Chapter 17

Designing Nonrecursive Digital Filters of Finite Length

The most direct method for designing a digital filter of finite length is to truncate the infinitely long impulse response sequence of the ideal filter. This leads to distortions of the desired filter characteristics, which can be only partially corrected by additional weighting of the impulse response. There are other procedures, however, which better approximate the desired filter frequency response function.

In practice, one of the following methods is generally used to design a nonrecursive digital filter:

- Fourier series approximation of the analog filter,
- sampling of the frequency response function of the analog filter,
- design of filters based on an acceptable error tolerance between the frequency response function of the digital filter and its analog ideal.

In this chapter, the frequency response function of the analog filter is designated by $\hat{H}(f)$, that of the digital filter by $H(f)$. $\hat{H}_k$ and H_k designate the sampled values of the analog filter $\hat{H}(f)$ and the digital filter $H(f)$, respectively. The following discussion of nonrecursive digital filter design is largely based on that of Oppenheim & Schafer (1975).

17.1 Designing Digital Filters by Fourier Series Approximation

The frequency response function of a digital filter has the following properties:

- The frequency response function is periodic with $\frac{1}{\Delta t}$:

$$H(f) = H(f + \frac{n}{\Delta t}) \ , \quad n = 0, \pm 1, \pm 2, \ldots \tag{17.1}$$

B. Buttkus, *Spectral Analysis and Filter Theory in Applied Geophysics*

- For real impulse response functions, the real and imaginary parts of $H(f)$ are even and odd functions, respectively, relative to $f = 0$:

$$|H(-f)| = |H(f)| \tag{17.2}$$

and

$$\Phi(-f) = -\Phi(f)\ , \quad 0 \le |f| \le \frac{1}{2\Delta t}\ . \tag{17.3}$$

The periodic frequency response function of a digital filter (see equation (17.1)) can be expressed as a Fourier series. According to equations (1.7) and (1.9),

$$H(f) = \sum_{n=-\infty}^{\infty} h_n e^{i2\pi n\Delta t f} \tag{17.4}$$

and

$$h_n = \Delta t \int_{-\frac{1}{2\Delta t}}^{\frac{1}{2\Delta t}} H(f) e^{-i2\pi n\Delta t f} df,\ n = 0, \pm 1, \pm 2, \ldots \tag{17.5}$$

The digital filter defined by Eq. (17.4) has the following transfer function:

$$H(z) = \sum_{n=-\infty}^{\infty} h_n z^{-n}\ . \tag{17.6}$$

(h_n) is the impulse response of the digital filter. If the series in equation (17.6) is truncated at $n = \pm N$, then

$$H_N(z) = \sum_{n=-N}^{N} h_n z^{-n}\ . \tag{17.7}$$

The frequency response function of this Fourier series approximation is, therefore,

$$H_N(f) = \sum_{n=-N}^{N} h_n e^{i2\pi n\Delta t f}\ . \tag{17.8}$$

The Fourier coefficients h_n are calculated using Eq. (17.5). The procedure can be illustrated by the following two examples:

Example 1: Determine the Fourier series approximation of length $2N + 1 = 11$ for the following analog low-pass filter:

$$\hat{H}(f) = \begin{cases} M & \text{for } -f_c \le f \le f_c \\ 0 & \text{otherwise}\ . \end{cases} \tag{17.9}$$