Subdividing Alpha Complex

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Abstract. Given two simplicial complexes $\mathcal{C}_1$ and $\mathcal{C}_2$ embedded in Euclidean space $\mathbb{R}^d$, $\mathcal{C}_1$ *subdivides* $\mathcal{C}_2$ if (i) $\mathcal{C}_1$ and $\mathcal{C}_2$ have the same underlying space, and (ii) every simplex in $\mathcal{C}_1$ is contained in a simplex in $\mathcal{C}_2$. In this paper we present a method to compute a set of weighted points whose alpha complex subdivides a given simplicial complex.

Following this, we also show a simple method to approximate a given polygonal object with a set of balls via computing the subdividing alpha complex of the boundary of the object. The approximation is robust and is able to achieve a union of balls whose Hausdorff distance to the object is less than a given positive real number ϵ.

1 Introduction

The notion of alpha complexes is defined by Edelsbrunner [6, 10] and since then it has been widely applied in various fields such as computer graphics, solid modeling, computational biology, computational geometry and topology [7, 8]. In this paper, we propose a simple algorithm to compute the alpha complex that subdivides a given simplicial complex. This can be considered as representing the complex with a finite set of weighted points. See Figure 1 as an example. Moreover, we also present a method to approximate an object with a union of balls via its subdividing alpha complex.

1.1 Motivation and Related Works

The motivation of this paper can be classified into two categories: the skin approximation and the conforming Delaunay complex.

Skin approximation. Our eventual goal is to approximate a given simplicial complex with the *skin surface*, which is a smooth surface based on a finite set of balls [7]. Amenta et. al. [1] have actually raised this question and the purpose was to perform deformation between polygonal objects. As noted in some previous works [2, 7], deformation can be performed robustly and efficiently with the skin surface. Our work here can be viewed as a stepping stone to our main goal.

As mentioned by Kruithof and Vegter [13], one of the first steps to approximate an object with a skin surface is to have a set of balls that approximate the object. For this purpose, we produce a set of balls whose alpha shape is the same as the object. It is well known that such union of balls is homotopy equivalent

K. Lodaya and M. Mahajan (Eds.): FSTTCS 2004, LNCS 3328, pp. 186–197, 2004.

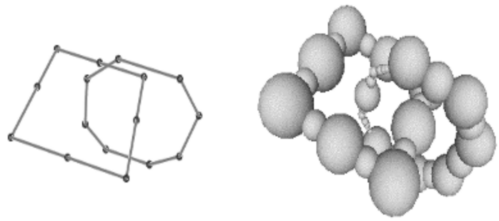

Fig. 1. An example of a subdividing alpha complex of a *link* embedded in $\mathbb{R}^3$. The right hand side of the figure shows a union of balls whose alpha complex resemble the input *link*, shown in the left hand side

to the object [6]. At the same time, we are able to produce a union of balls that approximate the object.

Approximating an object by a union of balls itself has applications in deformation. In such representation, shapes can be interpolated [15]. Some shape matching algorithms also use the union of balls representation [18]. We believe such approximation can also be useful for collision detection and coarse approximation [12].

Conforming Delaunay Complex(CDC). The work on conforming Delaunay complex(CDC) are done mainly for the unweighted point set in two and three dimensional cases [3, 4, 5, 11, 14]. As far as our knowledge is concerned, there is no published work yet on the construction of CDC for any given simplicial complex in arbitrary dimension. The relation of CDC to our work here should be obvious, that is, we compute CDC of weighted points in arbitrary dimension.

1.2 Assumptions and Approach

The assumption of our algorithm is a constrained triangulation of a simplicial complex $\mathcal{C}$ is given, that is, a triangulation of the convex hull of $\mathcal{C}$ that contains $\mathcal{C}$ itself. An example of this is the constrained Delaunay triangulation of $\mathcal{C}$ [19, 20].

Our approach is to construct the subdividing alpha complex of the l-skeleton of $\mathcal{C}$ before its $(l+1)$-skeleton. For each simplex in the l-skeleton, we add weighted points until it is subdivided by the alpha complex. In the process, we maintain the invariant that the alpha shape is a subset of the underlying space of $\mathcal{C}$. This is done by avoiding two weighted points intersecting each other when their centers are not in the same simplex. For this purpose, we introduce the *protecting cells*.

The main issue in this approach is that we do not add infinitely many points, that is, our algorithm is able to terminate. To establish this, we guarantee that, for each simplex, there is a positive lower bound for the weight of the added point. This fact, together with the compactness of each simplex, ensures that only finitely many points are added into the simplex. We will formalize this fact in Section 5.