\geq 0$, $i\neq j$. Therefore, the matrix A has to be the Metzler matrix. For the same reasons for $x(0,i)=0$ we have $\dot{x}(0,i) = Bu(0,i)\geq 0$, what implies $B\in\Re_+^{n\times m}$, since $u(0,i)\in\Re_+^m$ may be arbitrary for $i\in Z_+$.