A SIMPLE AND GENERAL GRAPH NEURAL NETWORK WITH STOCHASTIC MESSAGE PASSING

Ziwei Zhang[*], **Chenhao Niu**[*], **Peng Cui**
Department of Computer Science and Technology, Tsinghua University, Beijing, China
{zw-zhang16,nch16}@mails.tsinghua.edu.cn,cuip@tsinghua.edu.cn

Bo Zhang
Tencent
nevinzhang@tencent.com

Wei Cui
Squirrel AI Learning
cuiwei@songshuai.com

Wenwu Zhu
Department of Computer Science and Technology, Tsinghua University, Beijing, China
wwzhu@tsinghua.edu.cn

ABSTRACT

Graph neural networks (GNNs) are emerging machine learning models on graphs. One key property behind the expressiveness of existing GNNs is that the learned node representations are permutation-equivariant. Though being a desirable property for certain tasks, however, permutation-equivariance prevents GNNs from being proximity-aware, i.e., preserving the walk-based proximities between pairs of nodes, which is another critical property for graph analytical tasks. On the other hand, some variants of GNNs are proposed to preserve node proximities, but they fail to maintain permutation-equivariance. How to empower GNNs to be proximity-aware while maintaining permutation-equivariance remains an open problem. In this paper, we propose Stochastic Message Passing (SMP), a general and simple GNN to maintain both proximity-awareness and permutation-equivariance properties. Specifically, we augment the existing GNNs with stochastic node representations learned to preserve node proximities. Though seemingly simple, we prove that such a mechanism can enable GNNs to preserve node proximities in theory while maintaining permutation-equivariance with certain parametrization. Extensive experimental results demonstrate the effectiveness and efficiency of SMP for tasks including node classification and link prediction.

1 INTRODUCTION

Graph neural networks (GNNs), as generalizations of neural networks in analyzing graphs, have attracted considerable research attention. GNNs have been widely applied to various applications such as social recommendation (Ma et al., 2019), physical simulation (Kipf et al., 2018), and protein interaction prediction (Zitnik & Leskovec, 2017). One key property of most existing GNNs is permutation-equivariance, i.e., if we randomly permutate the IDs of nodes while maintaining the graph structure, the representations of nodes in GNNs should be permutated accordingly. Mathematically, permutation-equivariance reflects one basic symmetric group of graph structures. Although it is a desirable property for tasks such as node or graph classification (Keriven & Peyré, 2019; Maron et al., 2019b), permutation-equivariance also prevents GNNs from being proximity-aware, i.e., permutation-equivariant GNNs cannot preserve walk-based proximities between nodes such as the shortest distance or high-order proximities (see Theorem 1).

Pairwise proximities between nodes are crucial for graph analytical tasks such as link prediction (Hu et al., 2020; You et al., 2019). To enable a proximity-aware GNN, Position-aware GNN

[*]Equal contributions.

(P-GNN) (You et al., 2019)[1] proposes a sophisticated GNN architecture and shows better performance for proximity-aware tasks. But P-GNN needs to explicitly calculate the shortest distance between nodes and its computational complexity is unaffordable for large graphs. Moreover, P-GNN completely ignores the permutation-equivariance property. Therefore, it cannot produce satisfactory results when permutation-equivariance is helpful.

In real-world scenarios, both proximity-awareness and permutation-equivariance are indispensable properties for GNNs. Firstly, different tasks may require different properties. In social networks, for example, recommendation applications usually require the model to be proximity-aware (Konstas et al., 2009) while permutation-equivariance is a basic assumption in centrality measurements (Borgatti, 2005). Even for the same task, different datasets may have different requirements on these two properties. Taking link prediction as an example, we observe that permutation-equivariant GNNs such as GCN (Kipf & Welling, 2017) or GAT (Velickovic et al., 2018) show better results than P-GNN in coauthor graphs, but the opposite in biological graphs (please see Section 5.2 for details). Unfortunately, in the current GNN frameworks, these two properties are contradicting, as we show in Theorem 1. Whether there exists a general GNN to be proximity-aware while maintaining permutation-equivariance remains an open problem.

In this paper, we propose Stochastic Message Passing (SMP), a general and simple GNN to preserve both proximity-awareness and permutation-equivariance properties. Specifically, we augment the existing GNNs with stochastic node representations learned to preserve node proximities. Though seemingly simple, we prove that our proposed SMP can enable GNNs to preserve walk-based node proximities in theory (see Theorem 2 and Theorem 3). Meanwhile, SMP is equivalent to a permutation-equivariant GNN with certain parametrization and thus is at least as powerful as those GNNs in permutation-equivariant tasks (see Theorem 1). Therefore, SMP is general and flexible in handling both proximity-aware and permutation-equivariant tasks, which is also demonstrated by our extensive experimental results. Besides, owing to the stochastic nature and simple structure, SMP is computationally efficient, with a running time roughly the same as those of the most simple GNNs such as SGC (Wu et al., 2019) and is at least an order of magnitude faster than P-GNN on large graphs. Ablation studies further show that a linear instantiation of SMP is expressive enough as adding extra non-linearities does not lift the performance of SMP on the majority of datasets.

The contributions of this paper are summarized as follows:

- We propose SMP, a simple and general GNN to handle both proximity-aware and permutation-equivariant graph analytical tasks.
- We prove that SMP has a theoretical guarantee in preserving walk-based proximities and is at least as powerful as the existing GNNs in permutation-equivariant tasks.
- Extensive experimental results demonstrate the effectiveness and efficiency of SMP. We show that a linear instantiation of SMP is expressive enough on the majority of datasets.

2 Related Work

We briefly review GNN architectures and the permutation-equivariance property of GNNs.

The earliest GNNs adopts a recursive definition of node states (Scarselli et al., 2008; Gori et al., 2005) or a contextual realization (Micheli, 2009). GGS-NNs (Li et al., 2016) replace the recursive definition with recurrent neural networks (RNNs). Spectral GCNs (Bruna et al., 2014) defined graph convolutions using graph signal processing (Shuman et al., 2013; Ortega et al., 2018) with ChebNet (Defferrard et al., 2016) and GCN (Kipf & Welling, 2017) approximating the spectral filters using a K-order Chebyshev polynomial and the first-order polynomial, respectively. MPNNs (Gilmer et al., 2017), GraphSAGE (Hamilton et al., 2017), and MoNet (Monti et al., 2017) are proposed as general frameworks by characterizing GNNs with a message-passing function and an updating function. More advanced variants such as GAT (Velickovic et al., 2018), JK-Nets (Xu et al., 2018b), GIN (Xu et al., 2018a), and GraphNets (Battaglia et al., 2018) follow these frameworks.

[1] In (You et al., 2019), the authors consider the special case of shortest distance between nodes and name such property as "position-aware". In this paper, we consider a more general case of any walk-based proximity.

Li *et al.* (Li et al., 2018), Xu *et al.* (Xu et al., 2018a), Morris *et al.* (Morris et al., 2019), and Maron *et al.* (Maron et al., 2019a) show the connection between GNNs and the Weisfeiler-Lehman algorithm (Shervashidze et al., 2011) of graph isomorphism tests, in which permutation-equivariance holds a key constraint. Maron *et al.* (Maron et al., 2019b) and Keriven *et al.* (Keriven & Peyré, 2019) analyze the permutation-equivariance property of GNNs more theoretically. To date, most of the existing GNNs are permutation-equivariant and thus are not proximity-aware. The only exception is P-GNN (You et al., 2019), which proposes to capture the positions of nodes using the relative distance between the target node and some randomly chosen anchor nodes. However, P-GNN cannot satisfy permutation-equivariance and is computationally expansive.

3 MESSAGE-PASSING GNNS

We consider a graph $G = (\mathcal{V}, \mathcal{E}, \mathbf{F})$ where $\mathcal{V} = \{v_1, ..., v_N\}$ is the set of $N = |\mathcal{V}|$ nodes, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of $M = |\mathcal{E}|$ edges, and $\mathbf{F} \in \mathbb{R}^{N \times d_0}$ is a matrix of d_0 node features. The adjacency matrix is denoted as $\mathbf{A}$, where its i^{th} row, j^{th} column and an element denoted as $\mathbf{A}_{i,:}$, $\mathbf{A}_{:,j}$, and $\mathbf{A}_{i,j}$, respectively. In this paper, we assume the graph is unweighted and undirected. The neighborhood of node v_i is denoted as $\mathcal{N}_i$ and $\tilde{\mathcal{N}}_i = \mathcal{N}_i \cup \{v_i\}$.

The existing GNNs usually follow a message-passing framework (Gilmer et al., 2017), where the l^{th} layer adopts a neighborhood aggregation function AGG$(\cdot)$ and an updating function UPDATE$(\cdot)$:

$$\mathbf{m}_i^{(l)} = \text{AGG}(\{\mathbf{h}_j^{(l)}, \forall j \in \tilde{\mathcal{N}}_i\}), \mathbf{h}_i^{(l+1)} = \text{UPDATE}([\mathbf{h}_i^{(l)}, \mathbf{m}_i^{(l)}]), \tag{1}$$

where $\mathbf{h}_i^{(l)} \in \mathbb{R}^{d_l}$ is the representation of node v_i in the l^{th} layer, d_l is the dimensionality, and $\mathbf{m}_i^{(l)}$ are the messages. We also denote $\mathbf{H}^{(l)} = [\mathbf{h}_1^{(l)}, ..., \mathbf{h}_N^{(l)}]$ and $[\cdot, \cdot]$ is the concatenation operation. The node representations are initialized as node features $\mathbf{H}^{(0)} = \mathbf{F}$. We denote a GNN following Eq. (1) with L layers as a parameterized function as follows[2]:

$$\mathbf{H}^{(L)} = \mathcal{F}_{\text{GNN}}(\mathbf{A}, \mathbf{F}; \mathbf{W}), \tag{2}$$

where $\mathbf{H}^{(L)}$ are final node representations learned by the GNN and $\mathbf{W}$ denotes all the parameters.

