Bayesian Image Estimation from an Incomplete Set of Blurred, Undersampled Low Resolution Images*

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Abstract. This paper deals with the problem of reconstructing a high-resolution image from an incomplete set of undersampled, blurred and noisy images shifted with subpixel displacement. We derive mathematical expressions for the calculation of the maximum a posteriori estimate of the high resolution image and the estimation of the parameters involved in the model. We also examine the role played by the prior model when this incomplete set of low resolution images is used. The performance of the method is tested experimentally.

1 Introduction

High resolution images can, in some cases, be obtained directly from high precision optics and charge coupled devices (CCDs). However, due to hardware and cost limitations, imaging systems often provide us with only multiple low resolution images. In addition, there is a lower limit as to how small each CCD can be, due to the presence of shot noise [1] and the fact that the associated signal to noise ratio (SNR) is proportional to the size of the detector [16].

Over the last two decades research has been devoted to the problem of reconstructing a high-resolution image from multiple undersampled, shifted, degraded frames with subpixel displacement errors (see [3] for a review). Most of the reported work addresses the problem of estimating an $LM \times LN$ high resolution image from at least $L \times L$ low resolution images of size $M \times N$, that is, when the number of available low resolution images is at least equal to L^2, where L is the magnifying factor. In Molina et al. [12] a method for simultaneously estimating the high resolution image and the associated parameters within the Bayesian model is presented. Kim et al. [9] explore the conditions the shifts of the $L \times L$ low resolution images have to satisfy in order to solve the high resolution problem, at least from the least squares perspective. Elad and Feuer [6]

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study the same problem when combining Bayesian, Projection onto Convex Sets and Maximum Likelihood methodologies on high resolution problems. Baker and Kanade [2] also examine the impact of increasing the number of low resolution images, when proposing an alternative approach to the super resolution problem. However, not much work has been reported on the role played by the prior model when the system is incomplete, that is, when we have less than $L \times L$ low resolution images or when the shifts do not satisfy the conditions in [9] or [6]. In our previous work [10] we proposed a new method to solve the high resolution problem from an incomplete set of low resolution images when no blurring was present in the observation process. The method was based on the general framework for frequency domain multi-channel signal processing developed by Katsaggelos et al. in [8] (a formulation that was also later obtained by Bose and Boo [4] for the high resolution problem).

In this paper we extend the approach in [10] by considering that the low resolution images are also blurred, a case that frequently appears in Astronomy (see [11] for instance) and remote sensing. We also propose a method for estimating the high resolution image and the parameters associated to the model when blurring is present in the low resolution observations. The method performs well even when very few low resolution blurred images are available and they have different noise characteristics. Finally, we examine how the prior model compensates for the lack of information in the incomplete noisy and blurred low resolution observation set.

The rest of the paper is organized as follows. The problem formulation is described in section 2. In section 3 the degradation and image models used in the Bayesian paradigm are described. The application of the Bayesian paradigm to calculate the MAP high resolution image and estimate the hyperparameters is described in section 4. Experimental results are described in section 5. Finally, section 6 concludes the paper.

2 Problem Formulation

Consider a camera sensor with $N_1 \times N_2$ pixels and assume that we have a set of q shifted images, $1 \leq q \leq L \times L$. Our aim is to reconstruct an $M_1 \times M_2$ high resolution image with $M_1 = L \times N_1$ and $M_2 = L \times N_2$, from the set of low-resolution observed images.

The low resolution sensors are shifted with respect to each other by a value proportional to $T_1/L \times T_2/L$, where $T_1 \times T_2$ is the size of each sensing element (note that if the sensors are shifted by values proportional to $T_1 \times T_2$ or $q < L \times L$, the high-resolution image reconstruction problem becomes singular). In this paper we assume that the normalized horizontal and vertical displacements are known (see [4, 13] for details). When the displacements are unknown see, for instance, [7] and [17] for their estimations.

Let $\boldsymbol{g}_{l1,l2}$ be the $(N_1 \times N_2) \times 1$ observed low resolution image acquired by the $(l1, l2)$ sensor. Our goal is to reconstruct $\boldsymbol{f}$, the $(M_1 \times M_2) \times 1$ high resolution