One key property of the existing GNNs is permutation-equivariance.

Definition 1 (Permutation-equivariance)**.** *Consider a graph $G = (\mathcal{V}, \mathcal{E}, \mathbf{F})$ and any permutation $\mathcal{P} : \mathcal{V} \to \mathcal{V}$ so that $G' = (\mathcal{V}, \mathcal{E}', \mathbf{F}')$ has an adjacency matrix $\mathbf{A}' = \mathbf{PAP}^T$ and a feature matrix $\mathbf{F}' = \mathbf{PF}$, where $\mathbf{P} \in \{0, 1\}^{N \times N}$ is the permutation matrix corresponding to $\mathcal{P}$, i.e., $\mathbf{P}_{i,j}$ = 1 iff $\mathcal{P}(v_i) = v_j$. A GNN satisfies permutation-equivariance if the node representations are equivariant with respect to $\mathcal{P}$, i.e.,*

$$\mathbf{P}\mathcal{F}_{GNN}(\mathbf{A}, \mathbf{F}; \mathbf{W}) = \mathcal{F}_{GNN}(\mathbf{PAP}^T, \mathbf{PF}; \mathbf{W}). \tag{3}$$

It is known that GNNs following Eq. (1) are permutation-equivariant (Maron et al., 2019b).

Definition 2 (Automorphism)**.** *A graph G is said to have (non-trivial) automorphism if there exists a non-identity permutation matrix $\mathbf{P} \neq \mathbf{I}_N$ so that $\mathbf{A} = \mathbf{PAP}^T$ and $\mathbf{F} = \mathbf{PF}$. We denote the corresponding automorphic node pairs as $\mathcal{C}_G = \bigcup_{\mathbf{P} \neq \mathbf{I}_N} \{(i, j) | \mathbf{P}_{i,j} \neq 0, i \neq j\}$*

Corollary 1. *Using Definition 1 and 2, if a graph has automorphism, a permutation-equivariant GNN will produce identical node representations for automorphic node pairs:*

$$\mathbf{h}_i^{(L)} = \mathbf{h}_j^{(L)}, \forall (i, j) \in \mathcal{C}_G. \tag{4}$$

Since the node representations are used for downstream tasks, the corollary shows that permutation-equivariant GNNs cannot differentiate automorphic node pairs. A direct consequence of Corollary 1 is that permutation-equivariant GNNs cannot preserve walk-based proximities between pairs of nodes. The formal definitions are as follows.

[2]Since the final layer of GNNs is task-specific, e.g., a softmax layer for node classification or a readout layer for graph classification, we only consider the GNN architecture to its last hidden layer.

Definition 3 (Walk-based Proximities)**.** *For a given graph $G = (\mathcal{V}, \mathcal{E}, \mathbf{F})$, we use a matrix $\mathbf{S} \in \mathbb{R}^{N\times N}$ to denote walk-based proximities between pairs of nodes defined as:*

$$\mathbf{S}_{i,j} = \mathcal{S}\left(\{v_i \rightsquigarrow v_j\}\right), \tag{5}$$

where $v_i \rightsquigarrow v_j$ denotes any walk from node v_i to v_j and $\mathcal{S}(\cdot)$ is an arbitrary real-valued function.

Typical examples of walk-based proximities include the shortest distance (You et al., 2019), the high-order proximities (a sum of walks weighted by their lengths) (Zhang et al., 2018), and random walk probabilities (Klicpera et al., 2019).

Theorem 1. *Existing permutation-equivariant GNNs cannot preserve any walk-based proximity.*[3]

The formulation and proof of the theorem are given in Appendix A.1. Since walk-based proximities are rather general and widely adopted in graph analytical tasks such as link prediction, the theorem shows that the existing permutation-equivariant GNNs cannot handle these tasks well.

4 The Model

4.1 A GNN Framework using Stochastic Message Passing

A major shortcoming of permutation-equivariant GNNs is that they cannot differentiate automorphic nodes. To solve that problem, we need to introduce some mechanism as "symmetry breaking", i.e., to enable GNNs to distinguish these nodes. We sample a stochastic matrix $\mathbf{E} \in \mathbb{R}^{N\times d}$ where each element follows an i.i.d. normal distribution $\mathcal{N}(0, 1)$. The stochastic matrix can provide signals in distinguishing the nodes because they are randomly sampled without being affected by the graph automorphism. In fact, we can easily calculate that the Euclidean distance between two stochastic signals divided by a constant $\sqrt{2}$ follows a chi distribution χ_d:

$$\frac{1}{\sqrt{2}} \left|\mathbf{E}_{i,:} - \mathbf{E}_{j,:}\right| \sim \chi_d, \forall i, j. \tag{6}$$

When d is reasonably large, e.g., $d > 20$, the probability of two signals being close is very low.

Then, inspired by the message-passing framework, we apply a GNN on the stochastic matrix:

$$\tilde{\mathbf{E}} = \mathcal{F}_{\text{GNN}}\left(\mathbf{A}, \mathbf{E}; \mathbf{W}\right). \tag{7}$$

We regard $\tilde{\mathbf{E}}$ as the stochastic representation of nodes. By using the stochastic matrix and message-passing, $\tilde{\mathbf{E}}$ can be used to preserve node proximities (see Theorem 2 and Theorem 3). Then, we concatenate $\tilde{\mathbf{E}}$ with the node representations from another GNN with node features as inputs:

$$\begin{aligned} \mathbf{H} &= \mathcal{F}_{\text{output}}([\tilde{\mathbf{E}}, \mathbf{H}^{(L)}]) \\ \tilde{\mathbf{E}} = \mathcal{F}_{\text{GNN}}\left(\mathbf{A}, \mathbf{E}; \mathbf{W}\right), \mathbf{H}^{(L)} &= \mathcal{F}_{\text{GNN}'}(\mathbf{A}, \mathbf{F}; \mathbf{W}'), \end{aligned} \tag{8}$$

where $\mathcal{F}_{\text{output}}(\cdot)$ is an aggregation function such as a linear function or simply the identity mapping. In a nutshell, our proposed method augments the existing GNNs with a stochastic representation learned by message-passings to differentiate different nodes and preserve node proximities.

There is also a delicate choice worthy mentioning, i.e., whether the stochastic matrix $\mathbf{E}$ is fixed or resampled in each epoch. By fixing $\mathbf{E}$, the model can learn to memorize the stochastic representation and distinguish different nodes, but with the cost of unable to handle nodes not seen during training. On the other hand, by resampling $\mathbf{E}$ in each epoch, the model can have a better generalization ability since the model cannot simply remember one specific stochastic matrix. However, since the node representations are not fixed (but pairwise proximities are preserved; see Theorem 2), in this case, $\tilde{\mathbf{E}}$ can only be used in pairwise tasks such as link prediction or pairwise node classification. In this paper, we use a fixed $\mathbf{E}$ for transductive datasets and resample $\mathbf{E}$ for inductive datasets.

[3]Proposition 1 in (You et al., 2019) can be regarded as a special case of Theorem 1 using the shortest distance proximity.

4.2 A LINEAR INSTANTIATION

Based on the general framework shown in Eq. (8), we attempt to explore its minimum model instantiation, i.e., a linear model. Specifically, inspired by Simplified Graph Convolution (SGC) (Wu et al., 2019), we adopt a linear message-passing for both GNNs, i.e.,

$$\mathbf{H} = \mathcal{F}_{\text{output}}([\tilde{\mathbf{E}}, \mathbf{H}^{(L)}]) = \mathcal{F}_{\text{output}}(\tilde{\mathbf{A}}^K [\mathbf{E}, \mathbf{F}]), \tag{9}$$

where $\tilde{\mathbf{A}} = (\mathbf{D}+\mathbf{I})^{-\frac{1}{2}}(\mathbf{A}+\mathbf{I})(\mathbf{D}+\mathbf{I})^{-\frac{1}{2}}$ is the normalized graph adjacency matrix with self-loops proposed in GCN (Kipf & Welling, 2017) and K is the number of propagation steps. We also set $\mathcal{F}_{\text{output}}(\cdot)$ in Eq. (9) as a linear mapping or identity mapping.

Though seemingly simple, we show that such an SMP instantiation possesses a theoretical guarantee in preserving the walk-based proximities.

Theorem 2. *SMP in Eq.* (9) *with the message-passing matrix $\tilde{\mathbf{A}}$ and the number of propagation steps K can preserve the walk-based proximity $\tilde{\mathbf{A}}^K(\tilde{\mathbf{A}}^K)^T$ with high probability if the dimensionality of the stochastic matrix d is sufficiently large, where the superscript T denotes matrix transpose. The theorem is regardless of whether $\mathbf{E}$ are fixed or resampled.*

The mathematical formulation and proof of the theorem are given in Appendix A.2. In addition, we show that SMP is equivalent to a permutation-equivariant GNN with certain parametrization.

Remark 1. *Suppose we adopt $\mathcal{F}_{output}(\cdot)$ as a linear function with the output dimensionality the same as $\mathcal{F}_{GNN'}$. Then, Eq.* (8) *is equivalent to the permutation-equivariant $\mathcal{F}_{GNN'}(\mathbf{A}, \mathbf{F}; \mathbf{W}')$ if the parameters in $\mathcal{F}_{output}(\cdot)$ are all-zeros for $\tilde{E}$ and an identity matrix for $\mathbf{H}^{(L)}$.*

The result is straightforward from the definition. Then, we have the following corollary.

Corollary 2. *For any task, Eq.* (8) *with the aforementioned linear $\mathcal{F}_{output}(\cdot)$ is at least as powerful as the permutation-equivariant $\mathcal{F}_{GNN'}(\mathbf{A}, \mathbf{F}; \mathbf{W}')$, i.e., the minimum training loss of using $\mathbf{H}$ in Eq.* (8) *is equal to or smaller than using $\mathbf{H}^{(L)} = \mathcal{F}_{GNN'}(\mathbf{A}, \mathbf{F}; \mathbf{W}')$.*

In other words, SMP will not hinder the performance[4] even the tasks are permutation-equivariant since the stochastic representation is concatenated with the node representations of the other GNN followed by a linear mapping. In these cases, the linear SMP is equivalent to SGC (Wu et al., 2019).

Combining Theorem 2 and Corollary 2, the linear SMP instantiation in Eq. (9) is capable of handling both proximity-aware and permutation-equivariant tasks.

4.3 NON-LINEAR EXTENSIONS

One may question whether a more sophisticated variant of Eq. (8) can further improve the expressiveness of SMP. There are three adjustable components in Eq. (8): two GNNs in propagating the stochastic matrix and node features, respectively, and an output function. In theory, adopting non-linear models as either component is able to enhance the expressiveness of SMP. Indeed, if we use a sufficiently expressive GNN in learning $\tilde{\mathbf{E}}$ instead of linear propagations, we can prove a more general version of Theorem 2 as follows.

Theorem 3. *An SMP variant following Eq.*(8) *with $\mathcal{F}_{GNN}(\mathbf{A}, \mathbf{E}; \mathbf{W})$ containing L layers can preserve any length-L walk-based proximity (the length of a proximity is the maximum length of all its walks) if the message-passing and updating functions in the GNN are sufficiently expressive.*

The formulation and proof of the theorem are given in Appendix A.3. Results regarding the model expressiveness in permutation-equivariant tasks have also been explored in the literature (Keriven & Peyré, 2019; Loukas, 2019; Xu et al., 2018a).

Although non-linear extensions of SMP can, in theory, increase the model expressiveness, they also take a higher risk of over-fitting due to model complexity, not to mention that the computational cost will also increase. In practice, we find in ablation studies that the linear SMP instantiation in Eq. (9) works reasonably well on most of the datasets (please refer to Section 5.4 for further details).

[4]Similar to previous works such as (Hamilton et al., 2017; Xu et al., 2018a), we only consider the minimum training loss because the optimization landscapes and generalization gaps are difficult to analyze analytically.

5 EXPERIMENTS

5.1 EXPERIMENTAL SETUPS

Datasets We conduct experiments on the following **ten** datasets: two simulation datasets, **Grid** and **Communities** You et al. (2019), a communication dataset **Email** You et al. (2019), two coauthor networks, **CS** and **Physics** Shchur et al. (2018), two protein interaction networks, **PPI** Hamilton et al. (2017) and **PPA** Hu et al. (2020). We also adopt three GNN benchmarks: **Cora**, **Citeseer**, and **PubMed** Yang et al. (2016). We only report the results of these three benchmarks for the node classification task and the results for other tasks are shown in Appendix B due to the page limit. More details of the datasets are provided in Appendix C.1.

We summarize the statistics of datasets in Table 1. These datasets cover a wide spectrum of domains, sizes, and with or without node features. Note that Email and PPI datasets contain more than one graph and we conduct experiments in an *inductive setting* on these two datasets, i.e., the training, validation, and testing are split with respect to different graphs.

Table 1: The statistics of the datasets. For datasets with more than one graph, #Nodes and #Edges are summed over all the graphs and the experiments are conducted in an inductive setting.

Dataset	#Graphs	#Nodes	#Edges	#Features	#Classes
Grid	1	400	760	-	-
Communities	1	400	3,800	-	20
Email	7	1,005	25,571	-	42
CS	1	18,333	81,894	6,805	15
Physics	1	34,493	247,962	8,415	5
PPI	24	56,944	818,716	50	-
PPA	1	576,289	30,326,273	58	-
Cora	1	2,708	5,429	1,433	7
Citeseer	1	3,327	4,732	3,703	6
Pubmed	1	19,717	44,338	500	3

Baselines We adopt two sets of baselines. The first set is permutation-equivariant GNNs including GCN Kipf & Welling (2017), GAT Velickovic et al. (2018), and SGC Wu et al. (2019), which are widely adopted GNN architectures. The second set contains P-GNN You et al. (2019), the only proximity-aware GNN to date. We use the P-GNN-F version.

In comparing with the baselines, we mainly evaluate two variants of SMP with different $\mathcal{F}_{\text{output}}(\cdot)$: SMP-Identity, i.e., $\mathcal{F}_{\text{output}}(\cdot)$ as an identity mapping, and SMP-Linear, i.e., $\mathcal{F}_{\text{output}}(\cdot)$ as a linear mapping. Note that both variants adopt linear message-passing functions as SGC.

For fair comparisons, we adopt the same architecture and hyper-parameters for all the methods (please refer to Appendix C.2 for the details). For datasets without node features, we adopt a constant vector as the node features. We experiment on two tasks: link prediction and node classification. Additional experiments on pairwise node classification are provided in Appendix B.2. We repeat the experiments 10 times for datasets except PPA and 3 times for PPA, and report the average results.

5.2 LINK PREDICTION

Link prediction aims to predict missing links of a graph. Specifically, we split the edges into 80%-10%-10% and use them for training, validation, and testing, respectively. Besides adopting those real edges as positive samples, we obtain negative samples by randomly sampling an equal number of node pairs from all node pairs that do not have edges. For all the methods, we set a simple classifier: $\text{Sigmoid}(\mathbf{H}_i^T \mathbf{H}_j)$, i.e., use the inner product to predict whether a node pair (v_i, v_j) forms a link, and use AUC (area under the curve) as the evaluation metric. One exception to the aforementioned setting is that on the PPA dataset, we follow the

Table 3: The results of link prediction on the PPA dataset. The best and the second-best result are in bold and underlined, respectively.

Model	Hits@100
SGC	0.1187±0.0012
GCN	0.1867±0.0132
GraphSAGE	0.1655±0.0240
P-GNN	Out of Memory
Node2vec	0.2226±0.0083
Matrix Factorization	<u>0.3229±0.0094</u>
SMP-Identity	0.2018±0.0148
SMP-Linear	**0.3582±0.0070**

Table 2: The results of link prediction tasks measured in AUC (%). The best results and the second-best results for each dataset, respectively, are in bold and underlined.

Model	Grid	Communities	Email	CS	Physics	PPI
SGC	57.6±3.8	51.9±1.6	68.5±7.0	<u>96.5±0.1</u>	**96.6±0.1**	80.5±0.4
GCN	61.8±3.6	50.3±2.5	67.4±6.9	93.4±0.3	93.8±0.2	78.0±0.4
GAT	61.0±5.5	51.1±1.6	53.5±6.3	93.7±0.9	94.1±0.4	79.3±0.5
PGNN	<u>73.4±6.0</u>	<u>97.8±0.6</u>	70.9±6.4	82.2±0.5	Out of memory	80.8±0.4
SMP-Identity	55.1±4.8	**98.0±0.7**	<u>72.9±5.1</u>	<u>96.5±0.1</u>	<u>96.5±0.1</u>	<u>81.0±0.2</u>
SMP-Linear	**73.6±6.2**	97.7±0.5	**75.7±5.0**	**96.7±0.1**	96.1±0.1	**81.9±0.3**

splits and evaluation metric (i.e., Hits@100) provided by the dataset Hu et al. (2020).

The results except PPA are shown in Table 2. We make the following observations.

- Our proposed SMP achieves the best results on five out of the six datasets and is highly competitive (the second-best result) on the other (Physics). The results demonstrate the effectiveness of our proposed method on link prediction tasks. We attribute the strong performance of SMP to its capability of maintaining both proximity-aware and permutation-equivariance properties.
- On Grid, Communities, Email, and PPI, both SMP and P-GNN outperform the permutation-equivariant GNNs, proving the importance of preserving node proximities. Although SMP is simpler and more computationally efficient than P-GNN, SMP reports even better results.
- When node features are available (CS, Physics, and PPI), SGC can outperform GCN and GAT. The results re-validate the experiments in SGC Wu et al. (2019) that non-linearity in GNNs is not necessarily indispensable. A plausible reason is that the additional model complexity brought by non-linear operators makes the models tend to overfit. On those datasets, SMP retains comparable performance on two coauthor graphs and shows better performance on PPI, possibly because node features on protein graphs are less informative than node features on coauthor graphs for predicting links, and thus preserving graph structure is more beneficial on PPI.
- As Email and PPI are conducted in an inductive setting, i.e., using different graphs for training/validation/testing, the results show that SMP can handle inductive tasks as well.

The results on PPA are shown in Table 3. SMP again outperforms all the baselines, demonstrating that it can handle large-scale graphs with millions of nodes and edges. PPA is part of a recently released benchmark Hu et al. (2020). To the best of knowledge, SMP achieves the state-of-the-art on this dataset.

5.3 Node Classification

Next, we conduct experiments of node classification, i.e., predicting the labels of nodes. Since we need ground-truths in the evaluation, we only adopt datasets with node labels. Specifically, for CS and Physics, following Shchur et al. (2018), we adopt 20/30 labeled nodes per class for training/validation and the rest for testing. For Communities, we adjust the number as 5/5/10 labeled nodes per class for training/validation/testing. For Cora, Citeseer, and Pubmed, we use the default splits that came with the datasets. We do not adopt Email because some graphs in the dataset are too small to show stable results and exclude PPI as it is a multi-label dataset.

We use a softmax layer on the learned node representations as the classifier and adopt accuracy, i.e., how many percentages of nodes are correctly classified, as the evaluation criteria. We omit the results of SMP-Identity for this task since the node representations in SMP-Identity have a fixed dimensionality that does not match the number of classes.

The results are shown in Table 4. From the table, we observe that SMP reports nearly perfect results on Communities. Since the node labels are generated by graph structures on Communities and there are no node features, the model needs to be proximity-aware to handle it well. But P-GNN also fails

Table 4: The results of node classification tasks measured by accuracy (%). The best results and the second-best results for each dataset, respectively, are in bold and underlined.

Model	Communities	CS	Physics	Cora	Citeseer	Pubmed
SGC	7.1±2.1	67.2±12.8	92.3±1.6	76.9±0.2	63.6±0.0	74.2±0.1
GCN	<u>7.5±1.2</u>	<u>91.1±0.7</u>	<u>93.1±0.8</u>	<u>81.4±0.5</u>	**71.3±0.5**	**79.3±0.4**
GAT	5.0±0.0	90.5±0.5	**93.1±0.4**	**82.9±0.5**	<u>71.2±0.6</u>	<u>77.9±0.5</u>
PGNN	5.2±0.5	77.6±7.6	Out of memory	59.2±1.5	55.7±0.9	Out of memory
SMP-Linear	**99.9±0.3**	**91.5±0.8**	<u>93.1±0.8</u>	80.9±0.8	68.2±1.0	76.5±0.8

because it selects anchor nodes randomly and thus can only capture the proximities between nodes and cannot learn a classifier to separate nodes into different communities.

On the other five graphs, SMP reports highly competitive performance. These graphs are commonly-used benchmarks for GNNs. P-GNN, which completely ignores permutation-equivariance, performs poorly as expected. In contrast, SMP can manage to recover the permutation-equivariant GNNs and avoid being misled, as proven in Theorem 1. In fact, SMP even shows better results than its counterpart, SGC, indicating that preserving proximities is also helpful for these datasets.

Following P-GNN You et al. (2019), we also conduct experiments on pairwise node classification. We observe similar results as link prediction and provide the results in Appendix B.2.

5.4 Ablation Studies

We conduct ablation studies by comparing different SMP variants, including SMP-Identity, SMP-Linear, and the additional three variants as follows:

- SMP-MLP: we set $\mathcal{F}_{\text{output}}(\cdot)$ as a fully-connected network with 1 hidden layer.
- SMP-Linear-GCN$_{\text{feat}}$: we set $\mathcal{F}_{\text{GNN}'}(\mathbf{A}, \mathbf{F}; \mathbf{W}')$ in Eq. (8) to be a GCN Kipf & Welling (2017), i.e., induce non-linearity in message passing for features. $\mathcal{F}_{\text{output}}(\cdot)$ is linear.
- SMP-Linear-GCN$_{\text{both}}$: we set both $\mathcal{F}_{\text{GNN}}(\mathbf{A}, \mathbf{E}; \mathbf{W})$ and $\mathcal{F}_{\text{GNN}'}(\mathbf{A}, \mathbf{F}; \mathbf{W}')$ to be a GCN Kipf & Welling (2017), i.e., induce non-linearity in message passing for both features and stochastic representations. $\mathcal{F}_{\text{output}}(\cdot)$ is linear.

We show the results for link prediction tasks in Table 5. The results for node classification and pairwise node classification, which imply similar conclusions, are provided in Table 9 and Table 10 in Appendix B.3. We make the following observations.

- In general, SMP-Linear shows good-enough performance, achieving the best or second-best results on six datasets and highly competitive on the other (Communities). SMP-Identity, which does not have parameters in the output function, performs slightly worse. The results demonstrate the importance of adopting a linear layer in the output function, which is consistent with Theorem 1. SMP-MLP does not lift the performance in general, showing that adding extra complexities in $\mathcal{F}_{\text{output}}(\cdot)$ brings no gain in those datasets.
- SMP-Linear-GCN$_{\text{feat}}$ reports the best results on Communities, PPI, and PPA, indicating that adding extra non-linearities in propagating node features are helpful for some graphs.
- SMP-Linear-GCN$_{\text{both}}$ reports the best results on Gird with a considerable margin. Recall that Grid has no node features. The results indicate that inducing non-linearities can help the stochastic representations capture more proximities, which is more helpful on featureless graphs.

5.5 Efficiency Comparison

To compare the efficiency of different methods quantitatively, we report the running time of different methods in Table 6. The results are averaged over 3,000 epochs on a NVIDIA TESLA M40. The results show that SMP is computationally efficient, i.e., only marginally slower than SGC and comparable to GCN. P-GNN is at least an order of magnitude slower except for the extremely small

Table 5: The ablation study of different SMP variants for the link prediction task. Datasets except PPA are measured by AUC (%) and PPA is measured by Hits@100. The best results and the second-best results for each dataset are in bold and underlined, respectively.

Model	Grid	Communities	Email	CS	Physics	PPI	PPA
SMP-Identity	55.1±4.8	**98.0±0.7**	72.9±5.1	96.5±0.1	**96.5±0.1**	81.0±0.2	0.2018±0.0148
SMP-Linear	73.6±6.2	97.7±0.5	**75.7±5.0**	**96.7±0.1**	96.1±0.1	81.9±0.3	0.3582±0.0070
SMP-MLP	72.1±4.3	97.8±0.6	62.7±8.1	88.9±0.8	89.2±0.4	80.1±0.3	0.2035±0.0038
SMP-Linear-GCN$_{\text{feat}}$	72.8±4.2	**98.0±0.4**	74.2±3.9	92.9±0.6	94.3±0.2	**82.3±1.0**	**0.4090±0.0087**
SMP-Linear-GCN$_{\text{both}}$	**80.5±3.9**	97.3±0.7	73.4±5.5	89.8±2.0	91.7±0.2	79.7±0.3	0.2125±0.0232

graphs such as Grid, Communities or Email, which have no more than a thousand nodes, not to mention that the expansive memory cost makes P-GNN unable to work on large-scale graphs.

Table 6: The average running time (in milliseconds) for each epoch (including both training and testing), on link prediction task.

Model	Grid	Communities	Email	CS	Physics	PPI
SGC	25	28	58	210	651	704
GCN	25	35	75	214	612	784
GAT	36	43	140	258	801	919
PGNN	81	84	206	19,340	Out of Memory	6,521
SMP-Identity	26	37	96	284	751	840
SMP-Linear	28	26	84	212	616	832
SMP-MLP	23	28	83	237	614	831
SMP-Linear-GCN$_{\text{feat}}$	23	29	90	231	636	855
SMP-Linear-GCN$_{\text{both}}$	34	40	95	228	626	895

6 Conclusion

In this paper, we propose SMP, a general and simple GNN to maintain both proximity-awareness and permutation-equivariance properties. We propose to augment the existing GNNs with stochastic node representations learned to preserve node proximities. We prove that SMP can enable GNN to preserve node proximities in theory and is equivalent to a permutation-equivariant GNN with certain parametrization. Experimental results demonstrate the effectiveness and efficiency of SMP. Ablation studies show that a linear SMP instantiation works reasonably well on most of the datasets.

Broader Impact

GNNs have been a trending topic in the machine learning community for the past few years. Possible application scenarios of GNNs include social networks, biological networks, academic networks, information networks, etc. We expect our proposed SMP to find general applicability in all these scenarios, but the exact model performance may depend on the specific tasks and datasets. One advantage of SMP is its simple structure and superior efficiency, which makes it more suitable for large-scale graphs. Since SMP shares a similar backbone as other GNNs and we do not explicitly utilize any semantic information, we do not foresee that SMP will produce more biased or offensive content than the existing GNNs.

References

Peter W Battaglia, Jessica B Hamrick, Victor Bapst, Alvaro Sanchez-Gonzalez, Vinicius Zambaldi, Mateusz Malinowski, Andrea Tacchetti, David Raposo, Adam Santoro, Ryan Faulkner, et al. Relational inductive biases, deep learning, and graph networks. *arXiv preprint arXiv:1806.01261*, 2018.

Stephen P Borgatti. Centrality and network flow. *Social networks*, 27(1):55–71, 2005.

Joan Bruna, Wojciech Zaremba, Arthur Szlam, and Yann Lecun. Spectral networks and locally connected networks on graphs. In *International Conference on Learning Representations*, 2014.

Michaël Defferrard, Xavier Bresson, and Pierre Vandergheynst. Convolutional neural networks on graphs with fast localized spectral filtering. In *Advances in neural information processing systems*, pp. 3844–3852, 2016.

Justin Gilmer, Samuel S Schoenholz, Patrick F Riley, Oriol Vinyals, and George E Dahl. Neural message passing for quantum chemistry. In *International Conference on Machine Learning*, pp. 1263–1272, 2017.

Marco Gori, Gabriele Monfardini, and Franco Scarselli. A new model for learning in graph domains. In *Proceedings. 2005 IEEE International Joint Conference on Neural Networks, 2005.*, volume 2, pp. 729–734. IEEE, 2005.

Will Hamilton, Zhitao Ying, and Jure Leskovec. Inductive representation learning on large graphs. In *Advances in neural information processing systems*, pp. 1024–1034, 2017.

Weihua Hu, Matthias Fey, Marinka Zitnik, Yuxiao Dong, Hongyu Ren, Bowen Liu, Michele Catasta, and Jure Leskovec. Open graph benchmark: Datasets for machine learning on graphs. *arXiv preprint arXiv:2005.00687*, 2020.

Nicolas Keriven and Gabriel Peyré. Universal invariant and equivariant graph neural networks. In *Advances in Neural Information Processing Systems*, pp. 7090–7099, 2019.

Thomas Kipf, Ethan Fetaya, Kuan-Chieh Wang, Max Welling, and Richard Zemel. Neural relational inference for interacting systems. In *International Conference on Machine Learning*, pp. 2688–2697, 2018.

Thomas N Kipf and Max Welling. Semi-supervised classification with graph convolutional networks. In *Proceedings of the 6th International Conference on Learning Representations*, 2017.

Johannes Klicpera, Aleksandar Bojchevski, and Stephan Günnemann. Predict then propagate: Graph neural networks meet personalized pagerank. In *ICLR*, 2019.

Ioannis Konstas, Vassilios Stathopoulos, and Joemon M Jose. On social networks and collaborative recommendation. In *Proceedings of the 32nd international ACM SIGIR conference on Research and development in information retrieval*, pp. 195–202, 2009.

Qimai Li, Zhichao Han, and Xiao-Ming Wu. Deeper insights into graph convolutional networks for semi-supervised learning. In *Thirty-Second AAAI Conference on Artificial Intelligence*, 2018.

Yujia Li, Daniel Tarlow, Marc Brockschmidt, and Richard Zemel. Gated graph sequence neural networks. In *International Conference on Learning Representations*, 2016.

Andreas Loukas. What graph neural networks cannot learn: depth vs width. In *International Conference on Learning Representations*, 2019.

Jianxin Ma, Chang Zhou, Peng Cui, Hongxia Yang, and Wenwu Zhu. Learning disentangled representations for recommendation. In *Advances in Neural Information Processing Systems 32*, pp. 5712–5723. 2019.

Haggai Maron, Heli Ben-Hamu, Hadar Serviansky, and Yaron Lipman. Provably powerful graph networks. In *Advances in Neural Information Processing Systems*, pp. 2156–2167, 2019a.

Haggai Maron, Heli Ben-Hamu, Nadav Shamir, and Yaron Lipman. Invariant and equivariant graph networks. In *International Conference on Learning Representations*, 2019b.

Alessio Micheli. Neural network for graphs: A contextual constructive approach. *IEEE Transactions on Neural Networks*, 20(3):498–511, 2009.

Federico Monti, Davide Boscaini, Jonathan Masci, Emanuele Rodola, Jan Svoboda, and Michael M Bronstein. Geometric deep learning on graphs and manifolds using mixture model cnns. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, pp. 5115–5124, 2017.

Christopher Morris, Martin Ritzert, Matthias Fey, William L Hamilton, Jan Eric Lenssen, Gaurav Rattan, and Martin Grohe. Weisfeiler and leman go neural: Higher-order graph neural networks. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 33, pp. 4602–4609, 2019.

Antonio Ortega, Pascal Frossard, Jelena Kovačević, José MF Moura, and Pierre Vandergheynst. Graph signal processing: Overview, challenges, and applications. *Proceedings of the IEEE*, 106 (5):808–828, 2018.

Franco Scarselli, Marco Gori, Ah Chung Tsoi, Markus Hagenbuchner, and Gabriele Monfardini. The graph neural network model. *IEEE Transactions on Neural Networks*, 20(1):61–80, 2008.

Kakade Sham and Shakhnarovich Greg. Random projections. CMSC 35900 (Spring 2009) Large Scale Learning, 2020. URL `https://ttic.uchicago.edu/~gregory/courses/LargeScaleLearning/lectures/jl.pdf`. [Online; accessed 4-September-2020].

Oleksandr Shchur, Maximilian Mumme, Aleksandar Bojchevski, and Stephan Günnemann. Pitfalls of graph neural network evaluation. *Relational Representation Learning Workshop, NeurIPS 2018*, 2018.

Nino Shervashidze, Pascal Schweitzer, Erik Jan van Leeuwen, Kurt Mehlhorn, and Karsten M Borgwardt. Weisfeiler-lehman graph kernels. *Journal of Machine Learning Research*, 12(Sep):2539–2561, 2011.

David I Shuman, Sunil K Narang, Pascal Frossard, Antonio Ortega, and Pierre Vandergheynst. The emerging field of signal processing on graphs: Extending high-dimensional data analysis to networks and other irregular domains. *IEEE signal processing magazine*, 30(3):83–98, 2013.

Petar Velickovic, Guillem Cucurull, Arantxa Casanova, Adriana Romero, Pietro Lio, and Yoshua Bengio. Graph attention networks. In *Proceedings of the 7th International Conference on Learning Representations*, 2018.

Santosh S Vempala. *The random projection method*, volume 65. American Mathematical Soc., 2005.

Duncan J Watts. Networks, dynamics, and the small-world phenomenon. *American Journal of sociology*, 105(2):493–527, 1999.

Felix Wu, Amauri Souza, Tianyi Zhang, Christopher Fifty, Tao Yu, and Kilian Weinberger. Simplifying graph convolutional networks. In *International Conference on Machine Learning*, pp. 6861–6871, 2019.

Keyulu Xu, Weihua Hu, Jure Leskovec, and Stefanie Jegelka. How powerful are graph neural networks? In *International Conference on Learning Representations*, 2018a.

Keyulu Xu, Chengtao Li, Yonglong Tian, Tomohiro Sonobe, Ken-ichi Kawarabayashi, and Stefanie Jegelka. Representation learning on graphs with jumping knowledge networks. In *International Conference on Machine Learning*, pp. 5453–5462, 2018b.

Zhilin Yang, William W Cohen, and Ruslan Salakhutdinov. Revisiting semi-supervised learning with graph embeddings. In *Proceedings of the 33rd International Conference on International Conference on Machine Learning-Volume 48*, pp. 40–48, 2016.

Jiaxuan You, Rex Ying, and Jure Leskovec. Position-aware graph neural networks. In *International Conference on Machine Learning*, pp. 7134–7143, 2019.

Ziwei Zhang, Peng Cui, Xiao Wang, Jian Pei, Xuanrong Yao, and Wenwu Zhu. Arbitrary-order proximity preserved network embedding. In *Proceedings of the 24th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining*, pp. 2778–2786, 2018.

Marinka Zitnik and Jure Leskovec. Predicting multicellular function through multi-layer tissue networks. *Bioinformatics*, 33(14):i190–i198, 2017.

A THEOREMS AND PROOFS

A.1 THEOREM 1

Here we formulate and prove Theorem 1. First, we give a definition for preserving walk-based proximities.

Definition 4. *For a given walk-based proximity, a GNN is said to be able to preserve the proximity if, for any graph $G = (\mathcal{V}, \mathcal{E}, \mathbf{F})$, there exist parameters $\mathbf{W}_G$ and a decoder function $\mathcal{F}_{de}(\cdot)$ so that $\forall \epsilon > 0$:*

$$\left| \mathbf{S}_{i,j} - \mathcal{F}_{de}\left(\mathbf{H}^{(L)}_{i,:}, \mathbf{H}^{(L)}_{j,:}\right) \right| < \epsilon, \tag{10}$$

where

$$\mathbf{H}^{(L)} = \mathcal{F}_{GNN}(\mathbf{A}, \mathbf{F}; \mathbf{W}_G). \tag{11}$$

Note that we do not constrain the GNN architecture as long as it follows the message-passing framework in Eq. (1), and the decoder function is also arbitrary. In fact, both the GNN and the decoder function can be arbitrarily deep and with sufficient hidden units. Next, we rephrase Theorem 1 using the above formulation.

Theorem 1. *For any walk-based proximity function $\mathcal{S}(\cdot)$, a permutation-equivariant GNN cannot preserve $\mathcal{S}(\cdot)$, except the trivial solution that all node pairs have the same proximity, i.e., $\mathbf{S}_{i,j} = c, \forall i, j$, where c is a constant.*

Proof. We prove the theorem by contradiction. Assume there exists a non-trivial $\mathcal{S}(\cdot)$ which a permutation-equivariant GNN can preserve. Consider any graph $G = (\mathcal{V}, \mathcal{E}, \mathbf{F})$ and denote $N = |\mathcal{V}|$. We can create $G' = (\mathcal{V}', \mathcal{E}', \mathbf{F}')$ with $|\mathcal{V}'| = 2N$ so that:

$$\mathcal{E}'_{i,j} = \begin{cases} \mathcal{E}_{i,j} & \text{if } i \le N, j \le N \\ \mathcal{E}_{i-N,j-N} & \text{if } i > N, j > N \\ 0 & \text{else} \end{cases}, \quad \mathbf{F}'_{i,:} = \begin{cases} \mathbf{F}_{i,:} & \text{if } i \le N \\ \mathbf{F}_{i-N,:} & \text{if } i > N \end{cases}. \tag{12}$$

Basically, we generate two "copies" of the original graph, one indexing from 1 to N, and the other indexing from $N+1$ to $2N$. By assumption, there exists a permutation-equivariant GNN which can preserve $\mathcal{S}(\cdot)$ in G' and we denote the node representation as $\mathbf{H}'^{(L)} = \mathcal{F}_{\text{GNN}}(\mathbf{A}', \mathbf{F}'; \mathbf{W}_{G'})$. It is easy to see that node v'_i and v'_{i+N} in G' form an automorphic node pair. Using Corollary 1, their representations will be identical in any permutation-equivariant GNN, i.e.,

$$\mathbf{H}'^{(L)}_{i,:} = \mathbf{H}'^{(L)}_{i+N,:}, \forall i \le N. \tag{13}$$

Also, note that there exists no walk from the two copies, i.e. $\left\{v'_i \rightsquigarrow v'_j\right\} = \left\{v'_j \rightsquigarrow v'_i\right\} = \emptyset, \forall i \le N, j > N$. As a result, for $\forall i \le N, j \le N, \forall \epsilon > 0$, we have:

$$\begin{aligned} |\mathbf{S}_{i,j} - \mathcal{S}(\emptyset)| &\le \left| \mathbf{S}_{i,j} - \mathcal{F}_{\text{de}}\left(\mathbf{H}'^{(L)}_{i,:}, \mathbf{H}'^{(L)}_{j,:}\right) \right| + \left| \mathcal{S}(\emptyset) - \mathcal{F}_{\text{de}}\left(\mathbf{H}'^{(L)}_{i,:}, \mathbf{H}'^{(L)}_{j,:}\right) \right| \\ &= \left| \mathbf{S}_{i,j} - \mathcal{F}_{\text{de}}\left(\mathbf{H}'^{(L)}_{i,:}, \mathbf{H}'^{(L)}_{j,:}\right) \right| + \left| \mathbf{S}_{i,j+N} - \mathcal{F}_{\text{de}}\left(\mathbf{H}'^{(L)}_{i,:}, \mathbf{H}'^{(L)}_{j+N,:}\right) \right| < 2\epsilon. \end{aligned} \tag{14}$$

We can prove the same for $\forall i > N, j > N$. The equation naturally holds if $i \le N, j > N$ or $i > N, j \le N$ since $\left\{v'_i \rightsquigarrow v'_j\right\} = \emptyset$. Combining the results, we have $\forall \epsilon > 0, \forall i, j, |\mathbf{S}_{i,j} - \mathcal{S}(\emptyset)| < 2\epsilon$. Since ϵ can be arbitrarily small, the equation shows that all node pairs have the same proximity $c = \mathcal{S}(\emptyset)$, which leads to a contraction and finishes our proof. □

Notice that in our proof, $\mathcal{G}'$ can be constructed for any graph, so rather than designing one specific counter-example, we have shown that there always exists an infinite number of counter-examples by constructing an automorphism in the graph.

A.2 THEOREM 2

Here we formulate and prove Theorem 2. Note that some notations and definitions are introduced in Appendix A.1.

Theorem 2. *For the walk-based proximity $\mathbf{S} = \tilde{\mathbf{A}}^K(\tilde{\mathbf{A}}^K)^T$, SMP can preserve the proximity with high probability if the dimensionality of the stochastic matrix is sufficiently large, i.e., $\forall\epsilon > 0, \forall\delta > 0$, there $\exists d_0$ so that any $d > d_0$:*

$$P\left(|\mathbf{S}_{i,j} - \mathcal{F}_{de}\left(\mathbf{H}_{i,:}, \mathbf{H}_{j,:}\right)| < \epsilon\right) > 1 - \delta, \tag{15}$$

where $\mathbf{H}$ is the node representation of SMP in Eq. (9)*. The results hold for any stochastic matrix and thus is regardless of whether it is fixed or resampled.*

Proof. Our proof is mostly based on the standard random projection theory. Firstly, since we have proven in Theorem 1 that the permutation-equivariant representations cannot preserve any walk-based proximity, here we prove that we can preserve the proximity only using $\tilde{\mathbf{E}}$, which can be easily achieved by ignoring $\mathbf{H}^{(L)}$ in $\mathcal{F}_{\text{output}}([\tilde{\mathbf{E}}, \mathbf{H}^{(L)}])$, e.g., if we set $\mathcal{F}_{\text{output}}$ as a linear function, the model can learn to set the corresponding weights for $\mathbf{H}^{(L)}$ as all-zeros.

We set the decoder function as a normalized inner product:

$$\mathcal{F}_{\text{de}}\left(\mathbf{H}_{i,:}, \mathbf{H}_{j,:}\right) = \frac{1}{d}\mathbf{H}_{i,:}\mathbf{H}_{j,:}^T. \tag{16}$$

Then, denoting $\mathbf{a}_i = \tilde{\mathbf{A}}^K_{i,:}$ and recalling $\tilde{\mathbf{E}} = \tilde{\mathbf{A}}^K\mathbf{E}$, we have:

$$|\mathbf{S}_{i,j} - \mathcal{F}_{\text{de}}\left(\mathbf{H}_{i,:}, \mathbf{H}_{j,:}\right)| = |\mathbf{a}_i\mathbf{a}_j^T - \frac{1}{d}\tilde{\mathbf{E}}_{i,:}\tilde{\mathbf{E}}_{j,:}^T| = |\mathbf{a}_i\mathbf{a}_j^T - \mathbf{a}_i\frac{1}{d}\mathbf{E}\mathbf{E}^T\mathbf{a}_j^T|. \tag{17}$$

Since $\mathbf{E}$ is a Gaussian random matrix, from the Johnson-Lindenstrauss lemma (Vempala, 2005) (in the inner product preservation forum, e.g., see Corollary 2.1 and its proof in (Sham & Greg, 2020)), $\forall 0 < \epsilon' < \frac{1}{2}$, we have:

$$P\left(|\mathbf{a}_i\mathbf{a}_j^T - \mathbf{a}_i\frac{1}{d}\mathbf{E}\mathbf{E}^T\mathbf{a}_j^T| \leq \frac{\epsilon'}{2}(\|\mathbf{a}_i\| + \|\mathbf{a}_j\|)\right) > 1 - 4e^{-\frac{(\epsilon'^2 - \epsilon'^3)d}{4}}. \tag{18}$$

By setting $\epsilon' = \frac{\epsilon}{\max_i\|\mathbf{a}_i\|}$, we have $\epsilon > \frac{\epsilon'}{2}(\|\mathbf{a}_i\| + \|\mathbf{a}_j\|)$ and:

$$P\left(|\mathbf{S}_{i,j} - \mathcal{F}_{\text{de}}\left(\mathbf{H}_{i,:}, \mathbf{H}_{j,:}\right)| < \epsilon\right) > 1 - 4e^{-\frac{(\frac{\epsilon}{\max_i\|\mathbf{a}_i\|}^2 - \frac{\epsilon}{\max_i\|\mathbf{a}_i\|}^3)d}{4}}, \tag{19}$$

which leads to the theorem by solving and setting d_0 as follows:

$$4e^{-\frac{(\frac{\epsilon}{\max_i\|\mathbf{a}_i\|}^2 - \frac{\epsilon}{\max_i\|\mathbf{a}_i\|}^3)d_0}{4}} = \delta \Rightarrow d_0 = \frac{4\log\frac{4}{\delta}\left(\max_i\|\mathbf{a}_i\|\right)^3}{\epsilon^2\max_i\|\mathbf{a}_i\| - \epsilon^3}. \tag{20}$$

□

A.3 THEOREM 3

Here we formulate and prove Theorem 3. Note that some notations and definitions are introduced in Appendix A.1.

Theorem 3. *For any length-L walk-based proximity, i.e.,*

$$\mathbf{S}_{i,j} = \mathcal{S}\left(\{v_i \rightsquigarrow v_j\}\right) = \mathcal{S}\left(\{v_i \rightsquigarrow v_j | \textit{len}(v_i \rightsquigarrow v_j) \leq L\}\right),$$

where $\textit{len}(\cdot)$ is the length of a walk, there exists an SMP variant in Eq. (8) *with $\mathcal{F}_{GNN}\left(\mathbf{A}, \mathbf{E}; \mathbf{W}\right)$ containing L layers (including the input layer) to preserve that proximity if the following conditions hold: (1) The stochastic matrix $\mathbf{E}$ contains a unique signal for different nodes, i.e. $\mathbf{E}_{i,:} \neq \mathbf{E}_{j,:}, \forall i \neq j$. (2) The message-passing and updating functions in learning $\tilde{\mathbf{E}}$ are bijective. (3) The decoder function $\mathcal{F}_{de}(\cdot)$ also takes $\mathbf{E}$ as inputs and is universal approximation.*

Proof. Similar as Theorem 2, we only utilize $\tilde{\mathbf{E}}$ during our proof. We use $\mathbf{e}_i^{(l)}, 0 \leq l < L$ to denote the node representations in $\mathcal{F}_{\text{GNN}}\left(\mathbf{A}, \mathbf{E}; \mathbf{W}\right)$, i.e., $\mathbf{e}_i^{(0)} = \mathbf{E}_{i,:}$ and $\mathbf{e}_i^{(L-1)} = \tilde{\mathbf{E}}_{i,:}$. Our proof strategy is to show that the stochastic node representations can remember all the information about the walks.

Firstly, as the message-passing and updating function are bijective by assumption, we can recover from the node presentations in each layer all their neighborhood representations in the previous layer. Specifically, there exist $\mathcal{F}^{(l)}(\cdot), 1 \leq l < L$ such that:

$$\mathcal{F}^{(l)}\left(\mathbf{e}_i^{(l)}\right) = \left[\mathbf{e}_i^{(l-1)}, \left\{\mathbf{e}_j^{(l-1)}, j \in \mathcal{N}_i\right\}\right]^5. \tag{21}$$

For notation conveniences, we split the function into two parts, one for the node itself and the other for its neighbors:

$$\begin{aligned} \mathcal{F}_{\text{self}}^{(l)}\left(\mathbf{e}_i^{(l)}\right) &= \mathbf{e}_i^{(l-1)}, \\ \mathcal{F}_{\text{neighbor}}^{(l)}\left(\mathbf{e}_i^{(l)}\right) &= \left\{\mathbf{e}_j^{(l-1)}, j \in \mathcal{N}_i\right\}. \end{aligned} \tag{22}$$

For the first function, if we successively apply such functions from the l^{th} to the input layer, we can recover the input features of the GNN, i.e., $\mathbf{E}$. Since the stochastic matrix $\mathbf{E}$ contains a unique signal for different nodes, we can decode the node ID from $\mathbf{e}_i^{(0)}$, i.e., there exists $\mathcal{F}_{\text{self}}^{(0)}\left(\mathbf{e}_i^{(0)}; \mathbf{E}\right) = i$. For brevity, we denote applying such $l+1$ functions to get the node ID as

$$\mathcal{F}_{\text{self}}^{(0:l)}\left(\mathbf{e}_i^{(l)}\right) = \mathcal{F}_{\text{self}}^{(0)}\left(\mathcal{F}_{\text{self}}^{(1)}\left(...\left(\mathcal{F}_{\text{self}}^{(l)}\left(\mathbf{e}_i^{(l)}\right)\right)\right); \mathbf{E}\right) = i. \tag{23}$$

For the second function, we can apply $\mathcal{F}_{\text{neighbor}}^{(l-1)}$ to the decoded vector set so that we can recover their neighborhood representations in the $(l-2)^{th}$ layer, etc.

Next, we show that for $\mathbf{e}_j^{(l-1)}$, there exists a length-l walk $v_i \rightsquigarrow v_j = (v_{a_1}, v_{a_2}, ..., v_{a_l})$, where $v_{a_1} = v_i$, $v_{a_l} = v_j$ if and only if $\mathcal{F}_{\text{self}}^{(0:l-1)}\left(\mathbf{e}_j^{(l-1)}\right) = a_l = j$ and there exists $\mathbf{e}^{(l-2)}, ..., \mathbf{e}^{(0)}$ such that:

$$\begin{aligned} \mathbf{e}^{(l-2)} &\in \mathcal{F}_{\text{neighbor}}^{(l-1)}\left(\mathbf{e}_j^{(l-1)}\right), \mathcal{F}_{\text{self}}^{(0:l-2)}\left(\mathbf{e}^{(l-2)}\right) = a_{l-1}, \\ \mathbf{e}^{(l-3)} &\in \mathcal{F}_{\text{neighbor}}^{(l-2)}\left(\mathbf{e}^{(l-2)}\right), \mathcal{F}_{\text{self}}^{(0:l-3)}\left(\mathbf{e}^{(l-3)}\right) = a_{l-2}, \\ &... \\ \mathbf{e}^{(0)} &\in \mathcal{F}_{\text{neighbor}}^{(1)}\left(\mathbf{e}^{(1)}\right), \mathcal{F}_{\text{self}}^{(0:0)}\left(\mathbf{e}^{(0)}\right) = a_1 = i. \end{aligned} \tag{24}$$

This result is easily verified as:

$$\begin{aligned} (v_{a_1}, v_{a_2}, ..., v_{a_l}) \text{ is a walk} &\Leftrightarrow a_i \in \mathcal{N}_{a_{i+1}}, \forall 1 \leq i < l \\ &\Leftrightarrow \exists \mathbf{e}^{(i-1)} \in \mathcal{F}_{\text{neighbor}}^{(i)}\left(\mathbf{e}^{(i)}\right), \mathcal{F}_{\text{self}}^{(0:i-1)}\left(\mathbf{e}^{(i-1)}\right) = a_i, \forall 1 \leq i < l. \end{aligned} \tag{25}$$

Note that all the information is encoded in $\tilde{\mathbf{E}}$, i.e., we can decode $\{v_i \rightsquigarrow v_j | \text{len}(v_i \rightsquigarrow v_j) \leq L\}$ from $\mathbf{e}_j^{(L-1)}$ by successively applying $\mathcal{F}_{\text{self}}^{(l)}(\cdot), \mathcal{F}_{\text{neighbor}}^{(l)}(\cdot)$. We can also apply $\mathcal{F}_{\text{self}}^{(0:L-1)}$ to $\mathbf{e}_i^{(L-1)}$ to get the start node ID i. Putting it together, we have:

$$\mathcal{F}\left(\mathbf{e}_j^{(L-1)}, \mathbf{e}_i^{(L-1)}\right) = \{v_i \rightsquigarrow v_j | \text{len}(v_i \rightsquigarrow v_j) \leq L\}, \tag{26}$$

where $\mathcal{F}(\cdot)$ is composed of $\mathcal{F}_{\text{self}}^{(l)}(\cdot), 0 \leq l < L$ and $\mathcal{F}_{\text{neighbor}}^{(l)}(\cdot), 1 \leq l < L$. Applying the proximity function $\mathcal{S}(\cdot)$, we have:

$$\mathcal{S}\left(\mathcal{F}\left(\mathbf{e}_j^{(L-1)}, \mathbf{e}_i^{(L-1)}\right)\right) = \mathbf{S}_{i,j}. \tag{27}$$

We finish the proof by setting the real decoder function $\mathcal{F}_{\text{de}}(\cdot)$ to arbitrarily approximate this desired function $\mathcal{S}(\mathcal{F}(\cdot,\cdot))$ under the universal approximation assumption. □

[5] To let $\mathcal{F}^{(l)}(\cdot)$ output a set with arbitrary lengths, we can adopt sequence-based models such an LSTM.

B Additional Experimental Results

B.1 Additional Link Prediction Results

We further report the results of link prediction on three GNN benchmarks: Cora, Citeseer, and Pubmed. The results are shown in Table 7. The results show similar trends as other datasets presented in Section 5.2.

Table 7: The results of the link prediction task measured in AUC (%). The best results and the second-best results for each dataset, respectively, are in bold and underlined.

Model	Cora	CiteSeer	PubMed
SGC	**93.6±0.6**	**94.7±0.8**	**95.8±0.2**
GCN	90.6±1.0	78.2±1.7	92.4±0.9
GAT	88.5±1.2	87.8±1.0	89.2±0.8
PGNN	75.4±2.3	70.6±1.1	Out of memory
SMP-Identity	<u>93.0±0.6</u>	<u>92.9±0.5</u>	94.5±0.3
SMP-Linear	92.7±0.7	92.6±1.0	<u>95.4±0.2</u>
SMP-MLP	82.8±0.9	80.7±1.1	88.0±0.6
SMP-Linear-GCN$_{\text{feat}}$	86.7±1.4	81.1±1.4	90.5±0.6
SMP-Linear-GCN$_{\text{both}}$	80.1±2.5	80.0±2.0	81.1±2.0

B.2 Pairwise Node Classification

Besides standard node classification experiments reported in Section 5.3, we follow You et al. (2019) and further experiment on pairwise node classification, i.e., predicting whether two nodes have the same label. Compared with standard node classification, pairwise node classification focuses more on the relations between nodes and thus requires the model to be proximity-aware to perform well.

Similar to link prediction, we split the positive samples (i.e., node pairs with the same label) into an 80%-10%-10% training-validation-testing set with an equal number of randomly sampled negative pairs. For large graphs, since the possible positive samples are intractable (i.e. $O(N^2)$), we use a random subset. Since we also need node labels as the ground-truth, we only conduct pairwise node classification on datasets when node labels are available. We also exclude the results of PPI since the dataset is multi-label and cannot be used in a pairwise setting You et al. (2019). Similar to Section 5.2, we adopt a simple inner product classifier and use AUC as the evaluation metric.

The results are shown in Table 8. We observe consistent results as link prediction in Section 5.2, i.e., SMP reports the best results on four datasets and the second-best results on the other three datasets. These results again verify that SMP can effectively preserve and utilize node proximities when needed while retaining comparable performance when the tasks are more permutation-equivariant like, e.g., on CS and Physics.

Table 8: The results of pairwise node classification tasks measured in AUC (%). The best results and the second-best results for each dataset, respectively, are in bold and underlined.

Model	Communities	Email	CS	Physics	Cora	Citeseer	Pubmed
SGC	67.4±2.4	56.3±5.4	**99.8±0.0**	<u>99.6±0.0</u>	<u>99.2±0.3</u>	**95.5±0.7**	92.3±0.3
GCN	64.9±2.3	55.0±5.7	96.8±0.7	**99.7±0.1**	97.7±0.6	92.9±1.2	**94.8±0.4**
GAT	52.5±1.3	47.7±2.7	95.2±0.6	96.3±0.2	91.6±0.7	73.6±2.7	87.1±0.2
PGNN	<u>98.6±0.5</u>	<u>63.3±5.5</u>	90.0±0.5	Out of memory	85.5±1.2	49.8±1.8	Out of memory
SMP-Identity	**98.8±0.5**	56.9±4.1	<u>99.7±0.0</u>	<u>99.6±0.0</u>	<u>99.2±0.2</u>	95.2±1.1	91.9±0.3
SMP-Linear	**98.8±0.5**	**74.5±4.1**	**99.8±0.0**	<u>99.6±0.0</u>	**99.3±0.3**	<u>95.3±0.4</u>	<u>93.4±0.2</u>

B.3 Additional Ablation Studies

We report the ablation study results for the node classification task and pairwise node classification task in Table 9 and Table 10, respectively. The results again show that SMP-Linear generally

achieves good-enough results on the majority of the datasets and adding non-linearities does not necessarily lift the performance of SMP.

Table 9: The ablation study of different SMP variants for the node classification task. The best results and the second-best results are in bold and underlined, respectively.

Model	Communities	CS	Physics	Cora	Citeseer	Pubmed
SMP-Linear	99.9±0.3	**91.5±0.8**	**93.1±0.8**	**80.9±0.8**	**68.2±1.0**	76.5±0.8
SMP-MLP	**100.0±0.2**	90.1±0.5	92.3±0.8	79.3±0.8	67.0±1.5	76.8±0.9
SMP-Linear-GCN$_{\text{feat}}$	**100.0±0.0**	89.8±0.7	92.9±0.8	78.9±1.2	67.8±0.6	**77.3±0.6**
SMP-Linear-GCN$_{\text{both}}$	**100.0±0.2**	77.4±4.2	87.1±3.5	69.2±2.5	49.8±3.1	68.1±4.1

Table 10: The ablation study of different SMP variants for the pairwise node classification task. The best results and the second-best results are in bold and underlined, respectively.

Model	Communities	Email	CS	Physics	Cora	Citeseer	Pubmed
SMP-Identity	98.8±0.5	56.9±4.1	99.7±0.0	**99.6±0.0**	99.2±0.2	95.2±1.1	91.9±0.3
SMP-Linear	98.8±0.5	**74.5±4.1**	**99.8±0.0**	**99.6±0.0**	**99.3±0.3**	**95.3±0.4**	93.4±0.2
SMP-MLP	98.7±0.3	65.4±6.3	94.3±0.6	97.6±0.4	90.3±3.0	67.7±13.7	93.4±0.4
SMP-Linear-GCN$_{\text{feat}}$	**99.0±0.4**	60.2±9.3	95.6±0.7	98.3±0.7	96.1±0.7	88.8±1.6	**94.8±0.2**
SMP-Linear-GCN$_{\text{both}}$	98.8±0.4	61.6±6.0	95.2±0.7	97.8±0.8	94.3±1.9	83.5±3.9	94.1±0.7

B.4 Comparison with Using IDs

We further compare SMP with augmenting GNNs using a one-hot encoding of node IDs, i.e., the identity matrix. Intuitively, since the IDs of nodes are unique, such a method does not suffer from the automorphism problem and should also enable GNNs to preserve node proximities. However, theoretically speaking, using such a one-hot encoding has two major problems. Firstly, the dimensionality of the identity matrix is $N \times N$, and thus the number of parameters in the first message-passing layer is also on the order of $O(N)$. Therefore, the method will inevitably be computationally expansive and may not be scalable to large-scale graphs. The large number of parameters will also more likely lead to the overfitting problem. Secondly, the node IDs are not transferable across different graphs, i.e., the node v_1 in one graph and the node v_1 in another graph do not necessarily share a similar meaning. But as the parameters in the message-passings depend on the node IDs (since they are input features), such a mechanism cannot handle inductive tasks well.

We also empirically compare such a method with SMP and report the results in Table 11. The results show that SMP-Linear outperforms GCN$_{\text{onehot}}$ in most cases, not to mention that GCN$_{\text{onehot}}$ fails to handle Physics, which is only a medium-scale graph, due to the heavy memory usage. One surprising result is that GCN$_{\text{onehot}}$ outperforms SMP-Linear on Grid, the simulated graph where nodes are placed on a 20×20 grid. A plausible reason is that since the edges in Grid follow a specific rule, using a one-hot encoding gives GCN$_{\text{onehot}}$ enough flexibility to learn and remember the rules, and the model does not overfit because the graph has a rather small scale.

C Experimental Details for Reproducibility

C.1 Datasets

- **Grid** You et al. (2019): A simulated 2D grid graph with size 20×20 and no node feature.
- **Communities** You et al. (2019): A simulated caveman graph Watts (1999) composed of 20 communities with each community containing 20 nodes. The graph is perturbed by rewiring 1% edges randomly. It has no node feature and the label of each node indicates which community the node belongs to.
- **Email**[6] You et al. (2019): Seven real-world email communication graphs. Each graph has six communities and each node has an integer label indicating the community the node belongs to.

[6] https://github.com/JiaxuanYou/P-GNN/tree/master/data

Table 11: The results of comparing SMP with using one-hot IDs in GCNs. OOM represents out of memory. — represents the task is unavailable.

Task	Model	Grid	Communities	Email	CS	Physics	PPI	Cora	Citeseer	Pubmed
Link Prediction	$\text{GCN}_{\text{onehot}}$	91.5±2.1	98.3±0.7	71.2±3.5	93.1±1.3	OOM	78.6±0.3	86.8±1.5	81.7±1.1	89.4±0.5
	SMP-Linear	73.6±6.2	97.7±0.5	75.7±5.0	96.7±0.1	96.1±0.1	81.9±0.3	92.7±0.7	92.6±1.0	95.4±0.2
Pairwise Node Classification	$\text{GCN}_{\text{onehot}}$	—	98.9±0.5	67.3±5.6	97.6±0.2	OOM	—	98.2±0.3	94.4±1.2	98.9±0.1
	SMP-Linear	—	98.8±0.5	74.5±4.1	99.8±0.0	99.6±0.0	—	99.3±0.3	95.3±0.4	93.4±0.2
Node Classification	$\text{GCN}_{\text{onehot}}$	—	99.6±1.0	—	86.9±1.5	OOM	—	77.6±1.1	57.7±5.8	74.9±0.6
	SMP-Linear	—	99.9±0.3	—	91.5±0.8	93.1±0.8	—	80.9±0.8	68.2±1.0	76.5±0.8

- **Coauthor Networks**[7] Shchur et al. (2018): Two networks from Microsoft academic graph in **CS** and **Physics** with their nodes representing authors and edges representing co-authorships between authors. The node features are embeddings of the paper keywords of the authors.
- **PPI**[6] Hamilton et al. (2017): 24 protein-protein interaction networks. Each node has a 50-dimensional feature vector.
- **PPA**[8] Hu et al. (2020): A network representing biological associations between proteins from 58 different species. The node features are one-hot vectors of the species that the proteins are taken from.
- **Cora**, **Citeseer**, **Pubmed**[9] Yang et al. (2016): Three citation graphs where nodes correspond to papers and edges correspond to citations between papers. The node features are bag-of-words and the node labels are the ground truth topics of the papers.

C.2 Hyper-parameters

We use the following hyper-parameters:

- All datasets except PPA: we uniformly set the number of layers for all the methods as 2, i.e., 2 message-passing steps, and set the dimensionality of hidden layers as 32, i.e., $\mathbf{H}^{(l)} \in \mathbb{R}^{N\times 32}$, for all $1 \leq l \leq L$ (for GAT, we use 4 heads with the dimensionality of each head as 8). We use the Adam optimizer with an initial learning rate of 0.01 and decay the learning rate by 0.1 every 200 epochs. The weight decay is 5e-4. We train the model for 1,000 epochs and evaluate the model every 5 epochs. We adopt an early-stopping strategy by reporting the testing performance at the epoch which achieves the best validation performance. For SMP, the dimensionality of the stochastic matrix is $d = 32$. For P-GNN, we use the P-GNN-F version, which uses truncated 2-hop shortest path distance instead of the exact shortest distance.
- PPA: as suggested in the original paper Hu et al. (2020), we set the number of GNN layers as 3 with each layer containing 256 hidden units and add a three-layer MLP after taking the Hadamard product between pair-wise node embeddings as the predictor, i.e., $\text{MLP}(\mathbf{H}_i \odot \mathbf{H}_j)$. We use the Adam optimizer with an initial learning rate of 0.01. We set the number of epochs for training as 40, evaluate the results on validation sets every epoch, and report the testing results using the model with the best validation performance. We also found that the dataset had issues with exploding gradients and adopt a gradient clipping strategy by limiting the maximum p2-norm of gradients as 1.0. The dimensionality of the stochastic matrix in SMP is $d = 64$.

C.3 Hardware and Software Configurations

All experiments are conducted on a server with the following configurations.

- Operating System: Ubuntu 18.04.1 LTS
- CPU: Intel(R) Xeon(R) CPU E5-2699 v4 @ 2.20GHz
- GPU: NVIDIA TESLA M40 with 12 GB of memory
- Software: Python 3.6.8, PyTorch 1.4.0, PyTorch Geometric 1.4.3, NumPy 1.18.1, Cuda 10.1

[7] https://github.com/shchur/gnn-benchmark/tree/master/data/npz/
[8] https://snap.stanford.edu/ogb/data/linkproppred/ppassoc.zip
[9] https://github.com/kimiyoung/planetoid/tree/master/data